Students' Thinking and Writing in the Context of Probability

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This study examined changes in students' probabilistic thinking and writing during instruction emphasizing writing to learn experiences. A class of fifth-grade students with no previous experiences in writing during mathematics made significant gains in probability reasoning and writing; however the correlation between probabilistic thinking and writing was not significant. Analysis of focus students revealed that their writing changed from narrative summaries to reasoned patterns and generalizations. However some used invented representations without interpretation and were reluctant to write in mathematical contexts.

Research in writing across the curriculum has recently focused on content specific writing and learning (Langer, 1992; Waywood, 1994). In particular, researchers have found that writing in mathematics classes provides opportunities for students to refine, clarify, and organize their mathematical thinking (Azzolino, 1990); communicate what they know (Drake & Amspaugh, 1994); and generally construct a better understanding of mathematical relationships (Abel & Abel, 1988; Shepard, 1993). Some researchers have also begun to investigate the reverse perspective, that is, the extent to which *learning to write* can be enhanced by content specific *writing to learn* experiences (Freedman, 1995). However, the extent to which writing experiences in mathematics influence students' writing patterns has not been widely reported.

The present investigation, undertaken with a group of elementary students, examined both writing to learn and learning to write in the context of an instructional program in probability. More specifically, the study examined: (a) changes in students' probabilistic thinking; and (b) changes in students' writing during instruction in probability that emphasized writing to learn experiences.

Theoretical Considerations

The study is based on two theoretical positions. The first is a cognitive framework that describes elementary school students' probabilistic thinking (Jones, Langrall, Thornton, & Mogill, 1997). The second is a model relating writing and phases of conceptual learning (Shepard, 1993).

Framework for Describing Students' Probabilistic Thinking

The Probabilistic Thinking Framework is based on the assumption that thinking in probability is multifaceted and develops over time. Based on previous research the framework (e.g., Acredolo, O'Connor, Banks, & Horobin, 1989; English, 1993; Falk, 1983; Fischbein, Nello & Marino, 1991; Piaget & Inhelder, 1975; Jones et al., 1997) delineated four levels of probabilistic thinking for each of four key constructs: sample space, probability of an event, probability comparisons, and conditional probabilistic thinking over a two-year period, and appear to be consistent with neo-Piagetian theories that postulate the existence of levels of thinking that recycle during developmental stages (Biggs & Collis, 1991).

Students at level 1 are narrowly and consistently bound to *subjective* judgments. For example, a level 1 student examining a gumball machine containing 6 red and 3 yellow gumballs may choose yellow as the most likely to come out because "It's my favorite color." Such students see no need to apply quantitative reasoning in probability situations. Level 2 students are *in transition* between subjective and naive quantitative thinking. For example, a level 2 student faced with the same gumball machine will sometimes choose red because there are more red, but will at other times revert to subjective judgments and choose either red or yellow. Level 3 students characteristically use *quantitative* judgments when dealing with probability tasks. For example, a student at level 3 will consistently choose red in the gumball example above, and will explain that there are six red versus three yellow. Students at Level 4 use *numerical* reasoning and also express their probability thinking in terms of more precise probability measures. In the same gumball situation, a level 4 student will choose red because its chances are 6 out of 9, compared to only 3 out of 9 for yellow. The thinking levels of the Framework were used as the basis for constructing the instructional program and assessment instruments in this study.

Model of Writing and Phases of Conceptual Learning

Shepard's (1993) model focusing on writing and phases of conceptual learning in mathematics integrates three learning phases--initial, intermediate, and terminal (Shuell, 1990) with seven writing categories (Britton, Burgess, Martin, McLeod, & Rosen, 1975) that range from reporting to tautological writing. Shepard reduced the writing categories to six and generated six learning phases by splitting each of the original learning phases into an *early* and a *late* level. Shepard's learning-writing categories. Because the final two learning-writing phases are typically not accessible to elementary school children, this study utilized only the first four: (a) initial early (record); (b) initial late (generalized narrative), (c) intermediate early (low-level analogic), and (d) intermediate late (analogic).

In the *initial early* learning-writing phase, individuals simply record or summarize direct experiences without making inferences. In the *initial late* phase, individuals largely report or summarize experiences, but they also begin to identify patterns and formulate generalizations associated with the experience. For example, in exploring the possible sums when two dice are rolled, students in the early phase are likely to record the sums they actually find, while those in the late phase explain that some sums are more likely than others. The *intermediate early* phase is characterized by writing that identifies true generalizations but does not recognize relationships between these generalizations. In the *intermediate late* phase, not only are generalizations identified and described, but they are also related in a cogent way. For example, in the exploration involving the two dice, an individual in the intermediate early phase might record and justify that a sum of 7 is the most likely event, but unlike an individual at the intermediate late phase, may not recognize that the probabilities of sums from 2 to 12 are hierarchically ordered: increasing from 2 to 7, then decreasing from 7 to 12.

Research Questions

Methodology

This study sought to answer the following questions: (a) What changes occur in students' probabilistic thinking following an instructional program in probability that embraces writing to learn experiences? (b) What changes occur in students' writing within a probability context during an instructional program that emphasizes writing to learn experiences? and (c) Is there a relationship between learning-writing phases, as measured by Shepard's (1993) model, and levels of thinking in probability, as measured by the Probabilistic Framework (Jones et al., 1997).

Subjects

The population for this study was fifth-grade students from a university laboratory school. Students in this school represent a broad spectrum of cultural and socioeconomic backgrounds. One of the two grade 5 intact classes from this school (n=24) was randomly chosen to participate in the Probability Writing Program of this study. Both classes had previously participated in probability programs during grades 3 and 4 (Jones, Thornton, Langrall, & Miller, submitted; Jones et al., 1997). Four students from this class--here named Marion, Steffi, Jacques, and Terrance--served as focus studies. These students were "purposefully" selected (Miles and Huberman, 1994) at the *completion* of the intervention: the first student, Marion, made gains in both probability thinking and writing; the second, Steffi, made no gains in either writing or probability; the third student, Jacques made gains in probability thinking but a substantial gain in writing.

The Probability Writing Program

The Probability Writing Program (The Program) comprised ten 45-minute sessions. Two sessions were held each week over a period of five weeks. The teacher for The Program, was a final year undergraduate major in middle school education with specialization in mathematics. She had also served as a research and teaching assistant in the grade 3 and grade 4 probability projects. The Program, developed by the research team, consisted of probability and related writing tasks generated from the probabilistic framework (Jones et al., 1997). Each probability task was based on one or more of the key constructs of the Probabilistic Thinking Framework: 18% of the sessions focused on sample space, 27% on probability of an event, 42% on probability comparisons, and 13% on conditional probability. The tasks were chosen so that they would be accessible to students at all levels of the framework. Related writing prompts asked students to reflect on a probability task and to predict, explain, or justify their thinking. An example of a typical probability task and its associated writing prompts is presented in Figure 1.

An Instructional Writing Task

Q1:	What are the different ways to mark "T" or "F" on a 4-question true-false
	test? List all of the outcomes and explain how you found them.
Q2:	Given that the correct answer was TFTF, what was each student's
-	chance of getting a perfect paper by guessing? Justify your thinking.
Q3:	In our class, 2 students out of 25 earned a perfect paper.
	Does this surprise you? Write an explanation to support your response?

Figure 1. Sample probability task and writing prompts

The format for each of the two weekly sessions varied slightly. To begin the *first* session students responded to individual probability prompts in their journals. These prompts were generated by the teacher in response to students' journal entries from the previous week. This writing activity was followed by whole class discussion of a new probability task. Students made predictions, then broke into pairs and attempted to solve the new problem using both experimental and analytical approaches. That is, they first carried out a simulation of the problem, analyzed their results, and then tried to explain the situation using more formal probability reasoning. The students subsequently shared their thinking in a whole class discussion and completed a journal entry for the problem. The format of the *second* session of each week was identical to the first except that the initial writing activity was not undertaken. Each weekend the teacher assessed the students' journal entries, provided new prompts, and the cycle continued.

Instrumentation

The same Probability Thinking Protocol was administered in an individual interview setting at the beginning (September) and end (November) of the intervention. However, the Writing Protocols given at the beginning and end of the intervention were different. This was necessary to accommodate the fact that students had not previously undertaken writing during a mathematics class and to recognize their level of maturity in probability thinking prior to the intervention. The Probability Thinking Protocol was based on the Framework and comprised 20 items (See Jones et al., 1997). Five items were associated with sample space, four with probability of an event, seven with probability comparisons, and four with conditional probability. The items, linked to the four constructs of the framework, enabled researchers to explore students' thinking across each of the four probabilistic thinking levels.

A double coding procedure (Miles and Huberman, 1994) was used to assess students' thinking levels in relation to the Probability Thinking Protocol. Working independently, the first three authors coded pre and post intervention protocols to establish thinking levels for all students on each of the four constructs. Agreement was achieved on the coding of 93 % of the levels and variations were clarified until consensus was reached. Hence, each student's dominant (modal) level of probabilistic thinking was identified at the beginning and at the end of the intervention. The teacher for The Program administered the writing protocols in a whole class setting. The Initial Writing Protocol (Figure 2) was set in the context of a card game while the Final Writing Protocol (Figure 3) was set in the context of a sailboat race. In each case the writing prompt invited students to respond to a probability question and provide full justification of their thinking. These prompts were sufficiently open to invite a range of written responses across the Shepard's (1993) four phases. In scoring the writing protocols, the researchers again used the double coding procedure in a similar manner to that discussed above. The only difference was that each student was assigned a dominant (modal) learning-writing phase (1- initial early, 2- initial late, 3- intermediate early, and 4- intermediate late) rather than a probability thinking level. Agreement was achieved on the coding of 84% of the learning-writing phases.

Drawing the Cards

- Q1: Draw 2 cards from a set of 4 cards consisting of an Ace, King, Queen, and Jack. How many different outcomes are there? List the outcomes and write an explanation of your thinking.
- Q2: Jane says there are 6 ways to draw 2 cards from the set of 4 cards. John says there are 12 ways to draw 2 cards from the set of 4 cards. Explain how each of them could justify their answers.

Figure 2. Probabilistic writing protocol: Initial task

The Great Sailboat Race

You are going to play a game in which the sailboats are numbered 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12. The rules for playing the game are:

- You roll two dice and find their SUM.
- If the sum is 2, sailboat #2 moves forward 1 space.
- If the sum is 3, sailboat #3 moves 1 space.
- ... and so on up to a sum of 12.
- The first sailboat to move 4 spaces wins the race.

Play the game several times and then respond to the writing prompt.

Writing Prompt: Which sailboat do you think will win the race? Write a complete explanation to justify your response.

Figure 3. Probabilistic writing protocol: Final task

Data Sources

Data on students' thinking and writing in the context of probability were gathered from three sources: 1) assessments on probability thinking conducted at the beginning and end of the intervention; 2) assessments on probability writing conducted at the beginning and end of the intervention; 3) journals containing the writing of focus students throughout the intervention; and 4) researcher narratives of instructional sessions. Based on the data in 3) and 4), each of the first four authors generated a Researcher Writing Summary (RWS) on one focus student each session. These RWSs described students' writing and identified any diagrammatic or symbolic representations used by them. The RWSs also contained on-going researcher annotations which sought to relate students' writing to the four phases of Shepard's (1993) model. *Data Analysis*

Both "within" and "cross-case displays" (Miles & Huberman, 1994, p. 90; p. 207) were used to guide the analysis of qualitative data on the four focus students. For each focus student, the 10 RWSs, one for each session, were assigned multiple codes largely arising from the writing levels associated with Shepard's (1993) model. Subsequently, a time-ordered matrix was generated for each focus student to display trends in writing over the 10 intervention sessions. These matrices were further analyzed to discern common patterns and relationships across the four focus students. Triangulation using both the researcher and the assessment data enabled alternative interpretations to be evaluated. Statistical analyses were also performed using assessment data on probability thinking and probability writing as dependent variables.

Because data were "ordinal," non-parametric statistical tests were used to compare probability thinking and writing levels prior to and following the intervention, and to examine relationships between these variables.

Results

The Effect of The Program: Quantitative Analysis

The students' thinking levels on the Probability Thinking Protocol were determined at two assessment points: prior to The Program (September), and following The Program (November). A Wilcoxin Signed Ranks Test (Siegel & Castellan, 1988) indicated that there was a significant difference between the pre and post assessment thinking levels in probability (pre assessment median = 2.5, post assessment median =3, T = 40.5, p < 0.05). The student's learning-writing phases in the context of probability were also assessed prior to The Program (September) and following The Program (November). A Wilcoxin Signed Ranks Test (Siegel & Castellan, 1988) indicated that there was a significant difference between the pre and post assessment learning-writing phases (pre assessment median = 1, post assessment median = 2, T =139, p < 0.01).

Spearman correlation coefficients were determined to assess degree of association between learning-writing phase and probability thinking level. The pre assessment measure was significant (r = 0.41, p < 0.05), but the post assessment measure was not significant (r = 0.21, p > 0.20).

The Effect of The Program: Qualitative Analysis

Two learning patterns with respect to writing in the context of probability were discerned by analyzing RWSs. Students (a) moved from writing narratives of probability simulations to writing about patterns and generalizations drawn from probabilistic situations and (b) often substituted symbolic representations, without interpretation, for explanations.

Narratives to Patterns

Marion, whose writing moved from phase 1 to phase 4, showed the greatest change toward writing that incorporated patterns and generalizations. Marion's response to the Initial Writing Protocol (Figure 2) was typical of many phase 1 students prior to the intervention. When asked to list all possible outcomes when two cards were drawn from a set of 4 cards consisting of an Ace, King, Queen, and Jack, Marion doubled up on two pairs (see boldface below), listing 14 outcomes rather than 12. More importantly, she made no attempt to respond to the writing prompt which asked her to interpret and justify her response. Her report merely listed her outcomes as shown below.

2. Q, A 3. K, Q 4. A, J **5. A, K 6. K, A** 7. Q, 9. A, Q 10. Q, K 11. J, A **12. K, A 13. A, K** 14. J, Q 1. K, J 7. Q, J

8. J, K

By the end of lesson four in the instructional program, Marion was attempting to look for patterns and provide more explanation. Consider, for example, Marion's response to writing prompt Q3 in Figure 1. Marion wrote, "It isn't surprising at all -there are 16 ways but only one is right. A couple of people might have put TTTT. A couple of people might have put FFFF and a few more might have put FTTF." Marion correctly recognized that there were 16 outcomes and that only one was correct. Moreover, she also appeared to conjecture that two students might have written down each outcome. This suggests that she is using an approximation, that is, that 25 is roughly twice 16 and hence each outcome would come up twice. While Marion didn't provide sufficient written explanation to make her thinking clear, there was an attempt to describe a pattern and draw an inference from it.

By the post assessment, Marion's response to the Sailboat Race (Figure 3) exhibited phase 4 writing and showed that she was able to make true generalizations that were based on well reasoned arguments. She wrote, "Sailboat 7 would win because it had 6 outcomes (which she listed) and the others have less outcomes." Although her written response did not include numerical probabilities, she correctly ordered the sailboats according to their chance of winning, by making critical connections between the number of outcomes having a particular sum and the probability of the corresponding sailboat number.

Terrance's growth in writing is more typical of the students in this study. In essence, Terrance moved from using a writing style that was essentially confined to narratives to a writing style that incorporated patterns and even generalizations. More specifically, our analysis showed that during the first four lessons Terrance regularly provided lists of outcomes for experiments and began to provide some justifications rather than mere descriptions of what was happening. For example in lesson 4, when asked whether it was surprising that 2 students out of a class of 25 correctly guessed all four answers (Figure 1), Terrance correctly listed all of the 16 possible outcomes and wrote, "Yes, because there are 1 every 16 and 2 for 25." His thinking was similar to Marion's but was numerically more explicit in that he displayed the approximate proportion that he had in mind. However, like Marion he did not clarify his thinking nor did he attempt to generalize by suggesting that with 32 students you'd expect 2, with 48 you'd expect 3, and so on.

Teacher comments in Terrance's journal consistently encouraged him to "explain his thinking more fully" and to give reasons "why" he drew particular conclusions. This encouragement appears to have paid dividends because in later lessons the teacher wrote "excellent job" on two pieces of Terrance's writing. By the written posttest, Terrance was rated at level 3 in writing and had become more consistent in justifying his thinking. For example, in the Sailboat Race (Figure 3), Terrance concluded that sailboat 7 was more likely to win and wrote, "7 will win because it has more chances than 4 [for example] when the dice are rolled. [Sailboat] 7 has 6 ways 5+1, 1+5, 2+4, 4+2, 3+4, 4+3. [Sailboat] 4 has 3 ways 1+3, 3+1, 2+2." Although he assumed that his justification that 7 had a better chance than 4 was sufficient to show that 7 had the best chance, his thinking process is generalizable beyond 7 and 4.

Invented Symbolic Representations

Steffi, who was a level 4 thinker in probability even prior to instruction, exemplified students who preferred to use symbolic representations rather than written descriptions and explanations. Like most students in the study, Steffi was rated at phase 1 on the Initial Writing Protocol (Figure 2). Although she listed all 12 outcomes using systematic pairings like AK and KA, she didn't provide written explanation of the pattern used. Nor did she attempt to explain why there were twelve outcomes rather than six, an equally valid response. She simply wrote beside her diagrammatic listing, "We wrote them down," implying that the diagram said it all.

In subsequent lessons, Steffi consistently listed outcomes correctly in various probability situations, but in every case remained mute to the need for explanations or interpretations of the patterns. This occurred in spite of the fact that the teacher consistently urged her to write justifications of her thinking. For example, in lesson 4, Steffi came up with her own invented model for listing the outcomes when a student guessed the answers to the four-item true/false test (Figure 1):

 1. TTTT
 2. FFFF
 3. TFTF
 4. FTFT
 5. TFFF
 6. FTTT
 7. FTTF
 8. TFFT

 9. FFFT
 10. TTTF
 11. FFTT
 12. TTFF
 13. FFTF
 14. TTFT
 15. TFTT

 16. FTFF

Even though she appeared to use a pattern that reversed the positions of T and F, she did not respond to the written prompt seeking explanation. In the same task, when asked whether it was surprising that 2 out of 25 students guessed all four items correctly, Steffi provided a number sentence as her written response. Having previously written that there was a 1 in 16 chance of getting a perfect paper, she responded to Q3 (Figure 1) as follows, " No! because 16+16 = 32." The mathematical reasoning is valid but covert, because Steffi did not feel any compunction to identify, describe, or interpret the generalizations she was clearly able to make through symbolic representations.

Given this predisposition for symbolic representations, it is not surprising that Steffi was still rated at phase 1 on the Final Writing Protocol (Figure 3). Moreover her probability thinking was completely out of character. She merely presented a symbolic listing of the sailboats that won when she played the games. Then she wrote, "Sailboat number 5 is most likely to win because 5 came up most often when I played the game." Further, she wrote that 5 was her lucky number because she liked its shape. Her preoccupation with symbols and lack of justification was typical of her written responses, but her regression to subjective judgments and her reliance on direct experience rather than analysis was not typical of her level 4 probability thinking.

The use of symbolism by Jacques, initially a level 2 thinker in probability and a phase 1 writer, is similarly interesting. While Jacques' tendency to rely on symbolic representations was not as pervasive as that of Steffi, he frequently used stand alone diagrams rather than explanations. Like Steffi, Jacques remained at level 1 in writing largely because of this tendency to provide symbolic representations without explanation. In summary, the proclivity of these students to use symbolic representations without written explanations appears to be associated with a feeling that such representations completely meet the requirements of mathematical justification.

Discussion

Although students made significant gains in probability reasoning it is unclear whether these gains resulted from further experiences in writing about probability tasks, additional probability instruction, or a combination of both. The lack of correlation between learning-writing phases and probability levels and the characteristics displayed by focus students like Steffi and Terrance does not support the position that writing helped the students to refine, clarify, and organize their probability thinking (Azzolino, 1990), or to construct a better understanding of probability relationships (Abel & Abel, 1988; Shepard, 1993).

One explanation of this situation may be that 71% of the students in this study began the intervention at the lowest writing level, that is, level 1. Given the initial fixation of these students with narration rather than explanation, it may be argued that their writing had not developed to a sufficient level of sophistication to enhance the generalizations required in probabilistic thinking. Alternatively, because these students had previous instruction in probability, their growth in probabilistic thinking was already substantial and they had less need for the scaffolding that writing could provide. Further research is needed to examine both of these alternative positions.

Student gains in writing, based on Shepard's (1993) phase model, were not only significant; they were larger than the gains in probability thinking. Although the correlation between writing and probability thinking was not significant at post assessment, 63% of the students made a gain of at least one level in writing. Marion's progression from phase 1 to 4 on the Shepard model was not typical, but the shift from merely narrating probability situations to describing patterns and generalizations certainly was. Hence, this study provides some evidence that mathematics can provide a contextual platform for improving writing.

Although this study was limited by virtue of the fact that only one group participated, some implications can be drawn for writing in the area of mathematics. There is evidence in this study that children like Steffi were reluctant to write in the context of mathematics and required more time to recognize that symbolic representations needed interpretation. This suggests that teachers need to adopt an early and systematic approach to integrating writing and mathematics. Further, given the improvement in writing shown in this study, there is evidence that regular feedback and individual follow-up prompts from the teacher enhance both the quality and quantity of students' writing.

References

Abel, J., & Abel, F. (1988). Writing in the mathematics classroom. The Clearing House, 62, 155-158.

Acredolo, C., O'Connor, J., Banks, L., & Horobin, K. (1989). Children's ability to make probability estimates: Skills revealed through application of Anderson's functional measurement methodology. *Child Development*, 60, 933-945.

- Azzolino, A. (1990). Writing as a tool for teaching mathematics: The silent revolution.
 In T. J. Cooney & C. R. Hirsch (Eds.), *Teaching and learning mathematics in the* 1990's (pp. 92-100). Reston, VA: National Council of Teachers of Mathematics.
- Biggs, J. B. & Collis K. F. (1991). Multimodal learning and the quality of intelligent behavior. In H. A. H. Rowe (Ed.), *Intelligence: Reconceptualization and measurement* (pp. 57-76). Hillsdale, NJ: Erlbaum.
- Britton, J., Burgess, T., Martin, N., McLeod, A., & Rosen, H. (1975). The development of writing abilities. London: Macmillan.
- Drake, B. M., & Amspaugh, L. B. (1994). What writing reveals in mathematics. Focus on Learning Problems in Mathematics, 16 (3), 43-50
- English, L. (1993). Children's strategies for solving two- and three-stage combinatorial problems. *Journal for Research in Mathematics Education.*, 24, 255-273.
- Falk, R. (1983). Children's choice behavior in probabilistic situations. In D. R. Grey,
 P. Holmes, V. Barnett, & G. M. Constable (Eds.), *Proceedings of the first international conference on teaching statistics*, Volume II (pp. 714-716). Sheffield, England: University of Sheffield.
- Fischbein, E., Nello, M. S., & Marino, M. S. (1991). Factors affecting probabilistic judgments in children in adolescence. *Educational Studies in Mathematics*, 22, 523-549.
- Freedman, S. W. (1995). Crossing the bridge to practice. *Written Communication*, 12 (1), 74-92.
- Jones, G. A., Thornton, C. A., Langrall, C. W., & Miller, D. R. (1997). Using students' probabilistic thinking in instruction. Manuscript submitted for publication.
- Jones, G. A., Langrall, C. W., Thornton, C. A., & Mogill, A. T. (1997). A framework for assessing young children's thinking in probability. *Educational Studies in Mathematics*, 32, 101-125.
- Langer, J. (1992). Speaking of knowing: Conceptions of understanding in academic disciplines. In A. Herrington & C. Moran (Eds.), Writing teaching, and learning in the disciplines (pp. 69-85). New York: Modern Language Association of America.
- Miles, M. B., & Huberman, A. M. (1994). Qualitative data analysis: An expanded sourcebook. (3rd ed.). Newbury Park: Sage.
- Piaget, J., & Inhelder, B. (1975). The origin of the idea of chance in children. (L. Leake, Jr., P. Burrell, & H. D. Fischbein, Trans.). New York: W. W. Norton.
- Shepard, R. G. (1993). Writing for conceptual development in mathematics. *Journal of Mathematical Behavior*, *12*, 287-293.
- Shuell, T. J. (1990). Phases of meaningful learning. *Review of Educational Research*, 60, 531-547.
- Siegel, S., & Castellan, N. J., Jr. (1988). Nonparametric statistics for the behavioral sciences. New York: McGraw-Hill.
- Waywood, A. (1994). Informal writing-to-learn as a dimension of a student profile. *Educational Studies in Mathematics*, 27, 321-340.