Do they understand what we mean?: Assessment and communication.

Doreen Hartnall, University of Waikato

Communication is a vital part of assessment if student and assessor are to understand each other and not talk past each other. I consider why this misinterpretation should occur and what can be done to attempt to improve student's understanding of the intent of the question writer. Student answers on one question in particular are used to demonstrate some student misunderstandings.

Introduction

When considering how successful assessment tasks are in judging the process skills, it is necessary to consider whether the student reads (or reconstructs) the question as the writer intended. It is commonly assumed that the assessment process is clear, with questions asked in an unambiguous manner and all errors revealing students lack of knowledge of what is being tested. However much research suggests that this is not so and that children believed they were being asked a different question or thought they had to answer the implicit question which they perceived in the context rather than the explicit question asked (Donaldson 1978).

Words do have the power to communicate unambiguously if the listener has the cognitive structures necessary to link the ideas. However unless they are linked to real life examples there is a risk of concepts remaining as memorized items of use only in classroom mathematics. "...what students actually learn is not necessarily congruent with what their mathematics teachers want them to learn" (Ellerton and Clements 1991)

Macgregor found that phrases used for expressing comparisons in the vernacular were not standard and caused difficulty in constructing meaning for words such as 'than', 'as', 'more' and ' less'. Insignificant small words such as 'to', 'of' or 'by' were important to the sense, making skim reading which is stressed in other subjects, not sufficient for mathematical reading. There was a need to become fluent in formal English to be successful in understanding mathematical language. Some teachers have believed that the answer is to reduce the reading and writing component of mathematics lessons but this strategy is only successful in the short term. She also found that the order in which information was given did not always give the clue as to the order to attempt the problem. There is a need to actively teach strategies to students to aid in the selection and reorganisation of information in word problems. In addition she found that group work is only successful if students are able to understand the explanations shared (Macgregor 1993).

The research project Spoken Language and New Technology (SLANT) found the quality of talk in groups variable. Only exploratory talk was educationally valuable but the amount can be increased by practice on suitable tasks. Mercer sees a need for better methods for analysing continuous discourse and a willingness to bridge the gulf between qualitative and quantitative methodologies. He also stresses the importance of examining the language of classrooms in its cultural context and the analysis of genres (Mercer 1996).

Between 1988-92 there was a strong influence of process approach to maths teaching in many primary classes in Australia. (Ellerton and Clarkson 1992) ELIC (Exploring language in classrooms) and BLIPS (Basic learning in primary schools) may not have been enduring because "they were often used as add-on teacher-directed extension activities completed after the 'real' mathematics has been done". (Marks and Mousley 1990)

Language and cognition cannot be separated (Edelman, 1992). A child makes sense of things first and then of what is said. Language is used "to convey thoughts and feelings of individuals who already think independently of language". Therefore helping a student to make sense of the mathematics being learnt will help with prowess in using language.

Vygotsky regards learning as a social, culturally situated and guided activity and talk as a form of social action and social mode of thinking. Together people can construct knowledge and understanding. Neo-Vygotskians such as Bruner and Wood, have described adults' support of children's learning as scaffolding. Adults or other students can challenge thinking or provide counter-examples which cause perturbations in the student's existing schema. Mathematics teaching begins by finding out where a student "is at", i.e. their existing schemas about the topic. (There are no clean slates to work on in students minds!) Starting from there, the teacher needs to provide activites which will develop new insights into the topic which can easily be assimilated into students existing schema or will perturb it sufficiently to cause a new schema to be produced which can accommodate the new knowledge. Vygotsky says that concept attainment is complete only when a student can frame the concept in appropriate words (Vygotsky 1986).

Initiation-Response-Feedback exchanges are not discrete loops but can be linked together into longer spirals. Teachers can guide students to see connections and continuities and increase the problem solving abilities of children. Students need guidance (scaffolding) to enter educated discourse (Mercer, 1996). According to working group 7's report to the 7th International Congress on Mathematics Education 17-23 August 1992 the introduction to mathematical knowledge is like an introduction to a new culture; students are initially speechless. In many classrooms communication is asymmetrical, based on technical terms defined by the teacher with little meaning, just rituals. "Different symbolic forms can carry a multiplicity of interpretations that to the mathematician are 'the same', but to the learner seem entirely unrelated." (Susan Pirie quoted in Sierpinska and Steinbring 1992). Symbols and diagrams often do not become thinking tools but stay as knowledge containers which are not helpful in solving non-routine problems (Sierpinska and Steinbring 1992)

Frid's study of tertiary students ideas of mathematics (Frid 1993) is relevent to this study as these were the students who had successfully accomplished school mathematics. Over 50% believed mathematics consisted of rote learning of absolute truths with only one method possible. They believed understanding was being able to do examples and get correct answers. Mathematics was different to other subjects in that it was foreign to real world understandings. Some saw symbols as meaningless because either they had no meaning or the meaning had never been explained to them. Words were ascribed meanings which related to everyday meanings of words which often lacked mathematical precision. Frid described the teachers role as that of facilitator (by providing counter examples to stimulate analysis of ideas) and mediator between the students language interpretations and those accepted by the mathematical community. Everyday language can be a help or hindrance depending on how well students can integrate everyday language with the technical language. It can be refined by confronting with situations when everyday meanings are inadequate. Language is an essential component of building mathematical meanings from experience. The value of students own language in explaining their thinking must be acknowledged and encouraged.

Most errors made by children were reading, comprehension or transformation errors amd occured before any application of algorithms (Newman 1977). The highest category was comprehension which is clearly language based. What can teachers do to improve a learner's comprehension of mathematics text or ability to transform a word problem into an appropriate mathematical process? One suggestion is the use of carefully designed, often open-ended questions (Ellerton and Clarkson 1992).

The teacher must intervene but when, how and with what aim? Constructivist style teaching does not mean lack of control over constructed meanings. The benefits of "consulting with someone who knows" are immense. The choice of the initial problem is important if students are to be motivated to communicate. Writing is a vehicle to understanding especially expressive writing but it is important to realise that pupils might not understand what they tell us they understand.

When problems are considered by a cooperative group the discussion between students will advance the social construction of knowledge. Some open questioning by the teacher can help students when they appear to be about to give up or on an approach likely to end in a cul-de-sac, but care must be taken not to stifle innovative thinking. Teachers need to be aware that there are often many ways of solving the same problem some of which may be new to that teacher, and that sometimes an apparent digression from the task in hand may lead to equally valid learning. Therefore the teacher should not interfere too hastily. When conclusions are reached which are not generalisable then teachers can extend thinking by suggesting activities which will not fit that conclusion.

If teaching methods are to become more in tune with current research into how children learn, then assessment and instruction needs to be better integrated. (Driscoll 1995) describes an assessment feedback loop that can help align mathematical goals with teaching practice. Teacher questions can be not just assessment but also part of teaching. Assessment can occur while teaching is taking place where assessment is of new knowledge gained in the 'zone of proximial development' (Vygotsky 1978)

Method

As part of a larger study, I have examined in detail 306 student answer scripts to a question which aimed to test student abilities to use the mathematical processes of communication, problem solving and logic and reasoning. It was a question set on the 1996 moderation test for the internally assessed candidates sitting School Certificate that year. The question involved different ways to buy a video-recorder. The question was as follows:-

> An appliance store is selling new video recorders for $599.
> They offer two deals:
> • Trade in your old video recorder for $200, OR
> • take 15% off the price if you pay cash.
> Sulia's family is thinking about buying a new video, and her uncle has said he would buy their old recorder for $150.
> Which is the better deal for Sulia's family? Should they trade in the old video, or sell their old video to Uncle and use the cash to help pay for the new one?
> State what you are calculating at each step and show all your calculations.
> Clearly state your decision and say why you made this choice.

I have categorized their answers by a method similar to that described by Doig and Cheeseman. They say that "open assessment tasks can reveal conceptual understanding and mathematical strategies on a par with clinical interviews"(Doig and Cheeseman 1996). Though this question was not truly open, it had enough freedom of response for students to reveal their conceptual thinking.

Results

There was a rich diversity of answers which made categorizing difficult. Initially I went through all the papers and categorized each individual error, or in many cases number of errors. I found 55 different errors in total. Some of these occured frequently, some only occasionally and some were idiosyncratic. Only 8.8% had no errors of any kind. I went through the papers again and gathered similar errors into 6 main categories.

Category	Error	Percentage
1	Communication	29
2	Simplify to 3 options	14
3	Simplify to 2 options	20
4	Calculation	20
5	Muddled alternatives	5
6	No calculations	15

The first category were those with errors of communication. Many were able to correctly calculate the answers but failed to communicate what they were calculating at each stage. They did not explain why they made the decisions they did or how their calculations related to the problem or the decision reached. As this was an important process, clearly stated in the body of the question, this lost marks. 29% in total were in this category of inadequate or no communication. Of these 9.2% had no other errors so could have achieved full marks with more explanations of their reasoning. Most in this category also had other errors. The lack of communication in their answers may reflect a lack of emphasis in some schools on explaining in written language the steps taken to reach a decision. It also probably reflects an 'answer only is important' teaching style by some teachers and children. It is of interest to wonder if more of the students would have made fewer errors if they had more clearly laid out their thinking.

The second and third categories were those who had problems in comprehending the question and as a result simplified the question. 34% of the sample worked out an answer by constructing a simpler question from the one expected by the examiner. 14% changed the question to comparing three different options instead of two (category two) and 20% compared only 2 of the options (category three). Whether this is a problem of reading (not constructing the same meaning as that intended by the writer) or of a developmental stage (cannot understand the intention of the question and so makes some sense of it at a lower level) will be part of the research which is still continuing.

Category 2 errors consisted of making the question have 3 alternatives instead of 2. These consisted of (i) a discount for cash making $509.15 to pay (ii) sell to uncle making $449 to pay and (iii) trading in making $399 to pay. With these three as the alternatives most could decide that the best alternative was to trade in. Was there a difficulty in combining two different calculations into one option? and was this difficulty caused by a lack of understanding of the meaning of the question or by the complexity of the problem?

Category 2 example 1

·599 − 200 = $399
·599 x 15 ÷ 100 = $89.85c
·599 − 150 = $449.
The Sulia family are best to take the trade in deal because they will only have to pay $399.
(The 15% off if you pay in cash will only take off $89.85c leaving $509.15 to pay. The uncle's deal still leaves them to pay $449)

This student has clearly constructed a question different to the one intended by the writer. Has the student skim read the question looking only for the numbers and therefore read the question as having three options to choose between? Was the order of the last two questions important in deciding what the question was asking? Perhaps by phrasing the question slightly differently, or changing the order of the two questions at the end we could improve the number of students being successful on this question.

If that works it would prove that the problem is probably one of comprehension of the written word. However if that makes no difference to the number misconstructing this question in this way, then it would point to it being a developmental stage problem. These students would still be at a stage where this is the best interpretation that is comensurate with their existing understandings. The size of the group in category two and the consistency of their misconstructions makes it an interesting area to research further.

Category 3 consisted of three different means of simplifying the question to only two parts. (a) by a complete absence of the 15% discount. The question was simplified to trade or uncle's cash only. Perhaps the last sentence has been missed and not read so that the discount appears to be irrelevent to the question? However the calculations, the communications, the logic and reasoning are all without fault. (b) by a complete absence of uncle's cash. The question was simplified to trade or discount. Has the question reading been truncated and only the first few lines read? (c) a complete absence of the trade in. The question was simplified to discount or uncle's cash.

The fourth group consists of those students who made calculation errors and covers a wide range of errors. Some are possibly careless errors, such as miscopying a figure from the question or misreading a calculator answer, but most are misunderstandings as to how to calculate 15% of a figure or calculating 15% off the wrong amount. These included :-

- 15% as 100-15 =85
- 15% of $599 as 599/15=$39.93
- New price after 15% discount as 599/1.15=$520.87
- 15% of 599 as $599-15 =$584
- 15/599*100
- 599*100/15

All of these appear to be cases of misunderstanding or misremembering of taught algorithms. The next set are all cases of calculating the 15% off the wrong amount.

- 15% of $399
- 15% of $150
- subtracted the Uncle's $150 first and then calculated 15% of $449.

The last set are all idiosyncratic mistakes where no clear message is given as to method and it is not obvious how the answer is obtained. Extra questions such as could be given in an interview will be needed to ascertain the thinking behind these errors.

- 599-15%=$149.75
- 599-15%=23.25=575.75 with discount
- 15% discount =$1
- worked out discount by continual adding but made a mistake in addition.

20% of the candidates were in this fourth category.

A fifth category consists of those who muddled up the various alternatives. For example they mixed up trade and discount as if they were the same, or mixed up the discount and uncle's money, or believed they could both trade in and get the discount. In example 2 the uncle is seen as giving Sulia's family the 15% off instead of $150.

Category 5 Example 2

If she pay's this video store cash
it will only cost her $599x15% = $89.85d
$599 - $89.85 = $509.15d is all she
will have to pay altogether so I
think it would be a better deal if
she traded in her old one for
$200. Then she would only
have to pay $399 altogether.
It's a better deal than her
uncles on the 15% off I think!

Category 5 Example 3

This example mixes up trade-in with the 15% off.

Selling video to uncle
Selling = $150
$599 - $150 = $449 — $ 449 to pay

Trade In
599 - 15% = $89.85
$599 - $89.85 = $509.15 — $ 509.15 to pay

You should sell your current video recorder to your uncle

The next example is extremely confused. The 15% discount for cash is calculated but instead of being an extra saving and added to the $150 gained by selling to uncle it is subtracted from that amount making a much smaller saving. Are the words 'take off' being read as a subtraction activity whatever the circumstances?

Category 5 Example 4

If traded in $200 = 599 - 200 = $399
If sold for $150 = 599/100 x 15% = 89.95
$150 - 89.95 = leaves
$60.05. Salia's family would get a better deal if they sold their old video to her Uncle.

One even believed that it was possible to trade in, get the discount and then sell to uncle and end up with a new video for $20. Another similar case said they should sell to Uncle and trade in (making only $50 to pay) and then get a discount on that. 5% were in this category.

The final category consists of those who made an attempt but produced no calculations. Sometimes they produced a statement of which was cheapest but with no justification. This was the most common in this category which comprised 15% of the candidates. Sometimes the statements did not fit at all with the problem. eg “the new one is best or cheaper than the old one” or “the Mitsubishi is the better deal”or “buy her uncle’s and sell him her old one”. One answer appeared to use knowledge gained in economics rather than using mathematics skills.”Buy from the shop because if it doesn’t work they feel guilty”.

Discussion and Conclusion

The mistakes made on this question are indicative of problems with the language used to express the question. The realistic context encouraged students to attempt the question (only 16.7% made no attempt at this question) and the everyday language in which the question was framed gave them confidence. However their answers revealed misunderstandings of what was being asked. The use of "or" and "and" appear to have caused difficulties for some students as found by Macgregor (Macgregor 1993) with small words being overlooked or misinterpretted. Skim reading may have occured which missed the point of the question.

The ambiguity of words may have caused some of the difficulties where more than one meaning may exist for the same word or phrase especially when this may be class related. For example the phrase "better deal" may have meanings other than monetary to some cultures. Lower socio-economic groups may consider it better to pay off in small amounts even though the total cost is higher. Some terms such as "trade-in" may be unfamiliar to some students. The context may have got in the way of the mathematical calculation being assessed. (Hipkins 1996)

Certain words have been found to trigger certain ideas about the calculation called for and this can be unfortunate when those calculations are not the ones required. "Take 15% off" in this question triggers a subtraction reflex but this needs to be **added** to the uncle's $150 to find the total savings. "The level of difficulty of a word problem is related not only to its mathematical content but also to its linguistic form and semantic structure" (Gibbs and Orton 1994)

It may be that the wording of the question caused some of the misinterpretations of the problem. By writing the same question in different ways, by changing the order of the sentences or the length of the sentences, the researcher will attempt to improve students' achievement on this question. By interviewing students who made particular mistakes especially in categories 2 and 3, the researcher will try to discover what part of the question caused them to miscontrue the problem.

The analysis reveals a number of errors that students have made in dealing with this question and this information should be able to inform teachers as to likely errors in understanding. This feedback on the structural understanding of students as a group rather than as individuals, can improve teacher understanding of how students may be thinking and help them adjust their teaching accordingly. Teaching strategies to use when dealing with word problems, and guiding students in the reorganisation of information may improve student's success at this kind of problem. It has been suggested that the use of questioning techniques by the teacher which draw the student's attention to the main points in the problem, leads to higher level discussion and a deeper comprehension of the problem especially when student generated questions are encouraged. (Bean 1985)

The use of cooperative group work with teacher guidance may also achieve improvement. Using everyday language to describe mathematical thinking may improve the preciseness of the language used and help refine the meaning of words as situations arise where the everyday meaning of words is inadequate. Having to explain your reasoning to someone else makes it much more explicit and can often improve understanding. Socio-cognitive conflict has been shown to change student's thinking as they interact with other students with different ideas and understandings. (Mercer 1995)

Ellerton and Clarkson (1992) have indicated that the use of open ended questions can improve students' ability to comprehend the meaning of a word problem and choose the appropriate mathematical process. The use of realistic contexts with emphasis on answers that make sense in the real world, can lead to an improvement in the thinking taking place when dealing with word problems.(Verschaffel and Corte In press)

This study of student's scripts gives interesting insights into their thinking and probable misinterpretations of the language used to frame the question. Further research in the form of interviews are now necessary to find out what the student's were thinking when they misconstructed the question. Research will also continue into how assessment can be improved so that student and assessor do not talk past each other.

References

Bean, T. (1985). Classroom questioning strategies:directions for applied research. *The Psychology of Questions*. A. Graesser and J. Black. Hillsdale, New Jersey, Lawrence Erlbaum Associates: 335-358.

Doig, B. and J. Cheeseman (1996). When there isn't enough time for an interview: how to analyse open assessment tasks. *Technology in Mathematics Education,* P. Clarkson, Melbourne, Deakin University Press.

Donaldson, M. (1978). *Children's minds*. London, Fontana.

Driscoll, M. (1995). "The farther out you go....Assessment in the classroom." *The Mathematics Teacher* **88**(5): 420-425.

Ellerton, N. and P. Clarkson (1992). Language factors in mathematics learning. *Research in Mathematics Education in Australasia 1988-1991.* B.Atweh and J. Watson, Merga.

Ellerton, N. F. and M. A. Clements (1991). *Mathematics in Language : a review of language factors in mathematics learning*. Geelong, Victoria, Deakin University.

Frid, S. (1993). Communicating mathematics: A social sharing of language and decisions pertaining to truth and validity. *Communicating Mathematics*. M. Stephens, A. Waywood, D. Clarke and J. Izard. Hawthorn, Victoria, Australian Council for Educational Research.

Gibbs, W. and J. Orton (1994). Language and Mathematics. *Issues in Teaching Mathematics*. A. Orton and G. Wain. London, Cassell: 95-116.

Hipkins, R. (1996). Should contexts be used in science examinations? A critical review of current practice. *NZARE Conference*, Nelson.

Macgregor, M. (1993). Interaction of Language Competence and Mathematics Learning. *Communicating Mathematics*. M. Stephens, A. Waywood, D. Clarke and J. Izard. Hawthorn, Victoria, Australian Council for Educational Research.

Marks, G. and J. A. Mousley (1990). "Mathematics education and genre: Dare we make the process writing mistake again?" *Language and Education* **4**(2): 117-135.

Mercer, N. (1995). *The guided construction of knowledge:talk amongst teachers and learners.* Clevedon, Avon, Multilingual Matters Ltd.

Mercer, N. (1996). Language and the Guided Construction of Knowledge. *Language and Education*. G. Blue and R. Mitchell. Clevedon,Philadelphia, British Association for Applied Linguistics.

Newman, M. A. (1977). An analysis of sixth grade pupils' errors on written mathematical tasks. *Research in mathematics education in Australia*. M. A. Clements and J. Foyster. Melbourne, Victoria, Swinburne College Press. **2**: 269-287.

Sierpinska, A. and H. Steinbring (1992). Language and Communication in the Mathematics Classroom. *Proceedings of the 7th International Congress on Mathematics Education 17-23 August 1992.*, C. Gaulin, B. Hodgson, D. Wheeler and J. Egsgard, City, Les Presses de L'Université Laval, Sainte-Foy.

Verschaffel, L. and E. D. Corte (In press). "Teaching Realistic Mathematical Modeling in the elementary school. A teaching experiment with fifth graders." *Journal for Research in Mathematics Education.*

Vygotsky, L. (1978). *Mind in Society: the development of higher psychological processes*. Cambridge MA, Harvard University Press.

Vygotsky, L. (1986). *Thought and Language*. Cambridge, MA, MIT Press.