Teachers' Reported Use of Problem Solving Teaching Strategies in Primary Mathematics Classrooms

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The implementation of mathematical problem solving in New South Wales (NSW) primary classrooms is being explored. The results of part of a survey that has been administered to a sample of primary school teachers regarding their beliefs and practices in relation to problem solving is explored. Teachers report that they regularly use strategies such as whole class discussion including a focus on suitable problem solving strategies, concrete materials and teacher modelling. They rarely have calculators available for students, allow students to choose problems or spend much time on one problem.

There has been substantial advice to teachers to teach problem solving and to use problems as a focus of learning in mathematics (Cobb, Wood & Yackel, 1990; Schoenfeld, 1992). This advice has been provided in papers in research and professional journals as well as in curriculum documentation. Such advice has been accompanied by considerable efforts through preservice and inservice programs to change teaching practices from more traditional approaches to contemporary or reformed methods. In the reformed classroom, non-routine problems become the focus of learning (Clarke, 1993) rather than drill-and-practice exercises forming the basis of classroom activity.

If the success of advice and programs which focus on mathematical problem solving are to be ascertained, then research that investigates what teachers believe about teaching mathematics and what actually happens in teachers' classrooms is necessary. Teachers' espoused beliefs are influenced by their actual beliefs (Thompson, 1992), by their knowledge and interpretation of advice about teaching problem solving (Fennema, Carpenter & Peterson, 1989) and by their use and understanding of curriculum documents. Reported classroom practices are influenced by espoused beliefs, by actions in the classroom as well as by constraints such as parent and student beliefs and school culture. Figure 1 is a proposed model of the relationship between these factors.

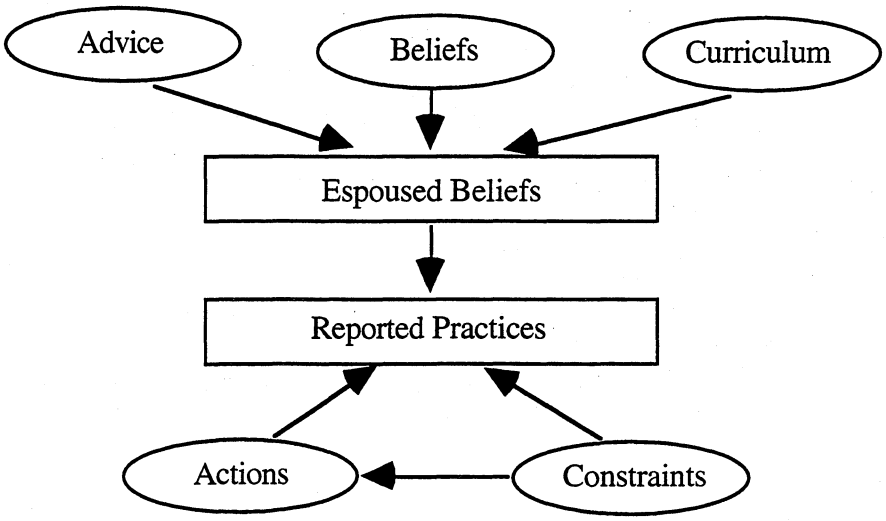

Figure 1. Factors which impact on espoused beliefs and reported practices.

Advice About Problem Solving

Problem solving can be a valuable vehicle for learning mathematics (Cobb et al, 1990). As students are challenged by problem solving tasks, it becomes necessary for them to think about what they already know; to consider strategies that may help them solve the problem; to engage in discussions with other students and the teacher in an effort to resolve conflicts; and to ultimately reorganise existing knowledge to accommodate new ideas.

Reviews of teaching practices (Koehler & Grouws, 1992) and strategies that promote mathematical thinking (Schoenfeld, 1992) reveal a variety of approaches to the teaching of mathematics. Koehler and Grouws (1992) outline five paradigms or different research programs which address approaches to teaching and learning mathematics. While these are varied they all "...accept the premise that students are not passive absorbers of information...[and]...view the teacher as an informed and reflective decision maker" (p. 123). Given these commonalities, the research findings recommend a variety of teaching strategies. In summary, the teacher's role is to plan for instruction, to develop appropriate tasks or problems, to encourage discussion and listen to responses and to modify instruction to meet the needs of individual students.

Establishing and maintaining an appropriate learning environment is another key role for teachers. This environment encourages mathematical thinking (Schoenfeld, 1992) and promotes students' mathematical communication (Weissglass, Mumme & Cronin, 1990). Social interaction is supported as students discuss, speculate and defend their views about mathematical ideas and problems. Teachers need to ensure that students feel comfortable enough to talk about their mathematical understanding without fear of ridicule (Lampert, 1990). In this environment, the teacher manages discussion but students take responsibility and share authority.

Problem solving tasks can be used in classrooms in many different ways. Wright, (1992) describes two approaches which teachers adapt according to their beliefs about student learning. One approach to problem solving is to present students with problems at the end of the topic after skills and procedures have been rehearsed. This approach is referred to as an 'ends' approach and is based on a belief that students need mathematical content and procedures before they can solve unfamiliar problems. Textbooks which place problems at the end of chapters reinforce this approach. Exercises and application problems are typically used in such classrooms.

Another approach, referred to as a 'means' approach, uses problems as the focus of learning. Problems are used to provide a stimulus for student thinking. Good problems create conflict in the minds of students and restructuring of understanding may be needed in order to resolve such conflicts. Unfamiliar and open-ended problems are favoured as the basis of mathematical activity. If this approach is adopted then problems would be used more frequently but fewer problems would be considered.

Beliefs About Problem Solving

It has been argued that teachers' decisions about how to teach mathematics are made on the basis of their knowledge and beliefs about how students learn mathematics (Koehler & Grouws, 1992; Thompson, 1992). Teacher thinking before, during and after lessons, plays an important part in instruction (Clark & Peterson, 1986) and belief systems influence perceptions, planning and actions in classrooms (Ernest, 1989). Teaching is a cognitively demanding activity in that teachers are constantly making decisions based on their beliefs, knowledge, judgements and thoughts and belief systems are important determinants of how individuals organise tasks and goals (Nespor, 1987; Cobb, 1986). The view of mathematics which teachers adopt and the models of teaching

and learning which they support will have a significant impact on classroom practice (Thompson, 1984; Ernest, 1989).

Studies have attempted to improve teachers' knowledge of student thinking in an effort to change beliefs and practice (Cobb et al, 1990; Fennema et al, 1989). In the research conducted by Fennema et al (1989) experimental teachers were given Cognitively Guided Instruction about childrens' thinking in addition and subtraction. As a result, these teachers spent less time on drill activities and more time on problem solving than a control group of teachers. The experimental teachers spent more time listening to student explanations and they accepted a wider variety of problem-solving strategies.

Curriculum Advice About Problem Solving

In NSW, it is mandatory to implement the current syllabus document that provides advice about content and approaches to teaching. The *Mathematics K-6 Syllabus* (NSW Department of Education, 1989) describes teaching and learning units under the topic strands of number, space and measurement. Developing processes and problem solving competence is addressed in the introductory section where there is a brief discussion about the role of problem solving. Advise is provided which suggests that problem solving can form the basis of mathematical activity in classrooms and that the role of teachers is to challenge student thinking and to develop problem solving competence in their students. The *Syllabus* suggests that in mathematics classrooms, problem solving tasks provide opportunities for students to engage actively in processes such as planning, estimating, looking for patterns, conjecturing, generalising and evaluating.

Teaching strategies are also described in the *Syllabus*. These include encouraging discussion in small groups, allowing students to develop their own problem solving strategies and recording procedures, providing concrete materials, designing tasks and choosing situations that relate to students' interests and experiences. Mention is also made of listening to student responses, supporting thinking when necessary, encouraging student reflection and guiding student's invention of their own problems. The document clearly supports much of the advice about problem solving that is contained in the literature.

This study is part of a larger investigation that aims to explore the level of implementation of mathematical problem solving in primary classrooms in NSW. Teachers' beliefs about the role of problem solving in learning mathematics as well as their classroom practices is being investigated. Data collection focuses on teachers' knowledge and beliefs about the discipline of mathematics, teaching mathematics and students' cognition since all of these factors impacts on what occurs in teachers' classrooms (Ernest, 1989).

Method

A survey has been used to gather data about teachers' espoused beliefs and classroom practices, use of particular mathematical tasks and resource materials as well as their concerns and perceived needs in relation to the implementation of problem solving approaches. Part of the survey was designed to ascertain the frequency of use of a list of possible teaching strategies. Teachers were also asked to record how frequently they use particular types of problems. The meanings of each of the terms 'exercise', 'application problem', 'open-ended problem' and 'unfamiliar problem' was outlined in the introductory section of the survey so that a common understanding could be achieved. This approach was taken in an effort to avoid the disparate meanings that are attributed to the term 'problem'.

There were twenty items that related to problem solving teaching strategies. These items were chosen on the basis of strategies mentioned in the literature (Clarke, 1995;

Koehler & Grouws, 1992; Van Zoest, Jones & Thornton, 1994) as well as in curriculum documentation (NSW Department of Education, 1989). Teachers were requested to rate these items under the headings of 'hardly ever', 'sometimes', 'often' and 'almost always'.

The survey was trialed with 25 teachers, five of whom were interviewed to check meaning of items and to clarify interpretation of instructions. Each of these teachers was clear about the definitions of the types of problems. The original survey used the categories 'hardly ever', 'about once a month', 'about once a week' and 'almost always' but these were considered too restrictive by the respondents. Changes in wording and lay-out were also made on the basis of feedback from trialing.

The survey was then administered to a voluntary group of 78 teachers in twelve schools in NSW. The researcher addressed each school staff at a meeting and invited them to complete the survey. The staff sizes of the schools ranged from 4 to 30 in both metropolitan and rural settings. To compare these responses to those from an informed community, tertiary mathematics educators, or lecturers, in NSW and Victoria were invited to complete the survey items relating to the frequency of use of teaching strategies. The prompt asked them to complete it on the basis of their reading of the problem solving literature. Twenty-one lecturers responded to this request.

Results

The responses for both groups were scored from zero for 'hardly ever' to three for 'almost always'. This method was employed so that means could be calculated for each item and the responses of each group compared. It is acknowledged that the differences between categories are not equivalent and therefore this process is used as a gross measure for comparison purposes only.

Less Frequently Used Teaching Strategies

Items that had means less than one, or *low* means, for the teachers included:

- you ensure that students work alone (alone);
- you have calculators available for students to use (calculators);
- you present unfamiliar and open-ended problems to the class with very little indication of how to solve them (little help);
- you provide a set of problems and the students are allowed to choose a problem they would like to work on (choose problem); and
- you allow the class or individual students to spend several lessons on the same problem (spend more time).

Items that had means less than one for the lecturers included:

- teachers ensure that the students work alone (alone);
- teachers explain in detail what the students have to do to solve problems (explain); and
- teachers set exercises to allow the students to practise their skills (exercises).

The first item is the only one in common.

To compare the distributions of each of the seven distinct items listed above, box plots have been drawn in Figure 2. There is a large difference in the distributions for the use of calculators. Giving students problems and allowing them to explore possibilities with little help from the teacher is certainly not a frequently used strategy for the teachers who were surveyed. Teachers reported that they set exercises much more frequently than lecturers believe the literature recommends.

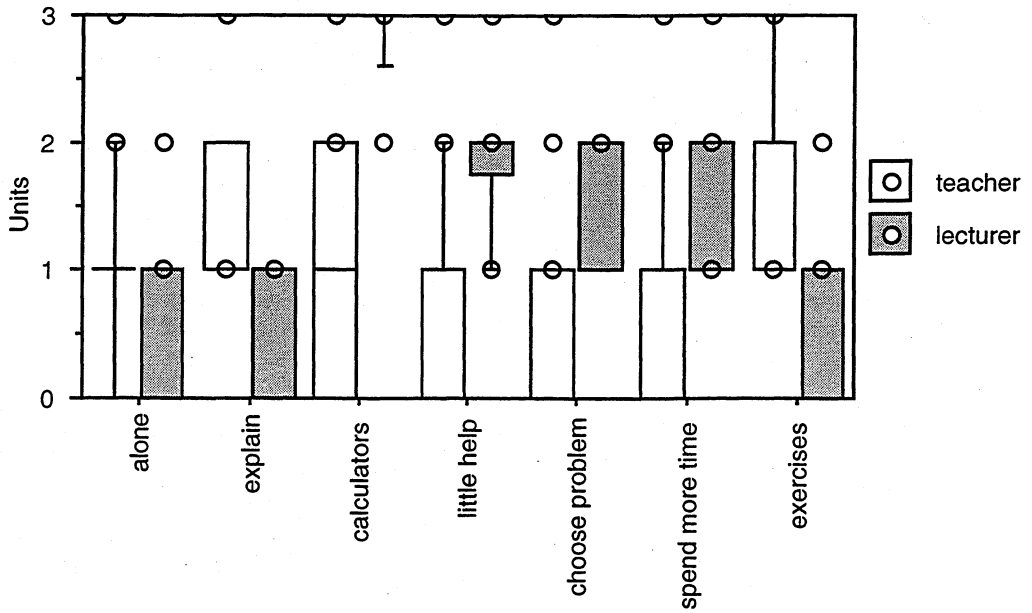

Figure 2. Box plots of items with *low* means for teachers and lecturers.

More Frequently Used Teaching Strategies

Items that had means greater than two, or *high* means, for both teachers and lecturers included:

- at the end of a problem solving lesson you lead a whole class discussion so that students can share solutions and strategies (discussion);
- you provide concrete materials for those students who need them (concrete materials); and
- you discuss useful problem solving strategies (strategies).

The item that had a *high* mean for the teachers alone was:

- you model the problem solving process to the class (model).

Items that had a *high* mean for the lecturers alone were:

- teachers have calculators available for students to use (calculators);
- teachers encourage the students to work in small, cooperative groups (groups);
- teachers encourage students to record their own procedures and methods of solving problems (record methods);
- teachers encourage students to pose their own problems (pose problems); and
- teachers use problems which arise from the school context or which relate to the students' experiences (experiences).

To compare the distributions of each of the nine distinct items listed above, for both teachers and lecturers, box plots have been drawn in Figure 3. Again there are differences in the distributions. Teachers reported a high frequency of use of modelling the problem solving process to the class. It is possible that this item may have been interpreted as teacher demonstration rather than specifically modelling problem solving. Encouraging small, cooperative groups and using problems that relate to students' experiences were used quite frequently by teachers although the means for these items were not greater than two. Encouraging students to record their own methods was used less frequently than lecturers believe to be desirable in mathematics classrooms. Encouraging students to pose their own problems was reported as a rarely used strategy by teachers.

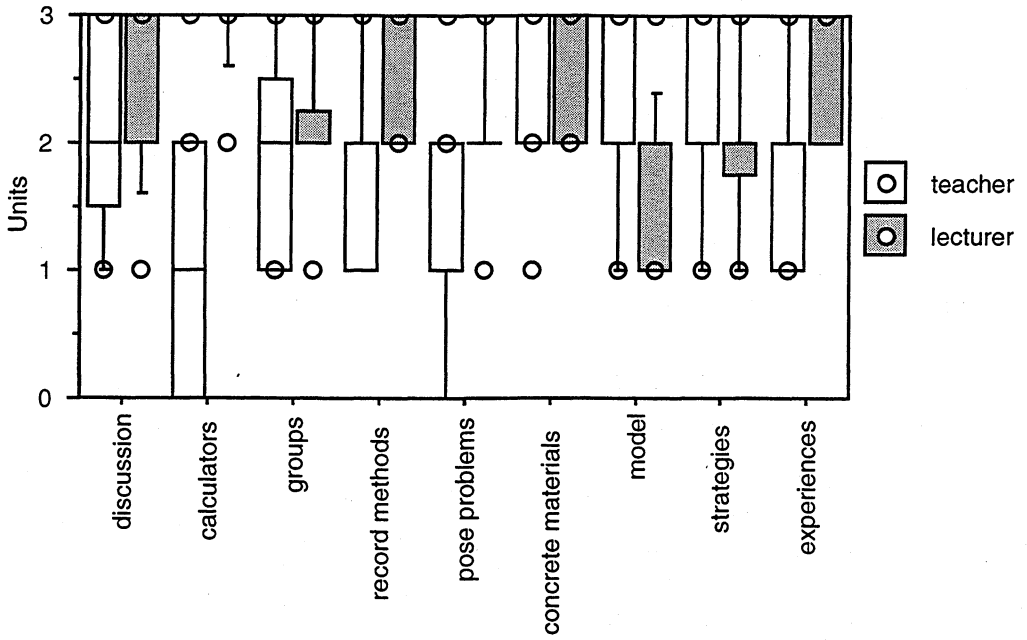

Figure 3. Box plots of items with *high* means for teachers and lecturers.

Frequency of Use of Different Types of Problems

The items that explored frequency of use of different types of problems revealed a wide range of responses from teachers. Table 1 lists the four items and the frequencies for each category. Overall, open-ended problems and unfamiliar problems are less frequently presented to students than application problems and exercises. However, it should be noted that there are clearly individual differences in frequency of use since a total of 24% of teachers pose open-ended problems 'often' or 'almost always' and 21% 'often' pose unfamiliar problems. Also, 7% of teachers 'hardly ever' and 28% 'sometimes' set exercises. These results demonstrate that some teachers appear to use open-ended and unfamiliar problems on a regular basis. This implies that they may use problem solving as a 'means' rather than as an 'end' in the learning of mathematics.

Table 1.
Frequencies of the teaching strategies for teachers which relate to problem types.

Teaching Strategy	Hardly Ever	Sometimes	Often	Almost Always
you present *application problems* which allow students to practise the skills they have just learnt	0	18	49	9
you pose *open-ended problems* to allow students to explore mathematical situations for themselves	8	50	16	2
you set *exercises* to allow the students to practise their skills	5	21	39	10
you pose *unfamiliar problems*	14	44	15	0

In response to these four items, lecturers recommended a higher frequency of use of open-ended and unfamiliar problems than the teachers reported. They also recommended a lower frequency of use of application problems and a much lower frequency of use of exercises. These differences between teachers and lecturers raise several questions about why all teachers have not responded to advice from the problem solving literature as well as from the *Syllabus*. It may be the case that implementing such

approaches is quite difficult given the constraints of classrooms or that lecturers are not in touch with the practical aspects of classroom and school life.

Implications

The strategies that lecturers have rated as being appropriate for regular use are recommended in advice given to teachers about problem solving in the reformed classroom as well as in curriculum documentation. These include whole-class discussion for sharing solutions and strategies; small, cooperative group discussion; provision of concrete materials and calculators; recognition of the need to encourage individual student recording of methods and procedures; and encouragement of student posed problems as well as using problems that relate to student interests.

Teachers still seem to be reluctant to implement some of these. It is clear from the results that most teachers see the value of whole-class discussion, teacher modelling and using concrete materials but teachers are not yet convinced that calculators are an integral part of the primary mathematics classroom. Also, there is less frequent use of individual student methods of recording and student created problems. This may suggest the persistence of a reliance by teachers on standard algorithmic procedures since teachers may believe that students need to learn the traditional or textbook method. It is also possible that teachers might feel threatened by unfamiliar methods and believe that it is important to maintain control of what children learn and how they represent their mathematics.

A small number of teachers seem to have responded to advice and do report using many of the teaching strategies that the literature claims promotes learning in mathematics. This does not of course guarantee that this is what is happening in their classrooms. Part of the larger investigation is to observe teachers as they implement problem solving strategies and to compare reported practices with observed practices. If teachers are in fact implementing these strategies then it is of interest to explore why other teachers have not responded to the same advice. Or, is it the case that teachers have different beliefs about teaching and learning mathematics? If teachers hold similar beliefs, it may be possible that constraints in some school cultures are much stronger than in others. Alternatively, particular school cultures may actively support a problem solving approach and encourage teachers to implement the strategies explored in this survey.

The results of the survey completed by these teachers provide valuable information about reported teaching strategies in relation to the use of problem solving in learning mathematics. A larger sample would help to confirm particular strategies that teachers have not adopted as recommended practice. There may be items where interpretation of meaning was unclear. This will be investigated in interviews that will be conducted with a sample of twelve of the respondents. From this small group, two teachers will be observed in an in-depth investigation of problem solving teaching practices. The school environments in which each of these teachers works will be explored so that constraints and support mechanisms can be identified.

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