Focussing on the Self as a Learner of Mathematics and Statistics

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A comparison of three consecutive implementations of a Mathematics and Statistics subject suggests that a focus on the self as a learner of statistics in addition to one on learning the course content may result in preferred outcomes. Modification of students' perceptions of themselves as learners can alter the acceptability of the experience of learning Mathematics and Statistics, whilst maintaining high performance levels. The body of the paper explores the exercises and manner through which the focus, on learning how to learn statistics, has been created and suggests ways in which the experiences of one class of students may be used to improve the experiences of successive generations of students.

Introduction

A constructivist epistemology of knowledge and the associated teacher-as-researcher research paradigm, have led to what this writer has experienced as a profound change in teaching practice and to the theme of this paper which is how teachers may incorporate a focus on learning how to learn Mathematics and Statistics into their teaching of the discipline.

Epistemology of knowledge

The key epistemological idea underlying the teaching and learning discussed in this paper, and the research process into it has been that knowledge is actively constructed (Von Glaserfeld, 1988) irrespective of the teaching pedagogy (Klein, 1995). What follows from this for the teacher is a need to assess what the student knows and how they have constructed that knowledge. This includes understanding the social context within which knowledge is constructed. The drive to know what knowledge has been constructed and how it may have been constructed has provided the impetus to finding and developing techniques which allow me, as the teacher, and the students to examine the processes involved in learning, and in learning Mathematics and Statistics. These techniques that will form the focus of this paper.

The reflective practitioner

How the techniques have been developed, as distinct from the impetus to have them, has been through the usage of a teacher-as-researcher or reflective practitioner research paradigm. Throughout the development and evaluation of successive implementations of a tertiary preparation course in Mathematics and Statistics, part of a program called Gateway, I have been involved in deliberate reflection upon what has transpired in the classroom. Qualitative and quantitative evidence to confirm or refute explanations as to what has been happening or experienced both inside and outside of the classroom has been gathered throughout the practice of teaching. The reflective process has been sometimes akin to ethnographic work relying on meticulous observation and description of events and interactions in the classroom. At other times the evidence to confirm or deny conjecture has been through questionnaires, the analysis of critical incidents, thematic analysis of written material gathered through student exercises or even independent evaluations. It has been this recent approach of being a teacher-researcher rather than a traditional researcher that has made a profound impact upon, and insight into, the teaching undertaken by the author of this paper.

Comparing Outcomes

To demonstrate that a focus on learning may result in comparable outcomes to those from courses which focus on technical content and processes a comparison of three

implementations of a tertiary preparation mathematics and statistics subject is presented (and reported elsewhere in greater detail (Porter *et al.*, 1994)). The implementations could be characterised as:

Base implementation: This, the first implementation provided base level outcomes to enable later comparisons. This implementation used the same pedagogical techniques as subsequent implementations and was consistent with what various authors would consider as constructivist approaches to teaching. Pedagogical techniques included experiential learning, small group work, activity based learning, classroom discussion and other language oriented activities and video materials. The focus was on implementing a curriculum developed from the ongoing work of unpacking statistical experts' knowledge (Porter *et al.*, 1993).

Curriculum focus: The second implementation modified the curriculum of the first, adding units and deleting units, sequencing them in order to provide students with better explanations and/or experience of data which could highlight the theme that, Statistics is about variability. The prime focus was on the mastery of technical content and processes.

Learning focus: The explicit focus of this implementation required that students examine the learning experience in addition to identifying and mastering the statistical ideas. All three implementations involved an examination of the process of learning but in the third implementation this was made explicit to the students. Whilst in early implementations students were placed in situations which would demand that they be responsible for extracting from an activity the important statistical ideas, by the third implementation students were made explicitly aware that the process of taking responsibility was one to be learned as well as the Statistical material itself. They were to reflect upon themselves as a learner in the process.

In Table 1 it is evident that base Mathematics and Statistics performance improved greatly in the curriculum focussed implementation, despite students' poorer pre-test scores. This was maintained when the orientation of the course shifted to include an explicit learning focus. The improvement in submission rate is also of interest with students seemingly more prepared to submit work. The decline in completion rate in the learning focussed course was explained in terms of outside influences eg eviction, illness interrupting students' study.

Table 1: Mathematics and Statistics Outcomes

		Implementation		
		Base	Curriculum focus	Learning focus
Number commencing		64	79	61
Number completing		47	62	37
Mathematics Pre-test	mean	82.4	64.0	62.8
	std	14.7	20.3	16.2
Course Completion %		73	78	61
Assignment submission rate %		76	95	100
Assignment mark /15	mean	7.4	8.8	10.7
	std	3.8	2.9	1.8
Statistics part of exam mark /15	mean	10.8	8.6	9.2
	std	5.1	2.6	2.4
Overall mark /100	mean	71.3	78	81.5
	std	13.6	10.1	6.3

The three implementations were sequential and did not follow an experimental

research paradigm with subjects being randomly allocated to different conditions. The teacher-as researcher paradigm used the reflective process and associated evidence to direct changes to teaching. As such, outcomes following the change are not interpreted as causative rather as viable reasons for the successful change in teaching. The evidence suggests that it is viable to take time in the mathematics and statistics classroom to focus upon issues related to learning as this does not necessarily detract from the outcomes of learning the discipline. This is important as far too often in teaching there appears to be too little time to cover the content let alone explore other issues such as learning.

As can be seen from Table 2, the curriculum focussed implementation whilst resulting in improved grades was accompanied by a decline is students' evaluations of the course. For the subject, which was part of a tertiary preparation program, to be considered a success it was seen as necessary for students to be more inclined to undertake further study rather than less inclined, mastery of content alone was considered inadequate. One of the advantages of including a focus on the learning process was that there was an improvement in student's attitudes toward the learning experience.

Table 2: Percentage of students responding to two most positive rating categories in formal evaluation.

		Implementation	
	Base	Curriculum	Learning
The lecturer sequences material in the unit very illogically (1) to very logically (5)	76	47	97
The lecturer stimulates me to think about the subject never (1) to always (5)	70	51	76
The lecturer has demonstrated understanding this subject very poorly (1) to very well(5)	86	85	91
I have felt enthusiastic about attending lectures in this subject never(1) to always (5)	63	45	88
The subject material has usually been very uninteresting (1) to very interesting (5)	55	40	82
The lecturer has presented the material very unclearly (1) to very clearly (5)	60	38	79
This lecturers interest in assisting me to learn is non existent (1) to very high(5)	70	66	88

Focussing on Learning

Garfield (1995) in listing the implications of constructivist theory for the learning of statistics concludes "students should be encouraged to assess their own learning as well as their notions of how they learn, by giving them opportunities to reflect on the teaching/learning process." This has certainly resulted in more positive experiences for these and subsequent classes of students.

The techniques for drawing the focus to the process of learning may be subtle or obvious, take moments or be part of an extended exercise, they may be indirect as a by product of a statistical exercise or deliberately induced. The focus may be on how student knowledge itself is constructed or on the affective nature of the process.

Whatever techniques are utilised however it is important for students to be aware that the learning process is considered as important as learning the material itself, as learning should continue when the teaching instruction stops. Without this being made explicit, students may, as with the 'curriculum" implementation of the course, fail to realise the tremendous advances that they have made, not because of the statistical concepts they had mastered, but because their approach to learning would allow them to master many more.

Techniques for providing the focus on learning how to learn

Setting the scene

In week one the students were asked the simple question 'What is the name of the course that you are about to undertake?' the unanimous response from all three classes of students was 'Mathematics and Statistics' rather than 'Learning Mathematics and Statistics'. This perception was rectified through an initial lecture explaining the approach to teaching and learning that was to be adopted. Usually a couple of students will indicate at the end of class that they 'feel better or more relaxed' after this explanation, their Mathematics anxiety having diminished a little.

During the introduction to the subject students were informed of the approach to learning that had been adopted.

> My teaching is premised on the assumption that each of you as learners will take what I have to say and put it together to form different constructions of knowledge, different perspectives on that knowledge. This premise is a basic tenet of constructivist theory and it contrasts with the notion that each of you as learners are passive vessels into which we pour knowledge, store it until it is again released in its uncontaminated form. The communication of what it is you understand is crucial if I am to help you develop appropriate representations of that knowledge. To aid that communication I have lots of class discussion and minor writing assignments...Approaching teaching from the perspective of constructivist theory, I require you as students to participate in activities and to make sense out of the ideas that those activities engender. In our work we will attach labels to ideas and repeat them week in week out so that you come to understand the language and ideas of statistics. I will not give you a lecture, sets of notes, readings...I want you to think about what you observe in class. I want you to piece together a jigsaw puzzle of ideas, to develop an understanding of what statistics entails. What are the most central ideas, what ideas are of lesser importance. You will need to learn to describe what it is we do and what we have observed, talk about it and write about it. (Introductory lecture)

The students were also introduced to the issues that confronted students in the previous two implementations. These teacher identified issues involved students: trusting their own ability to think about and do mathematical and statistical work; ascertaining the relevance of the material to their chosen discipline; ascertaining the personal relevance of the material to their lives; modifying expectations about the discipline when it was not as expected; dealing with uncertainty as characterised by 'tell me what it is I have to know, give me a rule or a formulae to follow'; being able to reflect upon what they had experienced in an appropriate manner, that is to be able to focus on the most important concepts or ideas rather than focussing on the easy concepts or the hard concepts, or upon the specific example rather than the generalisation or vice versa; in being able to recreate ideas rather than simply remembering them; and, being able to write about statistical ideas.

In the first class students are also given, in groups, a completed Maths test and asked to identify the strategies they could use for checking the answers were correct. This was a useful way of introducing students to the notion that there is more than one way of completing a task correctly although sometimes some methods may be easier or more precise than others. Many verbalise how they had always thought that they were wrong if their strategy was different to that being taught.

Exercises for focussing on learning

The snake: In the first or second class, students are also asked to draw a winding snake or stream (Pope, 1993) and on it to chart the various incidents that previously influenced their learning culminating in the experience that led to them applying to enter the Gateway program. This exercise was to focus them upon those incidents that had helped or hindered them in previous studies. At the end of the subject this was repeated with students being asked to focus on the journey through the subject, see for example Figure 1.

Personal challenges: Students are also asked to anticipate "What are the personal challenges that you think you will need to face and conquer in order to learn (1) your Mathematics?; (2) your Statistics? and, (3) the material in the other units in Gateway?" This exercise repeated at the end of the subject provided material which is then used at the commencement of the subsequent implementation.

Sharing student histories

The snake exercise and the personal challenges exercise have both generated data that has been taken back to subsequent classes. For the tertiary preparation groups anxiety appears to be the highest when the first piece of assessment is due. Students, feeling uncertain, want to know what they have to learn and what they are to give back to you. Using the stories gathered from previous students the new class is alerted to the commonality of their experiences, the paths that students follow, the range of emotion they may encounter and the fact that they, with perseverance, emerge at the other end of the course. A number of examples such as that in Figure 1 are shared.

Reflecting on one's current learning

During tests: It is the middle of a Maths test and the tension can be felt. Students are asked to stop, close their eyes and note their anxiety level. They reminded that the tests are to help them assess how much they have learned, why are they anxious?

Mid Course: Similarly, midway through the course at the start of class the instruction is as follows 'Close your eyes, breathe deeply for a few moments...now think about the challenges you have to face in learning your Mathematics and Statistics...pause...What are they...pause...Take a few minutes to write them down. Would anyone care to share what your challenges are and the strategies you are using to overcome them?' Strategies to deal with problems such as 'My partner is complaining that I spend to much time studying' are identified. Strategies such as 'I promised my partner that I would take a week holiday when I finished'. Other strategies to deal with the difficulty in understanding the Mathematics may include 'finding another textbook' or 'doing some examples everyday'.

Morning Tea: A scheduled 'morning tea'/ social gathering to focus on learning issues was included in one implementation.

Assessment for Learning

What students learn is often influenced by the nature of the assessment tasks. The assessment tasks in this course are an integral to the learning process. Full marks are given for three or four pieces of homework, if submitted. They are to provide students with feedback as to the quality of their work, clarification of ideas, development of writing in order that they have developed the appropriate quality and depth and breadth of ideas that are required for their major assessment work. In recent classes many students have chosen to complete more than the required pieces of assessment. The accompanying activity sheets will often suggest that students reflect not only upon the activities conducted in class but the experience of doing them, for example, "what did you learn about yourself whilst working in a group situation?"

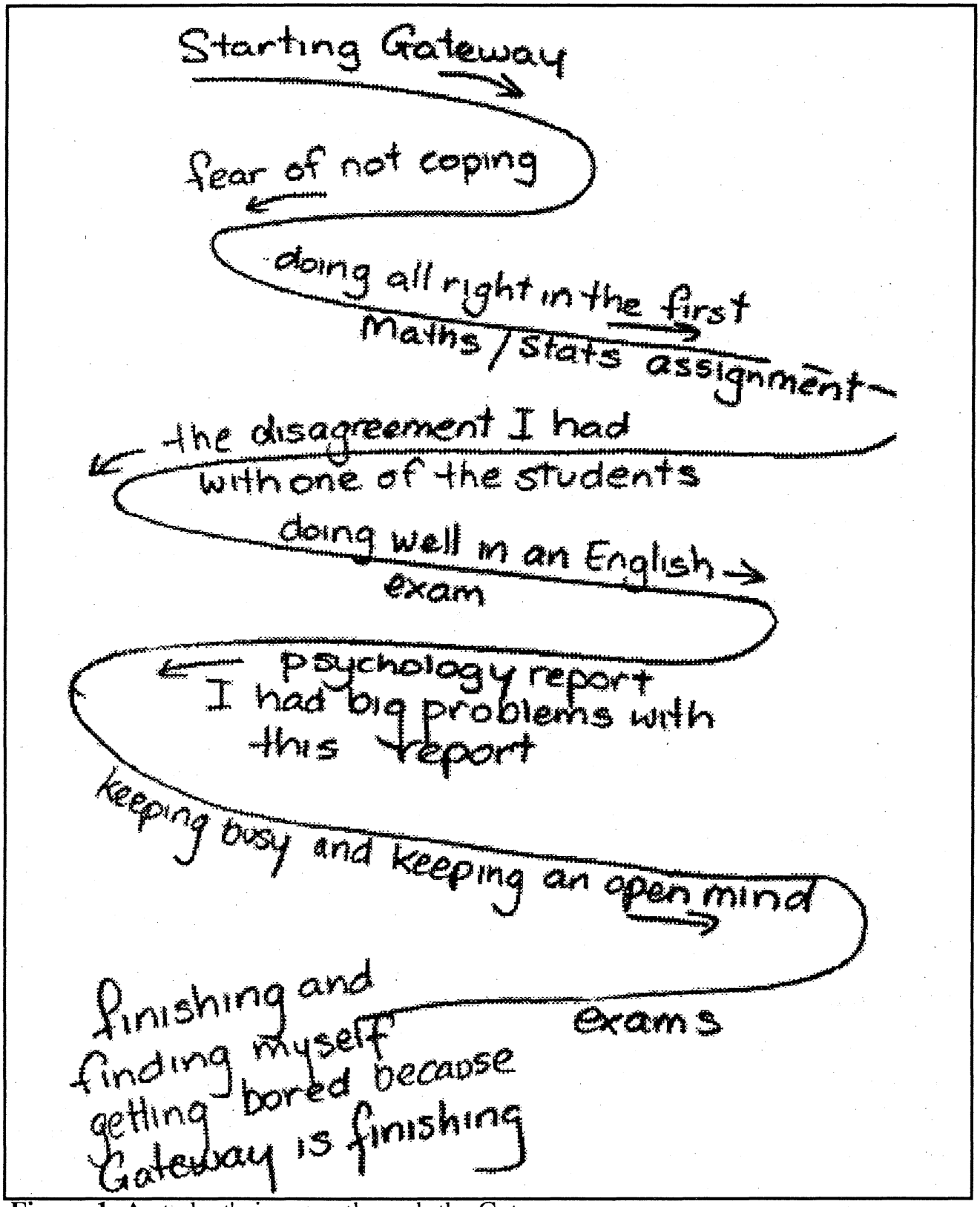

Figure 1. A student's journey through the Gateway program

Class Discussion

How are you going? Student homework in these courses has required that students reflect upon what has transpired in the week's statistical activity (the statistical exercises chosen provoke a wealth of statistical ideas and data), what ideas were generated, what procedures they used and why they did so? How are you going? What was the purpose of the work last week? are good starter questions to get students to enter the debate or provide information regarding the state of their own learning. A recent example of a

discussion, after a peer assessment session involved a student asking the question, 'Will you be giving us written feedback on our assignments?" Not satisfied with peer assessment, following a teacher generated set of criteria, observing other assignments which could provide ideas as to both content and layout, many students wanted to be told what was required by the teacher. This question was re-directed to illustrate a introductory week's discussion regarding the need for many students to be told precisely what to learn rather than to learn to select for themselves what is important in a situation, for the latter may invoke feelings of uncertainty and lack of trust in oneself. In the ensuing discussion some cited their awareness of the requirement that as part of the homework and peer assessment they were to identify for themselves what they believed was appropriate and to learn from the situation. Others become aware that they had self censored in their writing and learning "I didn't know you would want graphs or that we could put in our own data" . They had attempted to give the teacher what they thought the teacher wanted rather than responding to their own sense of appropriateness of material.

Providing a learning commentary during class exercises

Evidence of thinking statistically: Several of the class exercises provide the teacher with an opportunity to challenge students perceptions of themselves as mathematical or statistical thinkers. One exercise, involves students working through a worksheet developing ideas of location and spread, using both given criteria and self developed criteria to assess which they consider to be a good measure. When the answers are obtained, for example a zero for the sum of the deviation scores, or a large number for the sums of squares or an incomprehensible variance, students think they are wrong or stupid because they do not understand. Drawing attention to their thinking, indicating how it is that they are right and how they perceived the need for a better measure such as the standard deviation which they could understand, can be used to affirm the students as statistical thinkers.

Teacher talk: *T*eacher talk can help. My own experience was of saying to the Professor 'now why do you use this technique instead of that' only to be told what I already understood 'well the first is easier to understand'. That teachers can be waylaid by the same thoughts, the same self doubts as the novices can be instructive.

Developing the teaching role

Using knowledge of the learning experience

Knowing that students experience anxiety, self doubt or uncertainty during the process of learning has not necessarily been used as an excuse to remove the experience. Teachers in this program consider that one of the most important aspects of learning is learning how to make decisions about selecting information from the volumes of information available. Those decisions take place in an atmosphere of uncertainty, unlike the task of rote learning material that has been ordained as needing to be learned. An important aspect of learning occurs when one is not told what to learn, but asked to identify what is important in a situation and justify why it is important. One of the classes in this subject involves students in sorting two sets of seashells in order to determine if there was a difference caused by a maritime accident? In exercises such as this students learn about statistical issues such as measurement, the need to operationalise definitions and how comparisons may be made. In addition students may learn about how their learning is motivated. Student involvement in this task generally mirrors students' preparedness to make decisions about data. They start hesitantly, then have period of active sorting and then ask 'what does one do with the broken shells?' or 'what do we do with these shells when there are only a couple?' Decisions are required on the part of the teams, they are decisions which are seemingly too hard involving too many choices and it is often at this point that some members will feel suddenly bored, unable to continue, in conflict with others. The parallel with students undertaking thesis work and the decision to quit or continue is striking, the question as to whether or not to continue the search for solution is one often asked by students when they experience self doubt or a lack of

confidence. These are learning issues to be discussed.

Developing learning theory

For a teacher the process of examining what students have constructed, how they have learned and asking 'how could they have come to that construction?' is an illuminative process. It enables the development of personal theories of teaching and learning. For example, using the constructivist notion that reflection is an important part of constructing knowledge, students in these classes were asked to reflect upon what has transpired in class and to write about it. Little was I aware that for some the process of reflecting was one that in the classroom, at least, needed to be learned. Students needed to learn to focus on a coverage of important ideas rather than easy or hard ones, they needed to know how to remain focussed on the statistical ideas rather than be diverted to for example the enviromental theme embedded in a given topic. They needed to learn to focus not only on the detail of their own data but to recognise the generalisations possible from all data gathered. They needed to identify that the inability to write was not necessarily the inability to have good thoughts about the topic. For this teacher asking 'why did this student write this as homework?' and attempting to find the cause, rather than dismissing it as the student's problem or lack has led to many changes in teaching practice in order to accomplish certain ends. Developments in the teacher's personal theory of teaching and learning have occurred. An increased vitality in teaching has been experienced as each piece of assessment (once drudgery to mark) has become an new piece of data affirming or denying conjectures about how learning is taking place and suggesting further changes to practice.

Conclusions: Which exercises and when?

Since the first three implementations were completed several other groups of students have been taught statistics in a learning focussed manner. These groups include: more tertiary preparation students, Arts students, Law students; and staff and students in non credit introductory statistics workshops. Reflective practice, observing the impact of activities upon students, getting to know the interests of students groups, constantly listening for students' voices, asking them for their reactions, observing body language will all direct the extent to which the learning focus should be maintained and which exercises are best undertaken. Student commentary suggests that it is no one exercise that is instrumental in achieving that connecting experience, different students have insights with different exercises. A variety of interactions is warranted. What does appear important is that throughout the course there is a safe and accepting environment where there is ample opportunity for discussion of the subject matter and the experience of learning it. Many comments by students start "It fell into place when I overheard...".

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