Creating and Maintaining Knowledge-Building Communities of Practice During Mathematical Investigations

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In this paper, we describe a Pilot Study in which we investigated how a computer-mediated collaborative learning environment called CSILE could be used to mediate the establishment and the maintenance of a knowledge-building community of mathematics practice within two elementary school classrooms. We conclude that if CSILE is to successfully mediate knowledge-building during mathematical investigations, then it needs a human-computer interface which: (1) preserves the concreteness and the spontaneity of the students' synchronous, face-to-face investigative work and (2) helps to recreate or maintain the sense of community of practice that is engendered during face-to-face investigative inquiry.

Introduction

Within education, there are current trends towards thinking about learning and knowing as social as well as individual activities (Cobb, 1994; Lampert, Rittenhouse & Crumbaugh, 1995; Pea & Gomez, 1992). Acquiring knowledge is understood as a broadly social practice engaged in with peers and more knowledgeable others (Brown& Palicsar, 1989; Driver, Asoko, Leach, Mortimer & Scott, 1994; Lave & Wenger, 1991). Parallel to this, there have been recent trends within the discipline of mathematics towards viewing the doing of and thinking about mathematics as a social process of debate (Lakatos, 1976) or of shared meanings (Kitcher, 1984). These notions about learning and knowing and about the nature of mathematics suggest that to understand mathematics, one must understand the activities or practice of persons who are makers or users of mathematics, deviating from the more conventional view that understanding mathematics is equivalent to understanding the structure of concepts and the principles in the domain (Stein, Silver & Smith, 1996).

The above viewpoints are reflected in most mathematics education reform documents (e.g., National Council for Teaching Mathematics's *Professional Standards for Teaching Mathematics*; Australian Education Council's *A national statement on mathematics for Australian schools).* These documents place a great emphasis on changing the nature of classroom discourse to include authentic mathematical activity, collaborative mathematical thinking and "talk in the spirit of disciplinary work" (Lampert et al., 1995; Stein et al., 1996).

In order to provide students with authentic mathematical activity, most mathematics reform documents are suggesting that *instructional programs* need to focus far less on computations and much more on the development of number sense and on problem solving, mathematical modelling, and conjecture and proof. Major changes also are being advocated in *classroom practices* such as shifts away *from* (1) classrooms being simply a collection of individuals *towards* classrooms as mathematical communities of practice in which students engage in collaborative mathematical practice - sometimes working with each other in overt ways, and always working with peers and teachers as part of a shared community with shared norms for the practice of mathematical thinking and reasoning; (2) classrooms in which the teacher (or the computer or the textbook) is the sole authority for verification of answers *towards* classrooms where logic and mathematical evidence are used as the basis for verification; (3) merely memorizing algorithms and other procedures *towards* mathematical reasoning; (4) emphasis on mechanistic answer-finding *towards* conjecturing, inventing, problem-finding and problem-solving; and (5) treating mathematics as a corpus of isolated concepts and procedures *towards* connecting mathematics, its ideas and its applications in culture and society.

A great deal of cognitive instructional research seems to indicate that many existing cultures of schooling are quite antithetical to these changes (cf.Scardamalia & Bereiter, 1995). Thus, many educational commentators (e.g., Brown, Ash, Rutherford, Nakagawa, Gordon, & Campione, 1993; Marshall, 1988; and Scardamalia & Bereiter, 1995) are suggesting that in order for changes such as those being advocated by the reform documents to occur, new cultures of schooling need to be developed. One such new culture of schooling conceives of schools as *knowledge-building communities.* A knowledge-building community (e.g., a research team in the scientific community) is a group of individuals dedicated to sharing and advancing the knowledge of the collective. According to Bereiter (1994b), what defines a knowledge-building community is not formal association or physical proximity but rather a commitment amongst its members to invest their resources in the collective pursuit of understanding. Thus, in knowledge-building schools, the students are engaged in producing knowledge objects (e.g., ideas, theories, interpretations etc.) that can be discussed, tested, compared, hypothetically modified and so forth and the students see their main job as producing and improving such objects, not simply the completion of school tasks.

Some classrooms in Canada, the USA and Australia are being restructured into knowledge-building communities. Information technology has been recognized as an important tool for mediating the discourse in these classrooms (Brown et al., 1993; Brown & Campione, 1993; Scardamalia & Bereiter, 1995). However, because most existing information technology-based educational materials (e.g., CAI, ITS, educational games, simulations etc.) tend not to foster either collaborative learning or knowledge-building (Bereiter, 1994a; Nason, Ohlsson & VanLehn, 1995; Ridgway, 1988), new types of educational software such as CSILE have had to be designed for these knowledge-building classrooms.

CSILE (Computer Supported Intentional Learning Environment)

A standard CSILE installation has 6-8 networked computers per classroom, connected to a file server, which maintains the communal database and governs accessibility. The database consists of text and graphical notes, all produced by the students and accessible through database search procedures run locally or over the Internet. CSILE activities do not stand apart from the regular school program the way most computer activities usually do. They are integral to the whole classroom process. When off-line, the students are planning knowledge-building projects, seeking information from a variety of sources, and engaging in small group and whole class discussions of questions, ideas and findings. When on-line, the students are entering and following the plans, entering new information through text and graphic notes, and carrying on more pointed discourse on questions, ideas and findings. Anyone can add a comment to a note or attach a graphic note subordinate to another graphic note; indeed, constructive commenting on other students' notes is proactively encouraged and has been shown to be effective (Woodruff & Brett, 1993). Authors are notified when a comment has been made on one of their notes. Only authors can edit or delete notes.

The CSILE database, by capturing the learning processes of the students, can provide the above information for the teachers too, for in these classrooms, each computer workstation is connected to a multimedia database that contains the ongoing work of the class. In most classroom-based systems of this sort, each student's files are segregated into a separate folder or "account". CSILE takes the opposite tack and places all files (which in CSILE are called "notes") in a common area, viewable by all. Furthermore, facilities are provided which allow students to connect their notes to the notes of others. This permits people to more easily share information, answer each other's questions, provide advice, and so on. The strength of CSILE is that it objectifies the knowledge of the classroom and makes advancement of that knowledge a social activity. All questions, theories, ideas, information, and discoveries are preserved on the database for the analysis of the entire class. Unlike face-to-face conversation, which is transitory, CSILE provides a permanent record of the community's interactions. Furthermore, it eliminates the need for turn-taking, allowing all students to work simultaneously regardless of their location. CSILE databases can be accessed across the Internet, allowing anyone on the net to participate once they are given accounts and password information. These features permit a type of highly intensive peer collaboration across time and distance that would be impractical and chaotic without computers.

At present, CSILE is being used in a variety of curriculum areas. Anecdotal evidence from teachers using CSILE and from formal evaluation studies indicates that CSILE is excellent for nurturing collaborative learning and communities of practice in social studies, art, history, geography, language arts and science (Brett & Woodruff, 1995; Scardamalia & Bereiter, 1995). Thus, in these subjects, CSILE has been successful in facilitating the process of knowledge-building i.e., the construction of knowledge objects. For example, after completing CSILE-mediated natural science investigations, a group of Grade 7 Canadian elementary school children from a challenging urban setting were able to theorize and generate productive and challenging questions which were rated as being of high significance by practicing scientists (Woodruff & Meyer, 1995).

However, mediating such levels of collaboration, communities of practice and knowledge-building with CSILE in the domain of mathematics is proving to be much more difficult (Brett & Woodruff, 1995; Scardamalia & Bereiter, 1995). When they study mathematics, students usually seem to focus either on the personal (e.g., completing the task) or the utilitarian (e.g., using mathematics as a tool for resolving a real or imagined real-world issue) aspects of the discipline and overlook that mathematics also can produce abstract and/or creative knowledge objects (Baturo & Cooper, 1993). Thus, even when involved in CSILE-mediated mathematical activity, students tend not to collaboratively build mathematical knowledge objects (such as mathematical ideas, theories, interpretations etc) which can be further discussed, compared, tested, modified, rejected and so on. This problem is the focus of the study being reported in this paper.

The Study

In this study, we explored how CSILE could be used to mediate the public construction of mathematical knowledge objects during mathematical investigations. The participants in this study which was conducted in August-December 1995 were three groups of elementary school students at a inner-Toronto school[1] (Grades 3, 5 and 6), their two classroom teachers and researcher Nason. Nason introduced each of the investigation problems to the three groups of students and followed up each investigation by further visits to the classrooms and by CSILE-mediated discourse with the students and their teachers via the CSILE computer network.

The decision to ground these studies in a series of mathematical investigations was based on ontological, epistemological and motivational factors. In mathematical investigations, students are presented with open-ended problems or situations. They are not expected to produce "the right answer" but instead are required to explore possibilities, make conjectures and convince themselves and others of what they have found. The emphasis is on exploring the pieces of mathematics in all directions (Pirie, 1987). Mathematical investigations thus enable students to engage in authentic problem solving, mathematical modelling-type activities and in mathematics-in-the-making rather than being mere absorbers/consumers of mathematics (Onion, Burns, Thorpe, & Williams, 1990; Pirie, 1987). Furthermore, mathematical investigations can: (1) promote enjoyment of mathematics, (2) make math experience accessible to students by demystifying its subject-matter, (3) restore and develop students' faith in their own common sense, (4) provide situations where student-student and student-teacher discourse arise naturally, and (5) increase the students' willingness to 'have a go' since the threat of being wrong no longer hangs bleakly over them (Pirie, 1987). Mathematical investigations thus provide a most conducive context for the process of mathematical knowledge-building.

The major criteria for the selection of problems/issues that were investigated in this study were: (1) their appropriateness for the ages and interests of students, (2) their potential for mathematical knowledge-building, (3) whether the problem/issues could be adequately modelled by the graphic/iconic tools in the software; and (4) their potential for generating data which would inform the further development of the CSILE-mediated mathematics learning environment. One of the problems investigated was "Fiona the Frog" (Adapted from Onion, A., Burns, S., Thorpe, J. & William, D., 1990). In this problem, the children were presented with a picture of Fiona stuck down the bottom of a 10 metre deep well (See Appendix 1). They were told that each hour, she climbs up 1 metre and then falls back 0.5 metre. Their task was to investigate: *How long is it before Fiona is out of the well?* When they felt they had solved this problem, the investigation was extended to the following two questions:

(1) What about different depths of wells?
(2) Can you generate a rule for all depth of wells?

A simpler version of the problem in which Fiona climbs up 2 metres and then falls back 1 metre was available for the Year 3 children and for the upper grade children who reached an impasse with the original problem. Simpler versions were created for all of the problems/issues being investigated.

We conceived of the mathematical investigation process as comprising two components: *inquiry* and *justification*. For the most part, the *inquiry component* of each investigation was conducted via face-to-face discourse in whole-class and/or small group contexts (cf., Lampert et al.). The *justification component* was addressed when the students created discussion notes and subdiscussions on the CSILE database. The CSILE-mediated discourse focussed on: (1) the students' understanding of the problem being investigated; (2) what they did; (3) their solution(s) to the problem (and why they thought their solutions were good); (4) how might they change the problem; and (5) what had they learned from doing the investigation. They were also encouraged to make comments and ask questions about other students' notes and negotiate with other students about how the investigation should proceed. We hypothesized that these activities would help reinforce the community of practice that had been nurtured during the inquiry component. We also hypothesized that the asynchronous nature of these activities would facilitate reflection and further knowledge-building by all students but especially amongst those who need time for reflection and/or feel intimidated presenting "first-draft versions" of their ideas in synchronous, face-to-face situations.

During each of the investigations, data was collected and analysed from the following sources: the*CSILE Database notes, interviews of both the students and their teachers, ethnographic observation of the classrooms* and *samples of the students' work.* The CSILE data was qualitatively analysed in order to assess changes in the students': (a) patterns of collaboration, (b) quality of their mathematical knowledge-building, and (c) understanding of the nature and discourse of mathematics. The interviews of the students focussed on growth in the levels of subject-matter knowledge, and changes in practice, reflection and collaboration during the mathematical investigations. The interviews of the teachers conducted focussed on their perceptions about: (a) students' experiences during the investigations, and (b) changes in students' practice, reflection and collaboration during the mathematical investigations. The ethnographic observation of the classrooms focussed on: (a) how the teachers and the students utilized CSILE during the investigations, and (b) changes in students' practice, reflection and collaboration. Samples of the students' work were collected and analysed in order to ascertain: (a) growths in their levels of subject-matter knowledge, and (b) the quality and nature of the products of their mathematical investigations.

Findings

During the *inquiry component* of most of the mathematical investigations, we observed high levels of motivation and knowledge-building by all three groups of students. For example, in the "Fiona the Frog" investigation, all groups of children were able to create and utilize pictorial models, conduct animated discourse based around their pictorial models, and generate not only the solution to the original problem but also general rules for similar problems (e.g., wells with different depths and/or Fiona having different rates of climb and fall). The Year 3 children produced verbal rules; many of the Year 5 and 6 students were able to generate tabular and/or algebraic rules.

However, we found that the students were unable to maintain and/or transfer their sense of community and their high levels of mathematizing and knowledge-building into the electronic medium during the *justification component* of the investigations. On reflection, we felt that the limited success of the CSILE-mediated discourse could be attributed to three factors. First, when the students entered the CSILE environment, they had to leave behind any concrete/pictorial representations of the problem they might have been using during their face-to-face investigative inquiry, or at best, crudely draw those with CSILE's current graphics tools. Second, they had to turn the synthesis of a primarily oral/iconic process into a written text based form, with all the awareness of communication that writing takes. Because of these two factors, the students tended to have great difficulty in describing what they had done, justifying their solutions, explaining how the problem could be changed and in articulating what they had learned etc. Third, when they entered the CSILE environment, they had to leave behind the whole class/small group community which had been the impetus for much of the enthusiasm generated in the face-to-face work. It seemed that maintaining that sense of community required a powerful sense of purpose and direction, as well as skill in one's own metacognitive evaluation.

Discussion

From our findings, we concluded that if a knowledge-building software tool such as CSILE was to mediate knowledge-building successfully during mathematical investigations, it needed an interface which: (1) preserves the concreteness and the spontaneity of the students' synchronous, face-to-face investigative work and (2) helps to recreate or maintain the sense of community of practice that is engendered during face-to-face investigative inquiry.

We summized that the interface should include the following components: (1) a set of iconic mathematical tools, (2) a set of mathematical investigation knowledge-building scaffolds, and (3) a compact graphics design tablet and software package(e.g., *artPad II-Dabbler2*).

We feel that a set of iconic mathematical tools such as:

(1) Base 10 Dienes diagrams which can be composed and decomposed;
(2) a place value chart generator;
(3) a number line generator;
(4) balloon, dot, real-world and hopper/loop markers;
(5) geometrical shape drawers;
(6) slicers (e.g., for halves, thirds etc);
(7) tree and array model builders;
(8) an integrated spreadsheet and graph-making tool;
(9) a magnifying/contracting zooming tool;
(10) a stamping tool for the storage and reproduction of drawings, models and diagrams created by the students; and
(11) a tool that enables students to retrieve notes (including both "doodlings" and solutions/justifications) from previous mathematical investigations

would help to preserve the concreteness and spontaneity of the students' face-to-face investigative work and do much to maintain the sense of community. This belief is based on our observations of and discussions with the teachers and students involved in the study, insights gained from the research and development of iconic mathematical tools in the Australian Research Council-funded MENO Project (Nason & Martin, 1994), and from Kurt VanLehn's OLAE project (Martin & VanLehn, 1995) in which he found that giving students access to previously worked problems greatly assisted student learning by analogy.

In this study, we found that CSILE's present set of knowledge-building scaffolds did not provide an adequate structure for the consolidation and reconstruction of the students' inquiry actions or thinking. We feel that the interface needs both *inquiry* scaffolds(such as OUR PICTURE OF THE PROBLEM, WHAT WE HAVE TRIED SO FAR, and WHAT WE NEED TO KNOW) and *Justification* scaffolds (such as OUR PICTURE OF THE PROBLEM, WHAT WE DID, OUR SOLUTION(S) AND WHY WE THINK THEY ARE GOOD, HOW MIGHT WE CHANGE THE PROBLEM, and NEW LEARNING). The ideas for these scaffolds have been derived derived primarily from CSILE's present set of scaffolds, from research and curriculum literature about mathematical investigations (e.g., Lampert et al., Onions et al., Pirie) and from discussions conducted with the teachers in this study. We believe that the Inquiry scaffolds would facilitate smooth transitions between inquiry and justification during an investigation and together with the iconic tools help to maintain the direction and impetus of a mathematical investigation. We believe that the Justification scaffolds would nurture not only the consolidation and the refinement of knowledge objects built during the inquiry component of an investigation but also the building of new mathematical knowledge objects by both the teachers and the students whilst they are engaged in the process of justification.

In this study, we also observed that much of the animated and productive discourse that occurred during face-to-face investigative inquiry seemed to revolve around doodlings and drawings done by the students on sheets of paper or on the chalkboard. However, the students experienced great difficulty in transferring these doodlings and drawings onto the CSILE windows with the mouse and the keyboard and that what appeared in the windows seemed to lack the richness, insight, creativity and inclusiveness of their hand-made doodlings and drawings. Their CSILE notes thus tended not to encourage significant discourse. Compact graphic tablets and software packages (such as *artPadII-Dabbler2*) enable students to draw, doodle, paint or sketch as easily and as naturally as they do with pen, pencil, crayons or chalk. The inclusion of a graphic tablet and software package within the interface thus would make the process of transferring doodlings and drawings onto the computer screen much easier (or it would encourage groups of students to do their original doodlings and drawings on the computer screen). We therefore believe that the inclusion of a graphics tablet and software package into the interface would do much to maintain the spontaneity and concreteness of the students' synchronous face-to-face investigative work. Furthermore, in conjunction with the iconic mathematical tools, we believe that it would make it easier for students to conceptualize, modify and present their picture of the problem and comment on other students' pictures and/or conceptualizations of the problem. This should contribute much towards maintaining the sense of a knowledge-building community.

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Appendix 1

Mathematics Investigation 4A: Fiona Frog (from Onions et al., 1990)

Fiona is at the bottom of a well 10 metres deep.

Each hour, she climbs up 1 metre then fall back 0.5 metre.

Your task is to investigate:

How long is it before Fiona is out of the well?

Extensions

1. What about different depths of wells?
2. Can you generate a rule for all depth of wells?