Assessing the Impact of Graphics Calculators on Mathematics Examinations

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Graphics calculators will be allowed in VCE mathematics examinations from 1997, but questions must be set so that candidates who only have access to a normal scientific calculator are not disadvantaged. A sample of mathematics teachers, and two VCE examiners, were asked to assess the impact of the graphics calculator on thirteen multiple-choice questions used in a 1995 VCE mathematics examination. The teachers generally agreed with the assessments of the examiners, but where clear differences existed, tended to attribute less potential advantage to a graphics calculator user than did the examiners.

Introduction

In February 1995, the Victorian Board of Studies approved the use of graphics calculators in some year 12 Victorian Certificate of Education (VCE) mathematics examinations to take effect from 1997 (Board of Studies, 1995a). More recently, it has decided that graphics calculators will be allowed in *all* VCE mathematics examinations from 1997. In July 1995, the Secondary Education Authority of Western Australia announced that graphics calculators could be used in the mathematics Tertiary Entrance Examinations from 1998 onwards (Bradley, 1995). In Victoria only graphics calculators without symbolic processing capabilities have been approved, but Western Australia will allow the use of graphics calculators with limited symbolic processing capability (such as the HP 38G). Examination boards in other Australian states are also expected to eventually allow graphics calculators in their examinations but have yet to set timelines.

While the decision to allow graphics calculators in public mathematics examinations has been seen as a relatively radical step in Australia, it is merely reflecting a similar move in the UK, where all examination boards have allowed the use of graphics calculators in 'A' level examinations since 1994, and the US, where graphics calculators have been allowed in the Advanced Placement Calculus examination since 1993 and have been required since 1995. However, there does not seem to be a consensus as to the role that graphics calculators should play in examinations. In the UK, some examination boards have set examinations that are 'graphics calculator active'; that is, students are *expected* to have a graphics calculator in the examination and questions are set accordingly. However, other boards *allow* students to have a graphics calculator but set examinations that aim to be 'graphics calculator neutral'; that is, questions are set in a way that does not unduly advantage a student who has a graphics calculator compared to a student who only has access to a normal scientific calculator. In contrast, for the US Advanced Calculus examination which now assumes students have access to a graphics calculator, part of the examination is designed to be calculator active while the rest of the examination forbids the use of *any* calculator. In instructing examiners to set questions 'such that students are not advantaged by having a graphics calculator' (Board of Studies, 1995a, p. 14), the Victorian Board of Studies requires that, at least for the time being, VCE mathematics examinations will be graphics calculator neutral. Furthermore, this has to be done within the constraints of the existing syllabus. But how is this best achieved?

Unfortunately, in this regard, the literature on the use of graphics calculators in examining has mainly been concerned to date with identifying the associated problems (for example: Harvey, 1990; Boers & Jones, 1994; Kissane, Bradley & Kemp, 1994; Jones, 1995; Kemp, Kissane & Bradley, 1995; McCrae, 1995; Taylor, 1995; Tobin, 1995) and, in particular, offers little guidance as to how graphics calculator neutral examinations might be constructed. One possible and reasonably obvious approach, which has been adopted by the authors, is to start with the existing examination papers and see what difference having access to a graphics calculator might make. In analysing the 1994 VCE Mathematical Methods 3/4[1] Common Assessment Tasks 2 and 3 (Board of Studies, 1994a & 1994b), Jones (1995) found that the questions on these examinations ('CATs') appear to fall into three reasonably distinct categories. Firstly, there are those questions on which the graphics calculator could be classified as having *no impact* because it either contributes nothing to the completion of the task or no more than a scientific calculator. The second category of questions are those on which the graphics calculator *impacts*, by providing the user with an alternative, but still mathematically valid, method of solution. Finally, there are a category of questions which the graphics calculator effectively *trivialises* because it provides an alternative method of solution that requires little or no mathematical input from the user. In Jones' analysis, the focus was on determining the potential of the graphics calculator to assist in the *generation* of answers to questions. It ignored the potential of the graphics calculator as a device for *checking* answers generated by non-graphics calculator methods (see, for example, Kissane, Bradley & Kemp, 1994).

Using the same categorisation scheme, McCrae (1995) extended Jones' analysis to a second VCE mathematics subject, Specialist Mathematics[2], and produced table 1 which shows in summary form the percentage of marks that would have been affected if graphics calculators had been allowed in the 1994 VCE mathematics examinations[3].

Table 1. *Percentage of marks affected by graphics calculator use* (McCrae, 1995)

Examination	No impact	Impacts	Trivialises
Mathematical Methods CAT 2 (Multiple-choice)	69.7	12.1	18.2
Mathematical Methods CAT 2 (Short-answer)	35.3	64.7	0
Mathematical Methods CAT 2 (overall)	58.0	30.0	12.0
Mathematical Methods CAT 3	61.7	38.3	0
Specialist Mathematics CAT 2 (Multiple-choice)	90.9	3.0	6.1
Specialist Mathematics CAT 2 (Short-answer)	100	0	0
Specialist Mathematics CAT 2 (overall)	94.0	2.0	4.0
Specialist Mathematics CAT 3	83.3	10.0	6.7

On the basis of these results, it would appear that, while graphics calculators do have the potential to impact on a significant proportion of the questions currently used to assess Mathematical Methods 3/4 and Specialist Mathematics, the effect has not been to been to trivialise the majority of questions, but rather to broaden the methods available to answer many questions. As long as the potential impact on content validity is taken into account, such questions would appear to have a place in an examination which permits the use of graphics calculators. There are, though, a small number of question types that would be trivialised and which would no longer be useable in their present form in such an examination. But do VCE mathematics teachers agree with this assessment? And are there other types of questions that would need to be modified or replaced if the examination is to be graphics calculator *neutral*? These are the concerns of the present paper.

Method

The thirteen calculus multiple-choice questions from the 1995 Mathematical Methods 3/4 CAT 2 (Board of Studies, 1995b) were independently analysed by the authors, both of whom are VCE mathematics examiners[4], and twenty-seven VCE mathematics teachers. The calculus Mathematical Methods multiple-choice questions were chosen for analysis because in the previous year the corresponding questions had shown themselves to be the questions most likely to be trivialised by the introduction of the graphics calculator (see table 1). The teachers in the study were not chosen at random but were volunteers from schools where graphics calculators were known to be in regular use. The authors and twenty-four of the teachers were TI-82 users, two teachers were Casio fx-9700 users and one was a Sharp EL-9300 user. Although these three graphics calculators have similar capabilities, it was decided to exclude the three non-TI users from the sample so that there was no variation in calculator useage. Of the twenty-four teachers who were TI-82 users, two rated themselves as barely familiar with the non-statistics capabilities of the TI-82, five as somewhat familiar, thirteen as reasonably familiar and six as very familiar. The authors both rated themselves as being very familiar with the TI-82. The TI-82 has what can be regarded at present as standard features for a graphics calculator that will be allowed to be used in VCE mathematics examinations. The features relevant to the current investigation include the ability to draw and analyse graphs, evaluate derivatives and definite integrals numerically, and solve equations numerically. It does not have symbolic processing capability.

The questionnaire given to the teachers asked them to make an assessment of the impact of the graphics calculator on the thirteen multiple-choice questions using the following three categories:

A. The availability of a graphics calculator would have no impact on the question

B. The availability of a graphics calculator would have an impact, but the question could remain unchanged

C. The availability of a graphics calculator would have an impact and the question would need to be modified or replaced

These categories broadly follow those used by the authors to classify questions on the 1994 examination papers, but avoid giving a reason why the respondent might put the question in a particular category. The reasons for the teachers' classifications were solicited separately on the questionnaire and the preamble made it clear that 'impact on the question' meant whether a graphical calculator user would have an advantage over a student who only had access to a non-graphics, scientific calculator.

Results

For most questions, the classification assigned varied from respondent to respondent. A summary of the teachers' responses is given in table 2, together with those of the authors (the 'examiners').

Table 2. *Summary of teachers' and examiners' classifications of each question*

Question:	1	2	3	4	5	6	7	8	9	10	11	12	13
Category A	1	0	24	1	0	22	0	0	0	8	11	19	15
B	9	13	0	14	6	0	2	1	11	12	11	2	3
C	14	11	0	7	18	0	21	21	13	4	2	1	4
Examiner 1	C	B	A	C	C	A	C	C	C	B	B	B	A
2	C	C	A	C	C	A	C	C	C	C	C	B	C

To assist with data interpretation, the alphabetic categories A, B and C were numerically rated as A =1, B = 2 and C = 3. Thus, the higher the numerical rating given to a question, the greater the potential advantage attributed to the graphics calculator user on that question. An 'average' rating was then obtained for each question, for both the teachers and the examiners, and these were plotted against question number as parallel line graphs: one line for the teachers' ratings and one for the examiners' ratings; see figure 1.

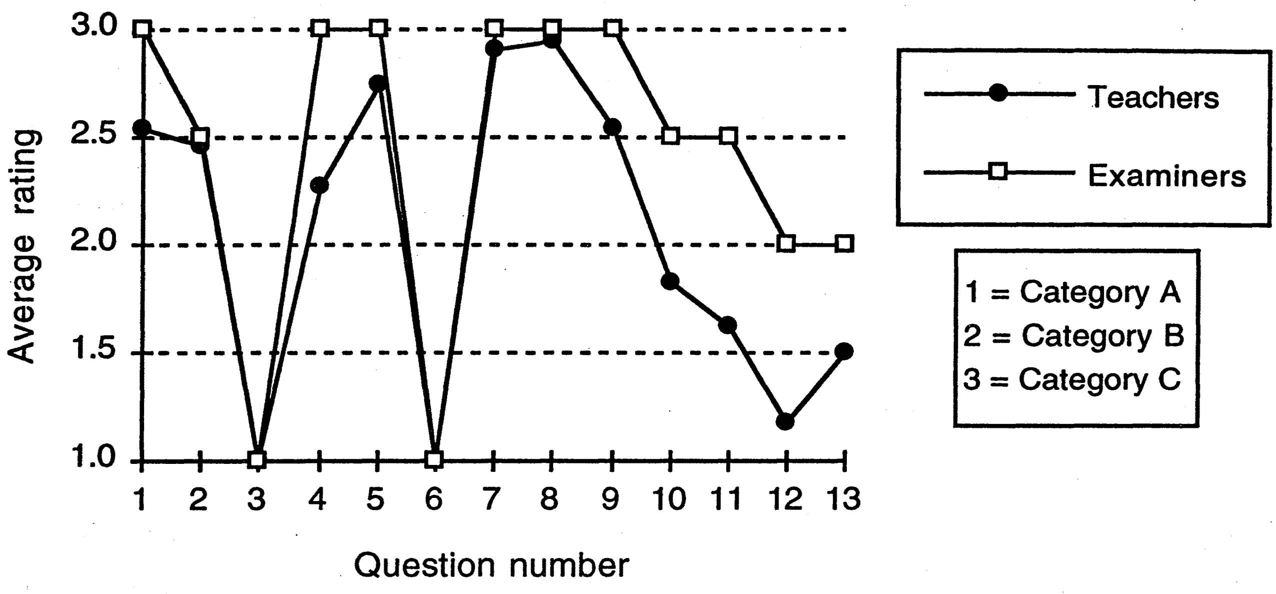

Figure 1. Parallel line graphs displaying average rating given to each question by the examiners and the teachers.

Analysis of Results

The overall pattern of results evident in figure 1 shows that, in general, the teachers and the examiners rated questions in a similar manner, but where differences existed, teachers tended to attribute less potential advantage to the graphics calculator user than did the examiners. According to the teachers' average ratings, at least three and up to six of the calculus multiple-choice questions on the 1995 Mathematical Methods CAT 2 could not be included on a graphics calculator neutral examination. By comparison, the examiners would be more conservative and exclude between six and nine of the questions[5].

Rating Agreement

Two of the questions (3 & 6) were rated A (graphics calculator no advantage) by all respondents, while three of the questions (5, 7 & 8) were rated C (graphics calculator a clear advantage) by most respondents. Typical of the latter group of questions is question 8 which asks candidates to select, from five alternative graphs, the one that best represents the graph with equation $y = 3\sin(2x) - 1$. Having a graphics calculator would clearly be an advantage in answering such a question, since little or no mathematical knowledge is required by the user to generate the desired graph .

Rating Disagreement

Clear differences (more than half a category) in the *average* ratings of the teachers and the examiners occurred for four of the remaining eight questions (namely questions 4, 10, 11 & 12). However, close inspection of table 2 shows that there are significant differences in the ratings of the individual teachers, either between themselves or with the examiners, for *each* of these eight questions. An examination of the reasons given by teachers for their ratings indicates that these questions can be classified into three groups for the purpose of our analysis.

The first group comprises questions 1, 2 and 4. Question 2 is representative of this group and is reproduced in figure 2. This question was classified as B by about half (54%) of the teachers and as C by the remaining teachers (46%), with a typical reason for choosing category B being that 'graphing each alternative would take much longer'. The teachers who chose category C also seemed to assume that a 'trial and error' approach would be used with a graphics calculator, but evidently weren't concerned about the time factor. Surprisingly, only one teacher suggested that an advantage of a graphics calculator would be that it could be used to verify an answer obtained in another way.

If the graph of $f: R \rightarrow R$ crosses the x-axis exactly three times, which one of the following rules could *not* be the rule for f?

A. $f(x) = x(x^2 - 4)$ B. $f(x) = x(x - 2)(x + 4)(x^2 + 1)$
C. $f(x) = (3 - x)(x^4 - 16)$ D. $f(x) = (x^2 - x - 6)(x - 4)$
E. $f(x) = (x^2 - x - 6)(x^2 - x - 12)$

Figure 2. Question 2, Mathematical Methods 3/4 Part I (Board of Studies, 1995b)

Question 4 was one of the questions in which there was a clear difference in teachers' and examiners' ratings, with the teachers favouring category B (graphics calculator some advantage, but no need to alter the question) whereas both examiners felt a graphics calculator would be of such an advantage as to require a change to the question (category C). This question is like questions 1 and 2 in that it shows a graphical relationship between quantities x and y and asks candidates to select the equation relating x and y from five alternatives. However, it is distinctive in that each alternative gives the *form* of an equation, for example $y = a \cos(bx)$ where a and b are positive constants, rather than a specific equation.

Questions 9, 10 and 11 constitute the second group of questions. Question 10, see figure 3, was the only question (out of the whole thirteen) for which the teachers' ratings were fairly evenly spread across all three categories, with 33% choosing A, 50% B and 17% C. Clearly, a graphics calculator could be used to generate the graph of the function, but a knowledge of terminology and some interpretive skills are still needed to obtain the answer. The variability in the teachers' ratings reflects the different weightings they gave to these factors. A typical reason given for choosing category B was that 'they could graph it but students would still need to know the terminology'.

A trigonometric function is given by $f: R \rightarrow R$ where $f(x) = 3\cos(2(x-\pi)) + 1$. The amplitude, period and range, respectively, of the function are

	amplitude	period	range
A.	3	π	R
B.	2	$\frac{2\pi}{3}$	[–4, 4]
C.	2	$\frac{2\pi}{3}$	R
D.	π	3	[–2, 4]
E.	3	π	[–2, 4]

Figure 3. Question 10, Mathematical Methods 3/4 Part I (Board of Studies, 1995b)

Similar considerations were evident in the rating of question 11, reproduced in figure 4. Here, a couple of teachers (8%) chose category C with the remainder equally split (46%) between categories A and B. A typical comment was that 'the calculator is an aid but interpretive skills needed'. No teacher suggested that the ability to graphically check one's answer would give a graphics calculator user an advantage. There was a clear difference in the average ratings of teachers and examiners on both questions 10 and 11, but once again in each case the teachers were more inclined to allow the question to remain unchanged than were the examiners.

If $f(x) = a \cos x + c$, where c is a positive real number, then $f(x) < 0$ for all real values of x if

A. $c > a$ B. $c < -a$ C. $c = 0$ D. $-a < c < a$ E. $c > -a$

Figure 4. Question 11, Mathematical Methods 3/4 Part I (Board of Studies, 1995b)

The last group of questions for analysis consists of the two remaining questions, numbers 12 and 13, both of which involve differentiation. Question 12, which is reproduced in figure 5, was rated as B by both examiners but as A by most (87%) of the responding teachers. This question could be answered with the support of a graphics calculator because the *graph* of the answer can be generated. Alternatively, the answer is identified if the numerical derivative is calculated at $x = 1$, since each option clearly has a different value at that point. It is possible that some teachers recognised these possibilities but considered them either too time-consuming or too sophisticated to constitute an advantage to graphics calculator users. However, there was no evidence of this in the teachers' comments and it is more likely that the graphics calculator's (numerical) differentiation capabilities were unknown to most of the teachers—something that will obviously change with increased useage.

The derivative of $2\sqrt{x}$ is equal to

A. $\frac{1}{2\sqrt{x}}$ B. $\frac{1}{4\sqrt{x}}$ C. $\frac{1}{\sqrt{x}}$ D. $\frac{4x^{3/2}}{3}$ E. 2

Figure 5. Question 12, Mathematical Methods 3/4 Part I (Board of Studies, 1995b)

Question 13 shows a graph of an 'unknown' function f and asks candidates to select the graph of the derived function f' from five alternatives. However, the graph is easily recognisable as that of $\sin x$ (on a restricted domain) and so a graphics calculator could be used to generate the answer and this is why one examiner rated it as category C. The other examiner rated it as an A, agreeing with the teacher who argued that if a student recognises that f is the sine function, then the answer is essentially given on the formula sheet anyway.

Discussion and Conclusion

From the analysis of the results, it would appear that the sample of teachers generally agreed with the way in which the examiners rated the impact of graphics calculators on the thirteen multiple-choice questions used in this study. Where clear differences in ratings occurred, the teachers tended to attribute less potential advantage to a graphics calculator user than did the examiners who tended to take a more conservative stance. In particular, if a graphics calculator could be of assistance in answering a question but considerable (mathematical) interpretive skills were still needed to obtain the answer, then the teachers did *not* regard the question as being unsuitable for inclusion on a graphics calculator neutral examination.

Although there is often the potential for a graphics calculator to be used for checking answers obtained by other means in the sort of questions analysed here, explicit mention of this aspect of the potential impact of graphics calculators was deliberately omitted from the explanation of the categorisation scheme given to the teachers. However, the opportunity existed for them to recognise this potential when they commented on their reasons for rating each question, but few did so. This may be because they generally failed to recognise this potential—in contrast to one of the more experienced graphics calculator users in the sample who added an overall comment that 'much of the advantage to the students is that they confirm part or all of their work'.

Alternatively, it may be that the teachers tended to discount the checking potential of the graphics calculator because of the critical factor that time plays in multiple-choice testing in VCE mathematics (there is an average of less than two minutes available to answer each question) and the mathematical sophistication needed to implement some checking methods.

Further research in the area is clearly needed, particularly as teachers become more experienced and skilful in the use of graphics calculators and relevant utility programs become readily-available for the various models. This research also needs to include an investigation of the impact of graphics calculators on *extended-answer* questions. You are often required to solve an equation or evaluate an integral as part of such questions and these procedures can be performed graphically and/or numerically on graphics calculators. There may also be more opportunity to check answers in these longer questions, with time not being as critical a factor, and more incentive since (unlike with multiple-choice questions) the correct answer is not generally given.

In conclusion, from the analysis presented in this paper, it would appear that the scheme used by the authors to categorise the potential impact of the graphics calculator on the sort of multiple-choice questions used in current VCE mathematics examinations yields similar results for both teachers and examiners, and that where ratings of questions do clearly differ the examiners tend to be more conservative. Examiners regard fewer of the current questions as being suitable, in their present form, for a graphics calculator neutral examination than do teachers. We must not be too distracted, however, by the pursuit of graphics calculator *neutral* examinations. To ensure that the great potential of graphics calculators is fully realised in the teaching and learning of mathematics, it is important that mathematics examinations become graphics calculator *active* as soon as possible.

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[1]Mathematical Methods 3/4 is a calculus-based subject, with some statistics and probability content. It is a prerequisite for almost all mathematics-related university courses in Victoria.

[2]Specialist Mathematics is a second calculus-based mathematics subject which is taken, in addition to Mathematical Methods 3/4, by students wishing to specialise in mathematics. It is a prerequisite for some tertiary engineering courses.

[3]There is a third VCE mathematics subject, Further Mathematics, which is a non-calculus based subject for students not needing mathematics in future years. No analysis is included for this subject because graphics calculators were found to have only minimal impact on the CATs.

[4]Jones is chief examiner for Further Mathematics and McCrae is chief examiner for Specialist Mathematics.

[5]Note that, for comparison with the first row of table 1, in both 1994 and 1995 there were thirty-three multiple-choice questions on the Mathematical Methods CAT 2, each worth one mark. Hence 3, 6 and 9 questions represent 9%, 18% and 27% respectively of the available marks.

References

Board of Studies. (1994a). *Mathematical Methods 3/4 Common Assessment Task 2: Facts, skills and applications task.* Carlton, Victoria: Author.

Board of Studies. (1994b). *Mathematical Methods 3/4 Common Assessment Task 3: Analysis task.* Carlton, Victoria: Author.

Board of Studies. (1995a, March). Mathematics. *VCE Bulletin*, pp. 13–14.

Board of Studies. (1995b). *Mathematical Methods 3/4 Common Assessment Task 2: Facts, skills and applications task.* Carlton, Victoria: Author.

Boers, M. & Jones, P. (1994). Students' use of graphics calculators under examination conditions. *International Journal of Mathematical Education in Science and Technology* *25*(4), 491–516.

Bradley, J. (1995, August). The impact of graphics calculators upon 'A' level mathematics examinations in England, Wales and Northern Ireland and public mathematics examinations in the USA (Report to the Secondary Education Authority of Western Australia). Perth: Murdoch University.

Harvey, J. (1990). Changes in pedagogy and testing when using technologies in college-level mathematics courses. In F. Demana & B. Waits (Eds.), *Proceedings of the Conference on Technology in Collegiate Mathematics*, (pp. 40–54). Columbus, New York: Addison-Wesley.

Jones, P. (1995). Graphics calculators in traditional year 12 mathematics testing. In A. Richards (Ed.), *FLAIR: Proceedings of the 15th Biennial Conference of the Australian Association of Mathematics Teachers* (pp. 221–227). Darwin, Northern Territory: Australian Association of Mathematics Teachers.

Kemp, M., Kissane, B. & Bradley, J. (1995). Assessment and the graphics calculator. In A. Richards (Ed.), *FLAIR: Proceedings of the 15th Biennial Conference of the Australian Association of Mathematics Teachers* (pp. 235–241). Darwin, Northern Territory: Australian Association of Mathematics Teachers.

Kissane, B., Bradley, J. & Kemp, M. (1994). Graphics calculators, equity and assessment. *Australian Senior Mathematics Journal 8*(2), 31–43.

McCrae, B. (1995). The use of calculators in Victorian Certificate of Education examinations: looking ahead (Preprint No. 10/95). Melbourne: Department of Science and Mathematics Education, University of Melbourne.

Taylor, M. (1995). Calculators and computer algebra systems—their use in mathematics examinations. *The Mathematical Gazette 79*(March) [No. 484], 68–83.

Tobin, P. C. (1995). Technological issues in the assessment of mathematics. In A. Richards (Ed.), *FLAIR: Proceedings of the 15th Biennial Conference of the Australian Association of Mathematics Teachers* (pp. 367–376). Darwin, Northern Territory: Australian Association of Mathematics Teachers.