# Preschoolers' Mathematical Patterning

Marina Papic
*Macquarie University, Sydney*
<marina papic@mq edu au>

Joanne Mulligan
*Macquarie University, Sydney*
<joanne mulligan@mq edu au>

This paper reports a study monitoring the development of 53 preschool children's patterning skills in two matched preschools, in one of which a 6-month intervention promoting patterning was implemented The development of an interview-based assessment comprising identification, repetition, visual recall, extension, transformation, and representation of patterns is described, and a preliminary analysis of the interview data is presented Children participating in the intervention program showed much greater improvement over the course of the 6 months than the non-intervention children

Patterning is critical to the abstraction of mathematical ideas and relationships, and the development of mathematical reasoning in young children (English, 2004; Mulligan, Prescott & Mitchelmore, 2004; Waters, 2004) The integration of patterning in early mathematics learning can promote the development of mathematical modelling, representation and abstraction of mathematical ideas It seems advantageous then, that initiatives in mathematics curricula and assessment in Australia and at international level are promoting the development of early mathematical patterning and reasoning (Clements, 2004; Doig, 2005) For example, the NSW mathematics K-6 syllabus (Board of Studies, NSW, 2002) has recently incorporated a Patterns and Algebra strand that emphasises pattern recognition in a variety of contexts Despite curriculum reforms and recent research interest in early algebraic thinking (Kieran, 2004; Warren, 2003), there is a paucity of research on the development of young children's mathematical patterning, and on the development and effectiveness of early childhood programs promoting patterning skills

## Background to the Study

In mathematics education there is a general consensus that patterning involves "observing, representing and investigating patterns and relationships in social and physical phenomena and between mathematical objects themselves" (Australian Education Council, 1991, p 4) Mathematical patterns encountered in school range from number sequences and spatial arrays to algebraic generalisations and geometrical theorems A pattern may be defined as a numerical or spatial regularity, and the relationship between the various components of a pattern constitute its structure Pattern and structure are thus at the heart of school mathematics Early algebraic thinking in the elementary school may involve the development of thinking skills where the letter symbol is assigned to describe patterns Other activities, for example analysing relationships among quantities, noticing structure, studying change, generalising, problem solving, modelling, justifying, proving, and predicting, can be engaged without using the letter symbol (Keiran, 2004) The conceptual development of 'pattern' in a variety of situations engages these processes

At a more fundamental level, patterning is an essential skill in early learning, particularly in the development of spatial awareness, sequencing and ordering, comparison and classification This includes the ability to identify and describe attributes of objects and similarities and differences between patterns Children's reasoning skills are also considered essential to understanding and applying mathematics; the identification, extension and generalisation of patterns are critical processes of mathematical reasoning (English, 2004, p 13) Mathematical modelling also provides an opportunity for young

children's development of patterning skills, informal sharing of ideas, representation and justification of why patterns are formed The early development of argumentation in this process can lead towards more sophisticated mathematical processes of generalisation and proof in later years (Perry & Dockett, 2002)

## Research on Patterning in Early Mathematics

Research on elementary school children's mathematical concepts and problem-solving processes has integrated the role of patterning in studies on counting (Thomas, Mulligan & Goldin, 2002), number sense and subitising (Bobis, 1996), number patterns using calculators (Groves & Stacey, 1998), analogical reasoning (English, 2004) and problem solving (Diezmann, Watters & English, 2001) However few studies have focused explicitly on young children's development of patterning skills in early childcare settings Waters (2004) conducted a study focused on describing the development of children's knowledge of patterning within the preschool setting Case studies of two preschool programs revealed that while teachers' acknowledged the role of patterning, there were limited worthwhile patterning opportunities for children Waters found that the process whereby children initiated and described their own patterns from basic repetition to spatial surface patterns enriched the development of patterning skills Teachers needed to become more aware of the types, level and complexity of patterns The study suggests that research is needed to provide sound evidence of the power of mathematical patterning in order to validate the inclusion of patterning in early childhood programs, and to contribute to the void of knowledge surrounding pattern development (Waters, 2004, p 571)

Some studies have incorporated patterning as one component of early mathematical development A recent study of first grader's use of pattern and structure showed that the ability to recognise and use pattern generalises across a wide range of mathematical tasks and this could be described as a general cognitive characteristic (Mulligan et al , 2004) Children's identification and representation of the structure of patterns was critical to successful task solution The study indicates that early development of pattern and structure may be critical to improvement in mathematics generally Children's patterning knowledge has also been found to influence the development of analogical reasoning, and the ability to identify, extend and generalise patterns important to inductive reasoning In a longitudinal, cross-cultural study of children's mathematical reasoning, English (2004) employed an attributions and patterning task to assess children's mathematical ability The study advances our knowledge of the development of mathematical reasoning but broader measures of analogical reasoning may provide further insights into links between specific patterning abilities and reasoning

The study of patterning has also been explored through research on early childhood programs for enhancing mathematical development Klein and Starkey (2003) investigated children's patterning abilities as one aspect of the Berkeley Math Readiness Project where children were required to copy, complete and extend linear patterns However the project focused more generally on developing a preschool mathematics curriculum and informing curriculum standards Ginsburg (2002) describes the effectiveness of a comprehensive early childhood mathematics curriculum 'Big Mathematics for Little Kids' The goal is to help children think mathematically beyond the play situation, building on everyday mathematics but incorporating traditional strands of the mathematics curriculum Patterning forms an integral part of such programs but the scope and depth of patterning activities in an early childhood program needs to be informed by research that describes explicitly, the informal and intuitive development of mathematical patterning

A study was therefore designed to describe the development of patterning skills from preschool through to formal schooling by raising two key research questions:

- Is there a link between a child's ability to pattern and their development of pre-algebraic and reasoning skills?
- Can an intervention program focused on identification and application of patterns, show long-term benefits for children's overall mathematical development?

This paper presents some preliminary results of this study

## Method

Two matched preschools participated in this study, which took place between June and December 2003 In one preschool (the "intervention" school), an experimental program promoting children's patterning skills was implemented An interview-based assessment of children's patterning skills was developed and administered in the beginning and end of the period

### *Setting and Participants*

Two matched preschools in the southwestern area of Sydney participated in the study Both preschools were privately owned and operated with government financial support, and they drew children from similar cultural, linguistic and socioeconomic backgrounds Each preschool had the same number of child placements (38) and similar staffing (one qualified early childhood teacher, one certified child carer, and four other support staff members with similar experience) The preschools were also similar in terms of level of funding, size and type of equipment, program scope and development, and level of parent involvement

The initial sample comprised 53 children aged between 3 years 9 months to 5 years, 27 from the intervention school and 26 from the non-intervention school), balanced for gender and broadly representative of the children in the final year of each preschool Of these 53 children who started in June, 33 were available for interview in December

### *The Intervention*

The researcher worked closely with the staff of the intervention preschool in developing, implementing and monitoring the intervention program This program built on children's existing ideas about patterns and capitalised on their interests and play situations The analysis of the initial assessment also provided a basis for designing program activities

The non-intervention preschool maintained its regular program independently of the researcher

### *Interview-Based Assessment Tasks*

Initial pilot work was conducted with six Kindergarten children, drawn from a large school-based project, who were experiencing difficulties in basic numeracy A common feature of their difficulties was their inability to copy a simple ABAB pattern with four blocks constructed as a tower A similar difficulty emerged in a study of first graders' use of pattern and structure across a range of mathematical tasks (Mulligan et al , 2004)

Nine assessment categories were identified from related research and the pilot work; tasks were designed to elicit children's intuitive ideas about patterns and to assess their ability to create, identify, extend, and copy simple patterns in a variety of forms (see Table

1) Trialing of all tasks was conducted with eight preschool children prior to commencement of the main study The tasks were then modified to accommodate children's use of equipment and interpretation of instructions The tasks had different response requirements and varied in the quantity, size or structure of the pattern For example, subitising patterns were presented in regular and irregular spatial arrangements and with varying numbers of dots

Table 1
*Categories and Descriptors of Patterning Tasks*

| Patterning category | Task descriptors |
|---|---|
| Concept of patterning | Imagine and draw a pattern<br>Identify and describe a pattern (stickers, counters, wrapping paper)<br>Design a pattern using concrete materials |
| Tower | Copy a 6 block ABABAB tower using blocks and colour<br>Continue a ABABAB block tower pattern<br>Identify screened parts of various ABABAB tower patterns<br>Copy from memory and draw from memory a given ABABAB tower pattern Repeat with ABBCCC pattern |
| Dot | Copy various triangular dot patterns (made with 3 and 6 counters) using counters and by drawing |
| Array | Copy various array patterns (made with 4, 5, and 6 counters) using counters and by drawing |
| Grid | Copy square and rectangular grid patterns made with 2, 3 and 4 squares by drawing |
| Subitising | Identify number of dots in regular and irregular dot patterns and dots within grids using 3, 4, 5 and 6 dots<br>Identify number of blocks in staircase block patterns made with 3 and 5 blocks |
| Numeral | Identify next numeral and colour in pattern of two and three numerals using two colours |
| Border | Complete a 4 x 4 AB border pattern using cut-out tiles |
| Hopscotch | Copy using square tiles a hopscotch template that is repeated three times Copy from memory<br>Complete using square tiles a hopscotch template rotated by 180°<br>Draw rotated hopscotch pattern by copying and from memory<br>Rotate hopscotch by 90°, then by 90° again<br>Identify position of hopscotch and teddy after 90° and 180°rotations |

The initial task explored children's images and identification of patterns in simple designs and objects without any specific expectations as to their responses Children explained what they visualized as a pattern and represented this in drawings In all the other tasks, with the exception of subitising, the child was required to copy the pattern with materials, draw or make it from memory, identify a screened element, or extend the pattern

The Subitising tasks determined whether children could recognise a small group of perceptual items and assign a number name The process also involved seeing parts in the whole, for example, a regular dot pattern of five was seen as 'two', 'one' and 'two' This

ability to subitise is considered fundamental in developing visual memory and pattern recognition (Bobis, 1996)

The four task categories Tower, Numeral, Border, and Hopscotch focused on the structure of a pattern with at least one element of repetition such as ABAB The hopscotch task also represented an ABABAB pattern rotated by 90° four times The tower tasks were inspired by Maher's longitudinal study (2002) but without focusing on combinatorial thinking The task categories of Dot, Array and Grid investigated whether there were differences in pattern recognition and representations using a spatial structure of equal sized units and spaces The design of these tasks was informed from studies on pattern, structure, and unitising with older students (Battista, 1999; Mulligan et al , 2004; Outhred & Mitchelmore, 2000)

### *Data Collection and Analysis*

Individual task-based interviews were administered in June and December of the final year of preschool The interviews were conducted by the first researcher at each preschool in a room separate from the main rooms They usually took place between 8 30 am and 1 00 pm and lasted 15-30 minutes each Each interview was conducted in two segments to accommodate children's attention span The order of the task and the procedures remained the same for all interviews, and all tasks were administered to all children Children were provided with paper, coloured pencils and a variety of concrete materials

The data collected included video-tapes and/or audio-tapes of the interviews and records of children's written and drawn responses Photographs were taken of children's responses to tasks using concrete materials Additional data were collected during weekly monitoring of the intervention program A case-study profile was assembled, comprising the teachers' program, observation records, videotapes and photographs of children's participation in activities, samples of children's work and anecdotes from teachers and parents

## Results and Discussion

The results presented below represent an initial analysis of children's responses to selected tasks; solution strategies and variations in children's representations are subject to further analysis

There was a wide variety of responses to the children' initial images of pattern including repetitions observed in nature such as symmetry in a spider's web, sequence of colour in a rainbow, a translation (footprints made by a dog), to geometric designs using colour and regular 2-dimensional shapes These were not further analysed

Table 2 indicates the percentage of correct responses to selected tasks that have been chosen to be representative of the eight remaining categories (see Table 1) Children from the non-intervention preschool were more successful at the June interview than those from the intervention preschool on all tasks except for the subitising task However, by the December interview, the intervention children were more successful than the non-intervention children across all tasks

In Table 2, the categories are ordered according to children's success on the tasks, from least difficult to most difficult The order of difficulty remained consistent across both preschools at both interview points, with the exception of a small variation between the Numeral and Dot tasks The tasks showed a wide variation in children's ability to identify patterns, depending on the mode of response and the type of representation For example,

the Array pattern was easier than the Tower pattern, but when the same structure was presented as a grid it proved more difficult

Table 2
*Percentage of Correct Responses to Representative Tasks in each Pattern Category, by Preschool and Assessment Date*

| Patterning category | Task descriptor | Intervention school | | Non-intervention school | |
|---|---|---|---|---|---|
| | | June (n=27) | December (n=19) | June (n=26) | December (n=14) |
| Border | Complete a 4 x 4 AB border pattern using cut out tiles | 74 | 100 | 81 | 86 |
| Array | Copy and draw array pattern (5 counters) | 59 | 95 | 73 | 86 |
| Tower | Copy a tower of 6 blocks in ABABAB pattern | 48 | 95 | 62 | 64 |
| Grid | Copy and draw rectangular grid pattern made with 3 squares | 37 | 84 | 42 | 50 |
| Subitising | Identify number of dots in regular dot patterns 3, 4, 5 & 6 | 19 | 84 | 19 | 21 |
| Numeral | Identify next numeral by colour (red or blue) in pattern (1 2 3 1 2 3) | 7 | 58 | 15 | 21 |
| Dot | Copy triangular dot pattern (using 6 counters) | 11 | 47 | 12 | 21 |
| Hopscotch | Draw hopscotch pattern rotated at 180° from memory | 0 | 42 | 4 | 36 |

The tasks in the categories Tower, Numeral, Border, & Hopscotch assessed children's ability to copy and extend a simple pattern The Tower and Border patterns repeated elements comprising of two colours (ABAB pattern), whereas the Numeral pattern incorporated both colour and numerals The Hopscotch task was the most difficult as it required the children to view the pattern from a different perspective and in a different spatial arrangement Children improved on the hopscotch tasks at both preschools However, correct responses were still minimal at the second interview (39% correct overall)

The Dot, Grid and Array tasks assessed children's ability to represent patterns using a spatial structure of equal sized units and spaces Children showed greater accuracy when the pattern was in a square or rectangular formation They found the triangular formation of dots much more challenging

The intervention children showed the greatest improvement on the Subitising task, probably because the intervention program included a large number of tasks and dice games that developed children's recognition of regular and irregular dot patterns By comparison, the non-intervention group showed little improvement on the Subitising task,

with most children maintaining their level of accuracy between the June and December assessments It appears that their regular preschool program did not expose them to any dice games and pattern recognition tasks likely to improve their subitising skills

Analysis of individual profiles of children who were successful across most tasks indicated that they could identify the spatial structure or the repeating element of a pattern, as well as observe similarity or regularity in a variety of modes These children could integrate more than one element at a time—for example, the simultaneous repetition of shape and colour

## Conclusions and Implications

This investigation was not intended as a controlled study and therefore the results do not permit generalisation However, it does seem clear that the intervention did lead to a substantial improvement in a wide variety of children's patterning skills More importantly, a sustained positive impact of the intervention program on the mathematical development of the intervention children, representing a range of abilities, was observed during the 6-month period of this study Moreover, results not reported here show that the improvement was maintained 12 months later, at the end of the first year of formal schooling

These findings support the notion that patterning is important in the overall development of mathematical representation and abstraction Children who performed poorly on patterning tasks at both interviews in preschool were identified as low achievers on other numeracy assessments at the end of the first year of formal schooling It appears that there is a strong link between a child's ability to pattern and their development of pre-algebraic and reasoning skills, as evidenced in the responses to the interview tasks Pre-algebraic thinking and mathematical reasoning may require the child to identify and represent the similarities and differences in objects and situations, the ability to perceive the structure of a pattern and to apply these skills using a variety of representations and modes

A recent international discussion group on mathematical thinking of young children (Hunting & Pearn, 2003) reported that advances in cognitive science had revealed evidence of greater mathematical capabilities than previously believed This finding supports educators' views that young children are more capable than current practices reflect, and that providing more challenging early educational programs can have a positive impact on school learning Unfortunately, analysis of survey and interview data from the teachers who participated in the study reported in this paper revealed a lack of awareness of the importance of patterning in mathematical reasoning and low confidence in teaching patterning

The development of an intervention program to promote patterning highlights the need to cater for children's interests and mathematical abilities and to engage children in challenging learning experiences The assessment schedule and documentation of the program can provide valuable professional support in the development and implementation of teaching strategies encouraging the learning of patterns and algebra in mathematics curricula

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