Teachers' Development of Substantive Communication about Mathematics

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Twenty preservice teachers participated in a collaborative action research project in which they taught mathematics lessons about shapes in 12 classrooms of a local primary school This paper reports on the development of teachers as they establish substantive communication about shapes in two upper primary classrooms The paper highlights common themes from analysis of the other students' reports Videotape or audiotape transcripts of the lessons and student work samples were available as well as my own classroom observation notes The preservice teachers' reflections were critical in their development of questioning skills Their desire to modify approaches and their growing knowledge of concepts built their self-confidence as teachers

Argumentation is seen as a critical feature in reform schools in the USA where teachers encourage mathematical thinking (Yackel, 2002) Wood (1999) and her team carefully analysed a sequence of lessons for a Year 2 classroom While not forgetting the contextual and social aspects of the classroom, one purpose was "to exemplify … the complex process of maintaining student participation in discussion and sustaining students in their mathematical thinking" (p 172) The pattern of interaction was:

> A child provided an explanation of her or his solution to the problem
> A challenge was issued from a listener who disagreed with the solution presented The challenger might or might not tell why he or she disagreed
> The explainer offered a justification for her or his explanation
> At this point, the challenger might accept the explanation or might continue to disagree by offering a further explanation or rationale for his or her position
> The explainer continued to offer further justification for her or his solution
> This process continued and other listeners sometimes contributed in an attempt to resolve the contradiction
> The exchange continued until the members of the class (including the teacher) were satisfied that the disagreement was resolved (Wood, 1999, p 179)

Wood (2003) compared this interaction with discourse patterns in conventional classrooms in which students tended to give the right answer or prescribed procedure and the teacher evaluated it while other students paid attention or checked their answers This approach is classified in the model of pedagogy promoted by the Quality Teaching in NSW Public Schools Project (NSW Department of Education and Training (DET), 2003) as having little "substantive communication" because they follow the typical "initiate-respond-evaluate" teacher-centred pattern in which the teacher asks a question, a student responds, the teacher makes an evaluative comment indicating correct or incorrect response and then moves on to the next question or lesson segment Students are merely required to "fill in the blank" or "guess what's in the teacher's head"

Substantive communication is sustained with logical extension or synthesis where the flow of communication carries a line of reasoning and the dialogue builds on statements or questions of another participant The communication (verbal, diagrammatic, gesture, etc) "is focused on the substance of the lesson It moves beyond mere recounting of experiences, facts, definitions or procedures and encourages critical reasoning such as making distinctions, applying ideas, forming generalisations and raising questions" (NSW

DET, 2003, p 23) The assessment rubric suggests quality is scored more highly if substantive communication is evident for more of the lesson However, Wood's (2003) model provides distinct contexts, responsibilities and thinking Mathematical thinking was revealed in the context of "strategy reporting" as recognising, comprehending, applying, and building with analysis Explainers told different strategies and clarified solutions while teachers accepted or elaborated these and other students listened to decide if their own strategies were different Synthesising and evaluating were more evident in "inquiry and argumentation" Explainers were giving reasons and justified or defended solutions while teachers asked questions and made challenges, provided reasons or asked for justification Listening students asked questions for understanding or clarification or disagreed and gave reasons for their challenges

During strategy reporting, teachers might prompt with a variety of statements like "How did you decide this? … Are there patterns? Is there a different way you can do this?" The teacher can facilitate inquiry and argument patterns of communication by asking questions such as "How are the two things the same? Does this make sense? … Does it always work? Why does this happen?" (Wood, 2003, p 440) These questions could be classified as structuring, opening-up or checking (Ainley, 1988)

Like Wood (2003), Hufferd-Ackles, Fuson and Sherin (2004) developed a framework for improved interactions showing shifts from the teacher to students in questioning, explaining mathematical thinking, sources of mathematical ideas and responsibilities for learning At the higher level, teachers expect students to initiate and question They may ask why questions and persist until satisfied with the answer Teachers will follow students' descriptions of their thinking carefully, encouraging more complete explanations and deeper thinking Students can defend and justify their answers and are more thorough in explaining Teachers permit students to interrupt explanations in order for students to explain or to own new strategies While still deciding what is important, the teachers use students' ideas and methods as the basis of the lesson Students will spontaneously compare and contrast and build on ideas Teachers expect students to be responsible for co-evaluation of everyone's work and thinking The teachers support students as they help one another sort out misconceptions and they help when needed Students may initiate clarifying other students' work and ideas

The teachers' role in supporting and overcoming misconceptions and in encouraging deeper explanations involves the teacher in having a purpose for giving information or asking questions Lobato, Clarke and Ellis (2005) refer to this as an aspect of the teacher "telling" (p 102) The teacher may initiate the solution path and communicate in a way that assists students to develop their conceptual knowledge This notion of telling is not the transmission mode of teaching procedures and telling facts to be learned by students but supporting learning by assisting the students to reflect and abstract from the activities It may involve teachers giving information, suggesting a strategy that will provide a way forward, providing an example to encourage students to rethink their actions, or asking a question This initiation is not a simple question as implied in the initiate-respond-evaluate pattern discussed by the NSW DET (2003) document

The Researching Numeracy Project Team (2004) in Victoria, Australia identified 12 practices involving questioning that contributed to improving student learning outcomes These were "Excavating, Modelling, Collaborating, Guiding, Convince me, Noticing, Focussing, Probing, Orienting, Reflecting/reviewing, Extending, Apprenticing" (pp 28-29) Questioning may vary in purpose yet extend substantive communication

Another perspective on this role of the teacher is to consider the teacher as "replacing" the everyday and pictorial representations provided by students with mathematical and more abstract representations (Brown & Hirst, 2004) While this is an efficient strategy, it may hinder participation of students in making meaning or integrating their knowledge It is similar to the initiate-respond-evaluate approach to teaching but provides ideas and "foregrounds mathematical practices such as 'representing' and 'explaining' and evaluates students' products in terms of mathematical norms that relate to 'meaning' and 'clarity'" (Brown & Hirst, 2004, p 11) It is similar to Lobato et al 's (2005) telling Opportunities are needed for students to "interweave" their own personal challenges, perseverances and discoveries or inventive ideas with the conventions of mathematics and hypothesising, testing, and validating (Brown & Hirst, 2004) Both replacement and interweaving value student participation and evaluate it within the classroom context rather than by a textbook or teacher

Developing Teachers' Confidence for Substantive Communication

One critical aspect in facilitating these communication patterns in classrooms is the development of students' sense of security and confidence in their learning Teacher confidence and knowledge of pedagogy and mathematics will help teachers to structure the tasks effectively to maintain students' confidence and positive affective states in the problem-solving classroom (Goldin, 2004)

Several researchers have undertaken co-researcher action research in schools in order to increase teacher confidence Johnson and Cupitt (2004) pointed out teachers are using group activities and concrete materials but "exploring the use of open-ended questions and encouraging students to describe the strategies they employ when solving mathematical problems, is more 'pedagogically challenging' to engage students in genuine mathematical inquiry and move into areas which do not come easily to them" (p 3) The researchers modelled and discussed lesson planning and reflection to encourage better "intellectual quality" in classrooms (NSW DET, 2003) They worked on increasing communication Hufferd-Ackles, Fuson and Sherin (2004) also modelled pedagogy and showed increasing teacher knowledge as the teachers were undertaking more effective classroom questioning Both sets of co-researchers showed that teachers' knowledge of pedagogy for problem solving and their confidence increased over the project

Teachers still need considerable professional development to develop their knowledge of the space strand of mathematics (Clarke, 2004) and hence to develop substantive and mathematised communication in classrooms (Ball, 2000) A NSW Australia project *Count Me into Space* (CMIS) aimed at improving teaching the space strand of Mathematics In this project, facilitating teachers undertook a day of professional development about how students learn space concepts They were introduced to some assessment tasks and they watched videotapes of classrooms in which (a) investigating and visualising and (b) describing and classifying were key points of focus They returned to their schools and team taught or supported four or five other staff members This project increased teachers' use of group work and concrete materials, questioning, and their understanding of the importance of investigating and visualising in learning space concepts (Owens, McPhail & Reddacliff, 2003) Teachers were more confident to have more open investigations and questioning Teachers were more aware of students' visualising and increasing knowledge and discussion about shapes One challenge in the space strand is to move teachers from emphasising the labels for shapes to describing shapes using attributes, analysing the roles

of the attributes and making logical arguments that justify the geometric relationships (Linquist & Clements, 2001)

Research Objective

The project reported in this paper focused on investigating how preservice teachers were developing their pedagogical skills for substantive communication in mathematics classrooms By focussing on the spatial concepts and processes of mathematics, about which teachers lacked confidence, critical aspects of the development of substantive communication would be highlighted Except for part of Johnson and Cupitt's concurrent study, space has not been a focus of action research studies on substantive communication or argumentation

Methodology

Procedure

Twenty preservice teachers in the third year of their primary teacher education or early childhood courses participated in the study as part of an elective subject They participated in tutorials for six hours on mathematics education for the space strand and substantive communication During this time, they watched videotapes prepared for the *Count Me Into Space* (CMIS) project and discussed how students learn about space They also watched two lessons, one on measurement, and evaluated these according to the Quality Teaching framework for deep knowledge, deep understanding and substantive communication Their readings included the Quality Teaching in NSW guide (NSW DET, 2003), Wood's (2003) paper, and excerpts from Hufferd-Ackles et al's (2004) paper

The teachers (cooperating class teachers and preservice teachers) were given a large number of sample lessons based on the CMIS project The lessons covered both two- and three-dimensional space The strength of these lessons was that they emphasised investigating and visualising as well as describing and classifying Each preservice teacher or pair of teachers met with one of the cooperating teacher in a good local primary school to select and modify six to ten lessons appropriate for the class The school's teachers attended two after school sessions on space and agreed to participate in the project

Data Collection and Analysis

Each preservice teacher kept a journal with anecdotal records and students' worksamples The preservice teachers evaluated each lesson using the readings and prepared a final report evaluating the teaching and learning All teachers were part of focus group discussions (preservice and class teachers separately)

This paper discusses two classrooms reflecting themes common in other classrooms as reported by the preservice teachers' anecdotal records, their own reflective comments and self-evaluations Not all preservice teachers achieved the same degree of communication and two of these classrooms were a contrast highlighting the aspects that affected the development of substantive communication in the other classrooms The chosen classrooms were selected for this report because I had access to taped data and transcriptions in addition to anecdotal and reflective comments The team-teaching teachers (one primary and one early childhood) in Class K (Years 5/6) used videotapes while the other teacher (early childhood) used audiotapes her Class M (Year 6) I viewed the videotapes and observed a lesson in each classroom

The qualitative data were analysed by annotating all interesting records with a comment which often linked to the literature All the preservice teachers provided good examples of different levels of communication in their reports From the taped material, I specifically noted how the teachers attempted to extend students' conversations From these annotations, some generalisations were drawn in order to better understand how beginning teachers can achieve substantive communication in their classrooms

Results and Discussion

Productive Reflection as a Key to Change

Class M's teacher reflected regularly using Wood's framework and listed questions that she asked to encourage strategy reporting and inquiry/argumentation The following summary was made after a lesson on making pentominoes (5 squares joined along their sides) and looking at their perimeters and areas

Strategy reporting

> What do you mean by they have to be different?
> Why do you think it's different?
> How am I going to work out the perimeter?
> Why do you think it's going to be 12?
> Who can tell me why it's 10 and not 12

Inquiry/argument

> How are we going to know if it's the same or different?
> Does anybody want to say anything different or agree or disagree with that?

She reflected on how her questioning increased strategy reporting and inquiry/argumentation The following transcript from the pentomino lesson shows how she encouraged students to interact and give their opinions (T stands for teacher)

T: Is that the same shape or a different one
D: Same
T: How come it's the same?
S: It's been rotated
E: It's different
T: Why do you think its different E?
E: Because the square we're looking at is in the top row not the bottom row
T: Someone else
V: They're the same because if you rotate it's on the right side not the left side
T: What happens, yep someone else
J: If you flip it over and rotate it once

At this point the teacher decided that there was general agreement about the two shapes being the same but one was reflected and rotated

Following another lesson that I observed, we realised that she had been pausing and the pauses were effective in generating student conversation even though the pauses were due initially to her uncertainty about the mathematics Later she used pauses deliberately

Listening for Student Questions and Developing Confidence to Handle Them

The above extract shows how she listened to students Her quiet teaching approach encouraged students to ask questions The next excerpt is from the same class

R: What's the difference between a rhombus and a diamond?
T: I'd like to know the difference Tell me what the difference is (pause) C?

C: A rhombus faces that way but this doesn't
T: What is a diamond?
C: A diamond's like that
T: Does anybody else want to say anything about that?
S: A diamond's like that The corners are equal …
T: What shapes make up a rhombus?
D: Triangles
E: If you look at the rhombus, it has the same shape as the diamond One side is down like that
F: It can be both a diamond and a rhombus …
T: What shape is this? (Holds the rhombus with one diagonal vertical)
Ss: A diamond
T: What shape is this? (Holds the rhombus with one side horizontal)
Ss: Rhombus
F: It's still a diamond
T: If I hold it like this and I have exactly the same shape drawn on a piece of paper, you look at it like this and if that is a diamond, and I turn it like that and you say it's a rhombus
K: It can't be
N: It can be
T: We have some confusion there and I think Miss O would back me up in saying when children are little they call it a diamond but we can now call it a rhombus as it is the proper mathematical name

The teacher took up the students' question and then provided helpful input (Lobato et als' "telling", 2005) by effectively using a diagram to challenge the students Finally she provided a logical explanation and the students in later lessons seemed more content to use the terms square and rhombus as appropriate

Reflection that Improves Planned and Spontaneous Questioning

The above excerpts were quite spontaneous Class K teachers also reflected on their use of questions and considered when they had missed opportunities for substantive communication Their reflections on the first two lessons mentioned times where students offered different opinions but they did not allow the conversations to continue They explained that this lack of open discussion was partly due to their own lack of confidence in knowing the possible answers In the second lesson, a student suggested that an isosceles triangle had acute angles However, a student pointed out that the isosceles triangle in front of them had a right-angle but the argument was not continued The teachers also failed to pursue a conversation about whether a shape was a parallelogram or a trapezium but they returned to this before the end of the lesson and substantive communication began

After these initial lessons, the teachers worked on the problem of asking questions so that students' ideas were pursued They improved their own knowledge by talking with each other and checking in books and with me They changed their questioning techniques and allowed more time for discussion by reducing content The teachers provided challenging questions for groups to consider before contributing to whole class discussion They reflected on the importance of cooperative groups for challenges when they realised that a student in an individual task of box-making would have been better with a partner They rearranged the desks so the group members could sit closer together for sharing

In later lessons, the students and teachers ensured they discussed whether there could be an obtuse-angled isosceles triangle and some good justifications were heard At first, the teacher (T1) and some students thought that the acute-angled isosceles triangle was called that because the base angles were acute Attempts to draw the 40°, 40°, 100° triangle were not helping them to visualise the obtuse-angled isosceles triangle

T1: So everyone got the isosceles triangle

J: You know how you call it an acute isosceles triangle, doesn't it *have* to be acute?
T1: That's exactly what I said to T2 before the lesson What does everyone else think? Do you think you can have obtuse angled isosceles triangle?
R: No then it would be scalene (Other students comment in the background)
T1: Then it would turn into a scalene
D: If both angles (pause) in the corners, it would go out like that (shows with hands)
T1: Would it still be a three-sided shape?
M: Every angle can't be obtuse anyway because it wouldn't work
T2: Why?
M: Wouldn't fit together real easy
T1: Did everyone hear what M said (pause) Does anyone disagree with that? (pause) Does everyone agree that all shapes should have an acute angle like it can't be made up of all obtuse angles?
C: Yeh it could
T1: What sort of shape do you think JM?
JM: Irregular hexagon (JM is asked to draw this)
JM: I'm not sure if I can … (JM draws a shape)
S3: That's got seven sides (referring to shape JM drew) …
S4: It's a heptagon …
T1: What are the angles like?
S5: Some are obtuse, some are acute
T1: Can you show me which ones are acute (S5 comes forward and then changes his mind)
S5: They just looked (turns shape around)
T1: So when you put it in another orientation
T1: M , … (do) you think you can make a shape with obtuse angles ?
M: Yep
D: Yes, but you can't make a triangle
J: What's an illegal heptagon?
T1: Irregular Who thinks this is an irregular heptagon? Why is it irregular?…
J: Not all sides are the same

Students continued to discuss other shapes on the paper deciding on whether they were irregular or not In this extract and later in the lesson, students were initiating conversation They were not just giving one property after another They were also building on each other's ideas (cf, Wood's "inquiry" & NSW DET's "substantive communication") The teachers carefully planned the next lesson The students discussed whether an isosceles triangle has a sharper vertex if the base is smaller The students made a connection between the sides of the triangle and its angles They continued their conversation about drawing the triangle with angles of 40º, 40º and 100º By this time, the teachers had realised that the non-equal angle of the isosceles triangle could be obtuse and argumentation continued until the drawn example could be explained As Hufferd-Ackles, Fuson and Sherin (2004) suggested, teachers as well as students are likely to gain knowledge and confidence through the project

Conclusion

The NSW QT levels are based on the proportion of the lesson using substantive communication but these levels fail to realise the nature of improvement (cf, Wood's model) and that less frequent substantive communication may be educationally significant The teachers described in this paper became aware of how to generate substantive communication and why it was significant At the start of this project, the teachers were confident about teaching in general but they were not confident with space concepts Over the course of the project, these preservice teachers through teaching, reflection, and sharing with each other and myself had increased their own personal knowledge and confidence

Teachers were careful about their questioning Teachers deliberately asked questions such as why or who agrees/disagrees and they learnt to extend the conversation time to allow for differences of opinion The teachers kept the level of knowledge high and encouraged students to tackle areas in which they had differences of opinion They supported students by restructuring the classroom, using concrete provisions and planning lesson steps carefully Teachers were co-learners with students When anxious, one of the preservice teachers initially tended to rush to a new question or topic whereas another paused The third teacher tried a question to make students think again Over time, the questions were better focussed and pauses deliberate The teachers were able to follow through with questioning until the inquiry led the class to a clear understanding The teachers' risk-taking was soon matched by that of the students As Goldin (2004) predicted, the confidence and sense of achievement of the teachers and students increased

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