A COMPARISON OF THE MATHEMATICAL UNDERSTANDINGS OF PRIMARY TEACHER EDUCATION ENTRANTS

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At times in the past ten years, politicians, educators and business people have deplored the decline in the mathematical ability of adolescents and older young people. Generally it has been the arithmetical skills which have been criticised.

This paper attempts to compare the understandings of primary teacher education students in their first weeks at University in 1994 with those of a similar group of students at two NSW Teachers' Colleges in 1964. The 1964 sample completed an achievement test and a test of mathematical understanding. As well, a small group were interviewed to determine any obvious attitudinal factors related to achievement and understanding.

The 1994 sample consisted of 162 students in their second week of a pre-service teacher education course. In this case, as the students were to be given an achievement test later in the semester, only the survey of mathematical understandings was administered. The responses were analysed using data related to the upper, middle and lower thirds of the scores. This provided information about the topics which students found difficult and enabled a comparison with the previous results.

Trends in education seem to swing from one extreme to another. Such has been the case in teacher education. Approximately twenty five years ago, the emphasis that had been placed on the mathematics discipline knowledge of students taking pre-service courses in preparation for teaching was gradually lessened with greater emphasis being given to general educational and professional subjects. The report of the Disciplinary Review into Teacher Education in Mathematics and Science signalled a return, at least in theory, to a greater emphasis on discipline knowledge. As well, in NSW, the Department of Education has stated that no one will be employed as a primary teacher unless they have completed two units of mathematics at the Higher School Certificate level or its equivalent. Lack of mathematics discipline knowledge is recognised as a constraint, not only on the primary teachers themselves but also on the students whom they teach. It is this constraint that this paper addresses.

The mathematical knowledge of pre-service primary teacher education students is of considerable interest to teacher educators, as it was in earlier times. While there have been attempts to quantify the mathematical knowledge of primary teachers, very little has been done to ascertain what teachers understand in mathematics. (Brown and Baird, 1993)

Grouws (1992) raises the issue as to whether a teacher's knowledge of mathematics affects his or her students' learning. He agrees that there is general consensus that teachers can only teach mathematics if their knowledge of it is sound but does not differentiate between practice of procedures as distinct from understanding of those procedures.

What, then, do primary teacher education students understand about elementary mathematics? Have there been changes in this understanding in the past thirty years?

The writer is in the unique position of possessing data on the mathematical understandings of pre-service primary teacher education students of thirty years ago. Two hundred and ninety two students from two NSW teachers' colleges were tested in 1964 in order to diagnose their deficiencies in mathematics and develop teaching programs which would be of assistance in remedying those deficiencies. The test consisted of three parts. The first part tested computational skill, the second part tested understanding and what is now referred to as 'number sense', while the third part tried to assess the attitudes of selected subjects. As well, the previous mathematical experience of the subjects was examined to see if there was any correlation between it and their performance in the tests of discipline knowledge and understanding.

The data obtained from the administration of the tests in 1964 indicated that about 40% of the subjects had a poor understanding of numeration, particularly amongst the weaker students. They also showed a poor understanding of the application of decimal place value notation to decimal fractions. While very few students made errors in the four operations with number, the concept of multiplication was not well understood. As many as 50% of the students had weaknesses in vocabulary, approximately 20% made errors in common fractions and approximately 40% made errors in decimal fractions associated with place value notation. About three quarters of the students had no understanding of percentages. At least 40% lacked knowledge of common measures such as the capacity of a cup. About half the students were unable to recognise common geometrical solids or indicate any real understanding of symmetry.

In keeping with the swings in trends in teacher education, it was thought that the re-administration of the test of mathematical understandings might provide some useful data for the pre-service course to be undertaken by primary teacher education students. As a result, it was hoped that the constraint placed on pre-service teachers by their weaknesses in mathematics could be removed. Accordingly, the original test of mathematical understanding was revised and administered to 162 students in their second week of their teacher education course.

The Survey

The revisions necessary were largely to cater for the introduction of metric measurement and decimal currency. In addition, because it is anticipated that probability will be included in the NSW K-6 Mathematics Syllabus shortly, four questions on chance were added to the original test. The processes for validation and reliability for the original test were assumed to apply for the revised test.

The analysis of the subtests is given in Table 1.

Item Numbers	Curriculum Area	Total Items
1-7, 12, 42	Number and numeration	9
8-11, 13-25, 39, 41	Operations	19
26-38, 74	Ratio	14
40, 43, 44, 56, 60-73	Measurement	18
45-55, 57-59	Space	14
75-79	Probability	4
	Total	78

Table 1. Analysis of subtests

The test is a test of 78 multiple choice items, each with three distractors. A separate answer sheet was provided for recording of responses. The time allowed was 1 hour, based on the original finding that no subject took longer than 55 minutes to complete the test.

The Sample

The sample consisted of 162 students in their second week of a primary pre-service teacher education course. Of these, 133 were female and 29 were male. These numbers constitute 82.1% and 17.9% of the total sample respectively. The corresponding percentages in the 1964 sample are 79.5 and 20.5 respectively. Table 2 shows the numbers and percentages of females and males in the two samples.

	1964		1994		Total	
	Females	Males	Females	Males	1964	1994
Number	232	60	133	29	292	162
Percentage	79.5	20.5	82.1	17.9	100	100

Table 2. Gender Distribution of Subjects

The mathematical experience of the subjects covered a wide range from basic mathematics through to the second highest level of mathematics at the HSC. Table 3 shows the numbers and percentages of subjects at each level of mathematical experience.

Exam Level	Subject	Females	Males	Total
HSC	3U	11	3	14
	2U	56	7	63
	MIS	25	5	30
	MIP	1	0	1
Sch Certificate		8	1	9
Unistart		6	5	11
Others and non-respondents		26	8	34
Total		133	29	162

Table 3. Mathematical Background of Subjects

The age groupings of the current sample vary considerably also and these are given in Table 4. It is important to note that the original sample consisted of all ex-high school students and therefore it was much more homogeneous in regards to age. The high proportion of mature aged entry students in the current sample is in keeping with the emerging interest in education in a region which has previously been considered disadvantaged and lacking in educational opportunities, particularly for females.

Gender	17 years	18 years	19 years	20-24 yr	25-29 yr	30-39 yr	40-49 yr	Total
Female	15	42	13	34	12	12	5	133
Male	1	0	4	16	3	4	1	29
Total	16	42	17	50	15	16	6	162

Table 4. Age Distribution of Subjects

The Analysis

As in the previous study, the data obtained from administering the Mathematics Understanding Survey to 162 subjects was analysed in terms of the performance of the upper, middle and lower thirds of the sample. The mean, median, quartiles, range and standard deviations for each third as well as for the total sample were computed. An item analysis was also completed by calculating the percentage of responses for each option in the multiple choice survey.

Results

The item analysis indicates that there are several items on which the current sample did better than the 1964 sample. These are Items 1, 3, 14, 21, 32, 33, 38, 53, 59, 60, 61, 66. Of these Items 1, 14, 32, and 66 were correctly answered by less than 50% of the students. The relevant percentages for the 1964 sample are given in parentheses.

Item 1. Number concepts, terminology, natural numbers. This item proved difficult for the majority of the students, only 24.07% (17.5%) giving the correct answer. The most popular incorrect response chosen by 47.53% (40.4%) included zero as a natural number. This is not surprising as mathematicians have not always agreed on the definition of natural numbers.

Item 14. Four operations, principle, multiplication. Only 26.54% (22.2%) of the students gave the correct answer for this item. Half (41.4%) the students opted for the looser and not always correct description of multiplication, that it is a process by which numbers are increased. That multiplication is "the process of combining numbers" was selected by 22.84% (33.6%) of the students.

Item 32. Ratio, principles percentage. The poor response to this item was surprising in view of the frequent occurrence of percentages. The correct response was given by 41.36% (36.0%) of the students, while the most popular incorrect response was that a percentage is "part of a hundred". This was selected by 46.91% (54.4%) of the students.

Item 66. Measurement, application, area. The correct response was selected by 45.06% (38.6%) of the students. "Square", "triangle' and "rectangle" were selected by approximately equal numbers.

The items on which the current sample performed markedly less well than the 1964 sample are numbers 4, 28, 30, 34, 37, 45, 50, 54, 55, 58, 64, 67, 68, 70, 71. Of these, Item 67 is the only one which proved too difficult for both samples of students.

Item 67. Measurement, principle, area, ratio. The correct response was given by only 6.7% (27.8%) of the students. The most popular incorrect response selected by 60.49% maintained the correct order of the ratio but overlooked the dimensions of area.

Items 1, 2, 5, 14, 19, 67 were poorly (< 25%) done both times. These were all items which required more precise mathematical terminology or understanding.

Items 7, 10, 12, 16 , 21, 23, 31, 35, 39, 40, 41, 42, 48, 49, 51 were well (>80%) done both times. These related to numeration, operations of division and subtraction, ratio, and geometrical terminology.

Item 60 is worth noting in that it is an item in which the current students did considerably better than the previous sample. The original item was in imperial measures and tested terminology and knowledge of common measures. This was revised to account for metric measure using the same principles. The correct response was given by 88.27% which contrasts with 26.4% of the 1964 sample.

The statistics for the two samples of students are given in Table 5.

Statistic	Upper Third	Middle Third	Lower Third	Total Group
Mean	58.28	47.83	33.52	46.78 (51.63)
Median	57	49	32	47.9 (53.1)
First quartile	55.3	45.7	29	40.8 (46.8)
Third quartile	60.25	51.5	39	55.73 (57.9)
Range	69-53	53-43	43-17	69-17 (66-17)
Stand. deviation	3.97	3.45	6.88	8.61 (8.47)

Table 5. Results for 1994 upper, middle and lower thirds (1964 in parentheses)

On a t-test, the difference between the two means is significant at the 0.001 level. These statistics indicate that the mathematical understandings of the students in the current sample are not as secure as for the 1964 students. The mean and median scores are lower and while the maximum score is higher than in 1964, there appears to be a weighting towards the lower third pulling the overall mean down. There is very little difference between the standard deviations of the two samples. The standard deviation of the lower third contrasts with that of the other two thirds. The quartiles indicate that for the upper, middle and lower thirds of students, only 25% do better than 55%, 45% and 29% respectively on the survey. This means that, for the lower third, 75% gave less than 29% of correct responses.

Discussion

Although there are several items in which the 1994 sample did better than the 1964 one, and also several items in which the current sample did well even though not as well as the previous sample, it is alarming that there are 48 items (64.9% of the items) on which the earlier sample were superior. Coupled with this is the lower mean and median in the distribution of current scores. This indicates that the mathematical understanding of pre-service teacher education students has deteriorated rather than improved in the past thirty years.

This general deterioration indicated by this study could be due to several factors. One factor is the difference between the two samples. The previous sample consisted of ex-secondary school students from colleges which drew their students mainly from geographical regions which were

not regarded as disadvantaged. They had an established tradition in valuing education. The current sample, on the other hand, has no such tradition as education has only recently become a reasonable expectation for many adults in the outer western Sydney region. Also 53.8% of the current sample are over twenty years of age with 13.6% over thirty. This means that most of these students have not been involved in formal schooling for some years and may be unable to remember different aspects of the items. It is also highly likely that when they were in school, much of their mathematics was learnt by rote and therefore easily forgotten.

Another factor could be due to the changes that have taken place in mathematics education over the past thirty years. In secondary schools, while the more advanced courses have not necessarily become less rigorous, at the more elementary level, courses have been introduced which do not demand the same degree of rigour which existed in previous courses. This could mean that the students who enter primary teacher education courses today do not have the same mathematical background as students of thirty years ago. The tendency to extend syllabi rather than allow time for consolidation over the past few years may also contribute to this general deterioration in standards.

Coupled with this is the fact that the current sample completed the survey in their second week in the course, whereas the original sample did theirs later in the year after some work in mathematics in their college courses. The survey was administered early in the course with a view to using the results to provide more effective teacher education subjects in the immediate instance.

The statistics presented concerning the performance of the upper, middle and lower thirds indicate that while the good students are still good - and, indeed, slightly better in one or two individual cases than in the previous sample - the weaker students do not do as well as a group as in 1964. The lower third has a greater range and standard deviation than the other thirds. This points to poorer performances being more widely spread over the range.

The level of student performance as highlighted by the median and quartiles is cause for great concern. Half the students in the upper third achieved only 57%, while half the middle third achieved 49% and half the lower third achieved 32%.

One factor which may be relevant with the lower third is time. One might reasonably expect the lower third to work more slowly than the upper third in completing work. Items 68 to 74 were omitted by 16% of the students or more. This indicates that some students may have not had sufficient time to attempt these items, thus reducing their scores.

Items in which the mathematical understanding of pre-service teacher education students has improved are those dealing with numeration, subtraction as repeated division, applications of percentages, number sense, some solid shapes, symmetry, common measures and the concept of perimeter. While there have been improvements, students still have weaknesses in some terminology, the concept of multiplication, the concept of percentages, and the relationship between the area and the perimeter of plane shapes.

Conclusions

On the survey scores, it would appear that the understanding of mathematics possessed by primary pre-service teacher education students has deteriorated in the past thirty years. It would be unwise to accept this conclusion without taking into consideration such factors as the difference in the samples, the changes which have taken place in school syllabi and the time factor.

The results also indicate that the range of scores achieved on the Mathematics Understanding Survey are skewed towards the lower end of the range. While the best students remain at the same level, the weaker students spread out more over the lower end of the range.

Whatever explanations are made for the poorer results of the 1994 students, the fact remains that their is poorer than the performance of students thirty years ago. Provision must be made in University teacher education courses to make up the deficit which is so obvious. This is particularly important for primary teachers because of the influence which they can have on the children in their classes. Even though it has not been shown that teachers' mathematical ability has any relationship to their success in mathematics, it would seem reasonable to suggest that a teacher who realises his or her deficiencies and tries to overcome them will be more confident in presenting the same or similar material to a class.

References

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