AN INVESTIGATION INTO KNOWLEDGE BASES OF PRIMARY AND SECONDARY MATHEMATICS TEACHERS: REPORT ON A PILOT STUDY

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In recent years much attention has been given to demands for curriculum reform in mathematics education. To date most attention has been placed on developing adequate frameworks for mathematics syllabuses and the need to obtain quality structures for assessing and reporting the mathematical attainments of students. Largely left out of these concerns, however, has been a focus on the understandings of mathematics teachers with respect to (1) content knowledge in mathematics and (2) content specific pedagogical knowledge in mathematics. The primary aim of this study is to obtain a view from the field of the current range and depth of mathematics teacher knowledge with respect to these domains. This pilot study has utilised data obtained from a purpose built survey instrument; 44 teachers in 10 schools (primary and secondary) comprised the sample. Analysis of results has indicated that over 50 percent of the teachers in the study may not be sufficiently prepared in mathematics content, and that almost two third of the teachers in the sample are concerned about their level of knowledge in contemporary teaching methodologies. Key differences with respect to these variables emerge between primary and secondary sectors and within the secondary sector. A study on a considerably larger scale is currently under way; this will provide results of greater validity and details concerning finer structures of relevance to this investigation. It is intended that this study provide much needed evidence relating to debates concerning the appropriate structures, content material and policy for pre-service and in-service mathematics education.

Building an understanding of the knowledge bases routinely accessed by mathematics teachers is a critical step in formulating models for teacher education and professional development. Current literature, however, is marked by a lack of clarity concerning the structure and content of these knowledge bases. For instance, Ball and McDiarmid (1990), in their discussion of the importance of subject-matter knowledge in mathematics do not seem to distinguish between the teacher knowledge of content, and the teachers knowledge of what Shulman (1986, 1987) has termed content pedagogic knowledge, viz the fusion of general pedagogic knowledge with specific elements within more general content domain topics. Instead, they naively assert that "teachers' intellectual resources and dispositions largely determine their capacity to engage students' minds and hearts in learning." (p439). As Carter (1990) points out this approach is wholly inadequate because it systematically downplays the substance of what teachers actually know or need to know about classrooms, content, and pedagogy and how that knowledge is organised. She concludes that "more discussion needs to be directed to what it means to teach, rather than simply to what is learned in which settings". Undue emphasis on content knowledge can therefore oversimplify the nature of teaching and give misleading signals to

policy makers (Goodson, 1993; Kennedy, 1993). For instance, the Speedy recommendations for teacher education (DEET, 1989), unjustifiably downplay references to the content pedagogic element in structures for teacher education programs and arguably put at risk the development of effective models for teacher education .

The purpose of this study is to assist in developing structural models for the cognitive operation of teachers by gathering information concerning the content of mathematics teacher knowledge bases. Previous studies, most notably that of the Board of Teacher Education in Queensland (1985) have focussed on teachers' formal qualifications rather than, as in this study, teachers' operationalised knowledge bases.

Knowledge bases for the mathematics teacher

Shulman (1986, 1987), in his influential studies on the formation of expertise in teaching, suggests that teacher knowledge needs to be considered first and foremost in any move to correct the "the resoluteness with which research and policy have so blatantly ignored ... aspects of teaching in the past". A key feature of the knowledge typology he proposes is the distinction made between knowledge of the teaching target (content knowledge), knowledge of pedagogical structures and procedures adequate to the task of teaching this material (pedagogical content knowledge), and curriculum knowledge. Essential to effective teaching, in Shulman's view, is the development of sophisticated conceptual and procedural knowledge structures within each of these knowledge domains. Shulman argues that typical teacher education programs focus too much attention on content knowledge at the expense of pedagogic content knowledge, and therefore fail to adequately equip beginning teachers. Review of the research literature on the knowledge bases of mathematics teachers reveals a significant weight is indeed given to mathematics content knowledge over pedagogic content knowledge (consider, for example, McNamara, 1991). This would support Shulman's general contention, and appears to be based on the unjustified belief that expert teachers directly combine content knowledge with general pedagogic constructs without recourse to systematic structures characteristic of content pedagogical knowledge.

In summary, recent research suggests that mathematics content knowledge is a necessary, but not a sufficient condition for good mathematics teaching. An outstanding task for mathematics education (teaching and curriculum, research, policy) must therefore be to make content pedagogic knowledge available for scrutiny, critical appraisal, and validation. New methods in order to accomplish these tasks may need to be developed. Making this aspect of mathematics teacher knowledge more visible would raise the status and also help to demystify expert teaching (Evans, 1993). Mathematics teacher knowledge would become

more recognisable as a body of rich and conceptually rigorous knowledge, rather than merely a serialised rendition of instructional 'methods'.

Method

This pilot study gathered data by survey questionnaire. A number of primary and secondary schools were approached seeking their cooperation in gaining access to teachers of mathematics at all levels. A total of 44 teachers in 10 schools (13 teachers in 4 primary and 31 teachers in 6 secondary) responded to the request. Years of teaching experience ranged from 2 to 32 years; 10 teachers serving up to 15 years, and 15 teachers with more than 15 years experience.

The questionnaire was designed to allow teachers to give an indication of their current status in relation to their mathematics content knowledge, mathematics pedagogic content knowledge and mathematics curriculum knowledge. This data was sought in an indirect fashion. For example, to tap into their content pedagogic knowledge, the teachers were asked to describe a recent mathematics lesson in which they used a successful teaching technique, and why it was successful. A further item sought an example of another technique which reflected their teaching style.

The teachers' curriculum knowledge was examined by asking them to acknowledge the sources of their teaching ideas and their involvement in mathematics in-service seminars. The mathematics content knowledge bases were investigated by asking teachers to describe the most interesting mathematics topics and the most challenging mathematics topics they had encountered and studied. A further indication of their knowledge bases was sought through self evaluation of their own teacher preparation and their perceived needs for upgrading of skills and knowledge. Data was collected also in relation to formal qualifications in mathematics, mathematics pedagogy, and years of experience.

Results

Mathematics Content Knowledge

Of the primary teachers surveyed, 54% said they judged themselves to be sufficiently well prepared in mathematics content knowledge, 8% said they were not, and 38% did not respond to the question. Of the secondary teachers, 45% were satisfied with their content knowledge, 19% were not, and a further 35% did not respond. Table 1 indicates the highest level of mathematics attainment.

When asked to indicate the most interesting mathematical topic studied 39% quoted a topic they attributed to tertiary level study. Topics most mentioned were within discrete mathematics and included operations research, applied linear algebra and optimisation theory. Calculus and analytical mechanics also were frequently mentioned. However, in a small

number of cases there was evidence that this material was not genuine tertiary content (percentages, "numbers in nature"). 5% of respondents indicated a topic drawn from the secondary curriculum (symbolic equations, and probability and statistics); and a further 27% of respondents indicated informal sources such as source books, television, science fiction and other materials (chaos theory and fractals, linear programming, complex numbers). Only one respondent (2%) indicated mathematics in-service as a source for interesting mathematics knowledge.

Table 1: Highest level of mathematics attainment by school sector

Highest level of attainment	Primary (%)	Secondary (%)	Total (%)
Mathematics major	0	13	9
Some tertiary level mathematics*	0	55	39
Secondary mathematics	85	16	36
No response	15	16	16

*This excludes curriculum/ methodology subjects

Similarly, when asked to indicate a topic of greatest mathematics challenge 45% proposed a topic derived from tertiary study (calculus, complex variables, numerical techniques, computer programming); 14% referred to a topic in the secondary area (algebra, trigonometry, calculus); 7% indicated informal sources such non specified personal resources such as "an old text book I bought for $1.50". One respondent nominated the multiplication algorithm as the most challenging topic encountered. Only one respondent (2%) indicated mathematics in-service as a source for challenging mathematics knowledge.

Pedagogic Content Knowledge

When asked to consider the adequacy of teacher preparation in mathematics 64% reported concern about the current level of their knowledge in important facets of contemporary methodology, eg assessment strategies, use of technology, and the role of language. 18% indicated the belief that they had been sufficiently prepared; and a further 18% did not respond to this question.

At the primary level it is assumed that the majority of teachers undertook mathematics education appropriate to primary school mathematics. It is interesting to note that some teachers (46%) did not acknowledge any qualifications in teaching mathematics either separate from or part of this initial award. At the secondary level, 55 % had mathematics teaching

qualifications, 26% did not possess qualifications to teach mathematics at this level, and the remaining 19% did not respond to the question.

Some subjects appeared not to appreciate the difference between content pedagogic knowledge and content knowledge.

Responses to Questions 1 (a), 1(b), 1(c) and 2(a), 2(b) provide the opportunity to examine and obtain corroborating evidence for the level of respondents' pedagogic content knowledge. Topics teachers chose covered a wide range of syllabus items in the areas of number, measurement, and algebra. Methods selected were as set out in Table 2. Responses to Question 2(a) indicated that these methodologies were not necessarily typical of normal teaching practice. Whilst 48% reported these strategies were typical, 16% said they were typical to some degree, 27% indicated they were atypical, and 9% did not respond to the question.

Table 2: Illustrative teaching methods nominated by school sector

Method	Primary (%)	Secondary (%)	Total (%)
Activity (concept formation)	40	38	39
Exposition	10	22	18
Activity (consolidation)	20	16	18
Problem solving	25	16	19
Games	5	5	5
Discussion	0	3	2

Table 3: Factors to which teaching success was attributed

Teaching success attributed to -		No. of responses	Total
Category	Sub-category		
Establishing links between knowledge domains	• Prior knowledge linked to target knowledge • Real life linked to classroom • Concrete linked to abstract	2 10 17	29
Understanding	• Deep • Surface	3 1	4
Teacher resources	• Simplicity • Feasibility (time) • Rapid feedback • Personal characteristics	2 1 1 1	5
Student Autonomy		8	8
Affect	• Interest and motivation • Student participation • Student enjoyment of lesson • Incentive to learn • Confidence • Relationship between affect and cognition	12 8 5 2 1 1	29
Non-scientific explanation (eg "it works!", "I love my subject" etc)		7	7

In order to obtain an indication of possible institutional influences on teachers concepts of successful teaching, the above factors were reclassified in terms of student learning (establishing knowledge links, understanding, student autonomy, student affect), teacher's management (teacher resources), and non-scientific factors (see Table 4).

Table 4: Mean number of attributed factors per teacher by school

School	Mean number of attributed factors per teacher		
	Student learning	Teacher's management	Non-scientific
Primary 1	3	0	0
Primary 2	1.375	0.125	0
Primary 3	2.5	0	0
Primary 4	2.333	0.667	0
Secondary 1	1	0	0
Secondary 2	1.77	0.29	0.14
Secondary 3	1.17	0	0.33
Secondary 4	3.14	0.14	0
Secondary 5	0.25	0	0.75
Secondary 6	1	0	0.4

Table 4 indicates that in terms of pedagogic content knowledge (1) there are clear differences between primary and secondary, (2) the knowledge profile is more even across primary schools than secondary schools, and (3) for the secondary schools in the sample there is marked clustering of teachers with similar pedagogic content knowledge profiles within some schools and not others.

Curriculum Knowledge

In Question 1(d) 47% of respondents claimed that the items nominated in Question 1 (a) as successful methods were original ideas. 21% were not able to give a source for the item. Sourcebooks and textbooks were used by a further 11% each, the remaining 10% used worksheets, journal articles or kits. Of the total sample, 14% did not respond to this question. However, when asked in question 1(e) to what extent the source material had been modified 38% reported that no modifications took place. This result is inconsistent with the 47% who reported the item was original. Some modifications to source material were reported by 59% of respondents. Only one subject reported significant modifications to the source material.

Results gathered from Questions 3(b) and 3(d) enable us to categorise the sources of mathematics content knowledge development applied in the curriculum context. For 3(b) the most interesting mathematics topic 43% referred to formal education (secondary or tertiary) as a source, 30% referred to personal sources such as television, general reading etc, and only 5%

referred to professional development programs such as in-service experiences or conferences. Similar results were obtained for 3(d) the most challenging mathematics topic.

When asked to describe a mathematics in-service experience which was of benefit, 52% were able to nominate a topic, however 48% reported that no in-service had been of benefit. Topics identified were wide ranging and included problem solving, assessment , and use of technology.

48% of the total sample thought that in-service courses should be credited towards formal qualifications at tertiary level, a further 16% offered conditional support for this suggestion, and 20% did not agree with the suggestion. 16% of the sample did not respond. Of those who thought formal qualifications would be desirable, most thought that Masters degrees (47%) or Graduate Diplomas/Certificates (37%) would be the appropriate award for credit.

Discussion

Mathematics Content Knowledge

Results indicated that 55% of teachers sampled were not prepared to say that in their own judgement they are sufficiently prepared in mathematics content for the tasks of classroom teaching; secondary teachers appearing to be more concerned than primary teachers. The adequacy of content knowledge from the perspectives of quantity and quality attainment therefore emerges as a major issue. Considering that the percentages of teachers who nominated tertiary level topics as providing interest and/or challenge were 39% and 45%, respectively, we can conclude that the content knowledge base of over 50% of the sample is at a sub-tertiary level. Underlining this conclusion is the finding that the highest level of content knowledge attainment of approximately one third (32%) of the secondary teachers in the sample was pre-tertiary. Further, when considering the source of mathematics knowledge it is notable that the impact of in-service programs is negligible (only one respondent commented favourably in this respect). This finding raises questions about the adequacy of the provision of in-service in mathematics content areas and points to teacher reliance on informal sources for extending mathematical knowledge.

Pedagogic content knowledge

Significant differences in the pedagogic content knowledge bases between primary and secondary level teachers were observed; this may not be surprising given the finding that 45% of the secondary teachers surveyed do not have formal qualifications in mathematics pedagogy whereas such studies are the norm for primary school teachers. In fact our study reveals that almost two thirds (64%) of teachers in the sample reported concern about the current level of their knowledge in important facets of contemporary mathematics teaching methodology.

Notwithstanding this concern, our results demonstrate that a wide range of teaching strategies are at times utilised in mathematics classrooms (Table 2) and that overall, teachers are able to articulate a rationale for these practices to a reasonable depth (Table 3). It is notable that the principal focus was placed on student learning rather than on conditions for teacher function, and that this is generally consistent with a constructivist paradigm for mathematics education (Confrey, 1987; von Glasersfeld, 1984; Yackel et al, 1990). However, key differences emerge (a) between primary and secondary, and (b) within the secondary sector. Primary level teachers seemed to be more articulate with reference to attributed factors for successful teaching than their secondary counterparts. Within the secondary sector, significant differences exist between schools in terms of teachers' ability to articulate rationale for effective teaching. Moreover, the profile of factors mentioned by teachers varied significantly across secondary schools; those schools which scored a significantly lower mean number of student learning factors per teacher ranked non-scientific factors more highly.

Curriculum knowledge

Relatively few teachers in the sample (11%) used source book and textbook materials in forming lessons. This suggests either that these materials were not considered adequate or they were not considered at all; in the latter case it is likely that teachers are unaware of the range of materials available or do not commonly have these at their disposal in the school. In this study evidence was found for both of these alternatives. In-service experiences provided another source of teaching ideas, and although half of the sample (52%) acknowledged that they were of some benefit, it appears that ideas derived from this source are not routinely operationalised in classroom teaching practice. The effectiveness of conventional in-service programs is therefore questioned. Considerable interest was shown by the sample in in-service models which incorporated credit for formal qualifications at tertiary level.

Conclusion

Results of this study have revealed that while there are strengths in mathematics teacher knowledge bases, particularly at primary school level, there are general grounds for concern in relation to teachers' content knowledge, pedagogic content knowledge, and curriculum knowledge; moreover, these concerns tend to be concentrated at the level of individual schools rather than at the level of individual teachers. Therefore, any attempt to improve the quality of teacher knowledge bases should take account of teacher relationships and the formation of the professional ethos within individual schools. A second major conclusion concerns teacher education at both pre-service and in-service levels; it is that reform processes need to balance needs with respect to content knowledge and pedagogic content knowledge. The case of

primary school teachers is instructive on this point: we found that they were significantly less well equipped in mathematics content, yet were considerably more proficient at content pedagogy. This finding correlates with the fact that all primary school teachers undertake mathematics methodology studies at tertiary level, whereas almost half of the secondary teachers surveyed have not undertaken such studies. The importance of the tertiary sector in the education of mathematics teachers is therefore underlined.

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SURVEY INSTRUMENT

1(a) Give an example of a recent mathematics lesson in which you used a teaching technique which you believe was particularly successful. Please indicate the topic, the year level (and subject if secondary e.g. Maths A, B, or C). Provide as much detail of the lesson as possible.

(b) In your judgement, why was this technique so successful?

(c) What was the original source for this technique?

(d) To what extent have you modified it to meet your own needs and situation?

(e) What factors do you consider provide for high levels of learning in mathematics?

2(a) When you reflect on your own teaching style, is this technique typical of the kinds of mathematics teaching strategies you generally use?

(b) Please give an example of another technique which reflects your teaching style.

3(a) What is the most interesting mathematics topic you have read about or studied?

(b) Where did you read about it or study it?

(c) What is the most challenging mathematics topic you have studied?

(d) Where did you read about it or study it?

4(a) Describe a mathematics in-service education experience which was of benefit to you as a teacher of mathematics? (Brief details of topics, provider and methods would be helpful.)

(b) What in-service education courses, seminars or workshops would you like to attend in order to benefit you as a teacher of mathematics?
(c) Should such in-service courses be credited towards a formal university qualification in mathematics education, e.g. Graduate Certificate, Graduate Diploma, Masters degree, etc?
(d) Which formal university qualifications would attract you when considering and choosing an in-service course appropriate to your career?
5(a) Do you feel that your education in mathematics has prepared you sufficiently for the classroom?
(b) What topics would you appreciate further study in e.g. number, algebra, measurement, chance & data, calculus, geometry?
(c) Describe your qualifications in the subject of mathematics itself (not teaching).
(d) Do you feel that you have had sufficient preparation in the area of teaching mathematics, e.g. alternative assessment strategies, SPS (Student Performance Standards), use of new technology such as graphical calculators and computer software, the role of language in learning mathematics?
(e) Describe your formal qualifications in mathematics teaching.
6. How many years experience have you had in teaching mathematics at various levels?