MATHEMATICAL KNOWLEDGE AND THE INTERMEDIATE SCHOOL TEACHER

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The results of a two-year professional development program in teaching mathematics for intermediate and secondary school teachers showed that the intermediate school teachers involved changed less than did the secondary school teachers. The limited mathematical knowledge of the intermediate school teachers was seen as a major constraint preventing them from making changes consistent with a constructivist approach to learning as recommended in a new national mathematics curriculum. Their limited mathematical knowledge tended to result in a narrow perception of mathematics and mathematics learning. They tended to reflect upon their classroom teaching more in terms of attitudes toward mathematics than in terms of values drawn from a broad mathematical knowledge. This reflection in terms of attitudes was unlikely to lead to improvement in their students' learning.

The authors have recently completed research on a two-year professional development project. Teachers Raising Achievement in Mathematics, or TRAM (Britt, Irwin, Ellis & Ritchie, 1993). This action research project involved 8 intermediate school teachers (students aged 11 and 12) and 10 secondary school teachers (students aged from 13 years). All teachers were asked to examine their own classroom practices and the effect that these had on student learning, decide what changes they would like to make, experiment with those changes, and then reflect on why changes were or were not effective in improving students' achievement in mathematics.

In including both intermediate and secondary school teachers we acknowledged the differences in the duties and training of these teachers. We also acknowledged the difficulties that many students have in moving between the two schooling systems. We hoped that having the groups work together would increase the teachers' understanding of difficulties that students experienced in moving between schools. Including both groups of teachers had initial advantages, but as the project advanced over two years, it also emphasised the differences between the groups.

Extensive qualitative and quantitative information was collected on the process and the results of the professional development project for the teachers and students involved. The majority of teachers were experienced. The years of experience ranged from 1 to 26 (mean 9.8) for intermediate school teachers and 2 to 28 (mean 12.5) for secondary school teachers. Results showed that secondary school teachers made more changes in their behaviour, toward a pedagogy more compatible with a constructivist view of learning, than had intermediate school teachers ($p < .05$). For example, secondary school teachers were more likely to 'tinker to see how (mathematical) ideas fit together' than were intermediate school teachers ($p < .01$) or 'have students write mathematics in their own words' ($p < .05$). Intermediate school teachers had as their most important objective "developing an awareness of the importance of mathematics in everyday life", while secondary school teachers rated this objective in fourth place, after "developing an attitude of enquiry", "developing an interest in mathematics", and "developing a systematic approach to problem solving". Case studies showed that the nature of experimentation and reflection done by the two groups of teachers was different. The reflection by an intermediate school teacher that indicated the greatest change was, "I now constantly ask the question, 'What is the mathematics in this activity:' " Secondary school teachers, on the other hand, reflected on, their increased ability to observe and help their students, their interest and ability to influence their colleagues and to influence the wider educational system.

These differences may be related to several factors. One is the job of the intermediate school teachers, who teach the same students for all subjects and who are therefore considered to be curriculum generalists rather than subject specialists as is the case for secondary school teachers. Another difference is the pre-service training of the two groups of teachers. Historically, in New Zealand, people wanting to be primary and intermediate school teachers went directly from their own secondary schooling to a teachers' college. Secondary school teachers, on the other hand, were expected to have a university degree in their specialist subject before having a briefer period of training to be teachers. The intermediate school teachers who were in this study for the full period had finished taking mathematics in Form 5 or Form 6 at age 15 or 16. The secondary school teachers, on the other hand, had an additional one or two years of secondary school mathematics and an average of 2.8 years of University mathematics.

While this difference in personal years of schooling in mathematics does not necessarily produce a difference in ability to teach mathematics, we argue that under the new national curriculum (Ministry of Education, 1992), which expects teachers to build mathematics lessons upon students' experiences and to integrate different areas of mathematics, the intermediate school teachers' limited background in mathematics was often a major handicap. This curriculum is underpinned by a constructivist view of learning. Although it provides broad achievement objectives and some sample activities, it does not tell teachers what to teach in the same way that some previous curricula have. Inherent in the curriculum is an expectation that teachers have a thorough knowledge of the appropriate mathematical content and an understanding of the connections among related mathematical ideas. Teachers who do not have this knowledge and understanding are likely to hold an instrumental or procedural view of mathematics in which mathematics is regarded as a set of tools comprising facts, rules and skills. They are therefore unlikely to be responsive to students in ways that reflect the constructivist approaches recommended in the curriculum. They are likely to present their instrumental views through a mechanistic teaching approach that uses demonstration to emphasise rules and procedures.

There has been considerable interest in the mathematical knowledge needed by pre-service and serving teachers (e.g. Ball & McDiarmid, 1990; Ball, 1993; Putnam, Heaton, Prawat & Remillard 1992). Putnam et al. discuss teachers' knowledge and beliefs about learning, teachers' knowledge of mathematics, and teachers' knowledge and beliefs about mathematics. Ball cites Schoenfeld as arguing that "learning to think mathematically means (a) developing a mathematical point of view - valuing the process of mathematization and abstraction and having the predilection to apply them. and (b) developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structure - mathematical sense-making." (p. 376).

Our findings and concerns are in agreement with those of these writers. We argue that teachers who have a broad and integrated knowledge of mathematics in advance of the mathematics that they teach are also more likely to have a perception of the nature of mathematics as "fallible, changing, and like any other body of knowledge, the product of human invention" (Ernest, 1991, p. xi). We argue further that such teachers are more likely to be able to use teaching approaches that emphasise problem solving so that students can construct appropriate knowledge, and be able to articulate for themselves a set of values related to mathematics and mathematics teaching which reflect this view of mathematics.

The two examples given below, from our research, show the difference that these factors can make when teachers attempt to improve their practice.

A Teacher with an Adequate Mathematical Background Who Was Able to Reflect on Her Teaching

One of these teachers was Jennifer ("Teacher 1" in Irwin & Britt, 1991). She was a teacher who had had a year of university mathematics, unusual among intermediate school teachers. She left the project after the first year so is not included in the final data, but her experiences in the first year were of interest. She had been teaching for three years in a traditional manner, with tightly structured lessons which included a quick quiz, a teacher-directed session, a worksheet to complete, and similar work for homework. She was always well prepared, and felt secure with this type of teaching. Her impetus for change came from a comment of her 15-year-old daughter, who complained, "why don't they ask us in exams what they have taught us, and not put it in a different way?" She realised that her daughter was learning procedures, and had difficulty transferring this knowledge to novel problems.

In her own class she wanted to try having students develop their own problems to help them see the generality and applicability of what they were learning. This would also give them a way of learning how mathematics could be applied to the ill-structured nature of problems that appeared outside of school. She chose to have the pupils in her class write their own problems for the topic being studied, using situations from outside school. She discovered that this was not something that her pupils could do immediately. She therefore used the steps of having them write a problem in groups, then put these problems on the board to see if others in the class could understand them, sorting out the information that a problem needed to contain if it were to be solved, and then helping students to rewrite the problem so that it could be solved by others in the class. Students then wrote problems individually, from their home experience, and these problems were used in class as the basis of future lessons.

This teacher's comments in her journal describe both her feelings about this innovation, and the students' response:

> "22.7.91 Feel as if I have made a breakthrough here. The 40 minute period was spent totally covering around 6 of the problems created by the children. I was surprised at the number of skills we covered by dealing with these problems, The class was very attentive and good discussion took place. Children who gave problems felt a great sense of importance.
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> I feel excited about outcomes today. I discussed with the class how they felt. I firstly asked whether they enjoyed the lesson. Overwhelming "Yes". They certainly appeared highly motivated. Secondly I asked whether they learnt more this way, and I wanted honest answers. They felt they had learnt many different skills and that they UNDERSTOOD the use of fractions.
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> I felt quite anxious at the beginning of the lesson as I did not have my normal mental, reinforcement, followed by teaching of a new skill, then a worksheet. I actually had nothing prepared. The children gave me what I needed and took greater responsibility for their learning. I feel their self esteem has improved through sharing their ideas."

This lesson was successful, and met the curriculum's criterion of working on problems that were meaningful to the students. It drew out and dealt with a variety of ideas and techniques for dealing with fractions, and the relationship between these ideas. Jennifer could teach this lesson entirely from student prompts (despite her anxiety) because of her firm conceptual and relational knowledge of the mathematics involved. She could see the potential relationships between the knowledge that her students had, and that which they needed in order to solve what were originally ill-structured problems (Resnick 1988). In reflecting on the lesson, she looked at what had happened in relation to values that matched what she valued about mathematics and mathematics learning. These included: students talking and writing mathematics, students understanding mathematics, students taking greater responsibility for their learning and students reflecting on their learning. The fact that students brought problems from home had the effect of providing in-built checking procedures, a feature of situated mathematics.

A Teacher Whose Limited Mathematical Knowledge Restricted Her Ability to Reflect on Her Teaching

Emily, another teacher in the project, had a limited mathematical background, and provides an important contrast to Jennifer who had taken a University level mathematics course. Emily said that although she learned from participating in the TRAM project, she never felt comfortable in it, especially not in the presence of the secondary school teachers. This teacher also ran tightly structured classes. These began with a skills test and were followed by work in ability groups, working on exercises, word problems, or with Emily on number concepts. Her lessons were heavily procedural. She saw herself mostly as the "explainer". However, in observed classes she let some serious misconceptions pass unchecked. For example, for much of the first year of the project she placed emphasis on learning to name place value. In an observed lesson on writing expanded numerals, she asked for the expanded version of 25002.2. A student wrote 200000 + 50000 + 2 + .2, and was commended for her effort before being asked to clean the board. It seemed that Emily's own limited knowledge of place value prevented her from recognising in the moment that the student's answer represented either a careless error or a serious misconception. Although she reported in an interview that she did quite a lot of problem solving, she used a book as the source of problems, and appeared not to be able to solve some of these problems herself. The confused and tentative nature of some responses that she gave did not stimulate the discussion and investigation that might have led students to adjust their understanding. She found that a group of students were having difficulty with the problem: "Mary is 6 years older than Jean. The sum of their ages is 20. How old is Mary?" Two students had written as their answers, "20 and 26". She did not ask whether or not this answer satisfied the constraints of the problem, or give the students help, for example, by offering the hint "*Suppose the girls were the same ages. What would their ages be?*" Instead she only said "Put your thinking caps on". She did not ask students who had successfully solved the problem to discuss their reasoning. There were few inbuilt ways for students to check their mathematics, as it was rarely tied to situations of importance to them.

During the period of the project she reported a growing commitment to incorporating everyday contexts and more games in her mathematics program. Nevertheless, despite some minor observed shifts, Emily herself maintained that she had changed very little. She believed strongly that learning tables and procedures came first, and that, for many students, there was little time for conceptual understanding. She suggested that, for her, there was a conflict between changes to a less structured style of teaching suggested by other project teachers and the new mathematics curriculum, and the culture of the school which had a strong focus on testing across the school. She felt that the students liked her tightly structured lesson format and that pressure for increased testing from the school administration had prevented her from moving away from her current approach. She commented:

> "...if you wait for understanding, sometimes you could be working at a very basic level, and I know understanding is important. When I went through school, I did subtraction using equal additions... But I didn't understand what I was doing. I had no idea where this ten came from or where I was giving it back to. I just knew I had borrowed ten and I had paid it back. But I could do subtraction and I always got it right and I felt good about that." (May 1992)

When interviewed about any changes she had made as a result of her participation in the project. Emily made the following comments.

> "I thought that I would get guidance but the project has actually wanted me to find out, me to do. And there's nothing wrong with that but ...ideas don't just come. It needs to be a sharing. So I was sort of hoping that I would come back from there motivated to do, whereas that hasn't actually been the case.
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> ...If the project is going to move from here into helping teachers, there's got to be an awareness of all that (fund raising, the need for school promotion resulting from the removal of school zones) because those are the realities of school, particularly at this level. more so than at high school where teachers get their (free) period most weeks."

This comment about high school teachers was repeated in an evaluation that she completed at the end of the project.

> "My confidence as a teacher was heavily undermined due to the project. I was given the impression by the high school teachers that we did no work and taught nothing and were particularly looked down on. From the project itself I got the impression that I was not working enough on maths or had the right emphasis on maths even though there was not enough time to do this with maths as well as teach everything else." (Nov 1992)

The most obvious effect of this professional development project on Emily was to arouse her old anxiety about mathematics. The demands of the new curriculum that she teach mathematics in a different way aroused emotional responses that had only just been kept in control in her role of teacher as explainer. She repeatedly objected to the presence of secondary school teachers, who made her feel "dumb". She used diversionary tactics to avoid doing mathematics in group sessions or being observed in her own classroom. Her behaviour fits Mandler's view (cited in McLeod, 1992) that the emotion aroused by her schema was stronger than her memory for mathematics which she may have learned.

> "...when I went through teachers college, they were right into the new maths and I cried a year through college.... A year of maths at college put me right off maths and it's taken me a long time to sort of say it's okay, I can cope again."

Emily's own anxieties and lack of confidence were reflected in her goals for the project. She wanted to have her students enjoy mathematics and she wanted them to know the basics so that they could have a sense of achievement. But she also wanted them to see the relevance of what they were doing and to develop an understanding of the processes.

Her class did appear to be a happy place to be overall. However, her happy class made very little progress. On the Chelsea Diagnostic Test of Place-value and Decimals (Hart, K., Brown, M., Kerslake, D., Kuchemann, D. & Ruddock, G., 1985), given at the beginning and at the end of the school year, 37% of the class increased one level or more, 44% stayed on the same level, and 19% scored at a lower level at the end of the year than at the beginning. The average rate of progress for the class was the lowest of any of the classes in the project, and considerably lower than the rate of progress shown by a large sample of English children (Hart, 1981).

This professional development project was predicated on the proposition that short-term in-service programs had no more than short-term effects on the teachers who attended them. If more important professional change was to take place, it had to happen because the teachers saw the need for it in their own teaching, through carefully observing their students, thinking about what different teaching methods might prove more successful, and most importantly, reflecting on the effect of their teaching. The report on this project (Britt et al. 1993) concluded that teachers' reflections lay at the heart of a successful professional development project. The teachers who made important changes in their practice did so by reflecting on their pedagogy against a background of mathematical understanding and a related set of values about mathematics and mathematics learning in order to see if their pedagogy was in fact enabling their students to construct a more complete understanding of mathematics. Without such a background, teachers like Emily could only reflect against that aspect of mathematics that was most important in her own learning, her negative feelings about the subject.

These two teachers fell near the extremes of those intermediate school teachers involved in this project. The needs of the first teacher could be met by a professional development program which depended on reflection for improvement of teaching. The needs of the second teacher were not met. What then do teachers like Emily need in order to improve their practice? We contend that they do not need one-day in-service programs that suggest new activities that they might do. When teachers have strong anxiety about mathematics, this anxiety needs to be acknowledged and addressed first, probably in the company of other teachers who have had this anxiety and learned ways of overcoming it. Then they need an extended in-service program that emphasises the relationships among different mathematical ideas and challenges them to find the mathematics in ill-structured situations. They need time to reflect on the mathematical ideas that emerge from their investigations, and the mathematical thinking that they use to reach successful solutions. Such a program needs to raise awareness of mathematics as a way of knowing about our world (Bishop, 1988). At regular intervals during such a program teachers need opportunities to articulate their own changing values about mathematical knowledge and about teaching mathematics. As teachers develop a deeper, integrated mathematical knowledge and change their beliefs and values, they will need time and collegial support to experiment with new approaches in their classrooms. Only then is it likely that changes in classroom teaching practices are likely to be sustained in ways that can lead to improved student learning.

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