Mathematical Errors in Fractions: A Case of Bruneian Primary 5 Pupils.

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This paper reports on an ongoing study of errors in the use of fractions exhibited by a sample of 396 Primary 5 pupils in Brunei Darussalam. Among the five types of errors identified, those demonstrating a lack of understanding of basic facts predominated. The quantitative data reported here were obtained from a diagnostic pre-test administered in a longitudinal study. The test was administered during the first phase of the study, and coupled with qualitative data obtained from pupil interviews. After having identified the different errors exhibited by the pupils, the study set out to investigate in which components of the fractions syllabus significant errors occurred. Data from the pre-test was shared with the 15 teachers whose pupils participated in this study in order to assist the subsequent teaching of fractions. The presentation will include a discussion of possible reasons for the identified errors.

Children encounter fractions and fraction-related concepts both in real-life and in classroom situations, and a firm understanding of fractions undoubtedly helps children make sense of a number of other ideas in their daily life. Despite the context in which children engage fractions, it is generally agreed that this topic provides teachers with insight into developments in children's understanding of and relations among numbers. These understandings are built on children's personal experiences, their intuitions, and formal knowledge they gained in the classroom. Though often complex in character, fractions provide pupils with important prerequisite conceptual foundations for the growth and understanding of other number types and algebraic operations in the later years of their school experience. Despite the critical conceptual link between mathematics strands such as space and measurement provided by fractions, the topic continues to present problems and difficulties for children in primary schools (Pitkethly & Hunting, 1996). Children tend to make all sorts of errors, not only in the computation of fractions but also in the basic concept.

The acknowledged difficulties in learning fractions are reflected and documented in a number of studies that have examined different aspects of this topic. As early as 1958, Hartung acknowledged that the fraction concept is complex and cannot be grasped all at once: It must be acquired through a long process of sequential development. Orton (1992) supported this view when he wrote that the concept of fraction develops over a long period, during which time children experience the different meanings of fractions in a variety of situations. Researchers have concluded that this complex topic causes more trouble for elementary and middle school pupils than any other area of mathematics (Bezuk and Bieck, 1993). Teaching fractions is therefore both important and challenging at the lower level of schooling. Consequently, in teaching fractions, teachers should provide experiences that involve other mathematical concepts including number, length, weight, and money, and these should be set in meaningful situations to which children can relate.

In the Brunei Education system, mathematics is considered a very important and core subject, and it has been seen and will continue to be a basic education requirement.

Fractions are one of the main topics in the lower and upper primary mathematics syllabus and despite the assertions from researches that it is a difficult and complex topic, children in Brunei are taught the fractions topic formally as early as Primary 3 right through to Primary 4, 5 and 6.

Research shows that pupils tend to hold misconceptions and make numerous errors in dealing with fractions (Ashlock, 1994; Engelhardt, 1977; Swedosh, 1996). The most cited common errors range from the most basic types such as the inability to arrange them in ascending or descending order, grouping errors, basic fact errors and incorrect operations, to more complex problems such as the inability to apply them in solving word problems. As well as these, pupils also tend to make careless errors. Despite correct operations performed earlier, some pupils record wrong answers to the questions given. Generally then, it appears that pupils' knowledge and skills in performing fractions tasks are learned without much depth conceptual understanding. Basically, when pupils are given computational items after each operation of fractions, they are able to solve them well. However, when mixed operations and solving word problems items are given together, problems occur.

This paper reports on one part of a longitudinal study titled *Mathematical Errors in Fractions: A Case of Primary 5 and Primary 6 Pupils in Brunei Darussalam*. In this study, Primary 5 pupils were tested using a 28-item paper-and pencil diagnostic pre-test. Soon after they sat for a written test, forty-eight pupils were selected for an interview, with the aim to probe more on the items on which they performed incorrectly. A total of 48 pupils, 25 males and 23 females and twelve from each of the four participating schools were interviewed (probed) on the items in which they were incorrect in the diagnostic pre-test. The 12 pupils represented the four top, four averages and four below average performers from each school. The information obtained from the diagnostic pre-test coupled with information from the pupils' interviews and information from literature reviews was used to brief the concerned teachers so that they could pay special attention to the areas of fractions in which their pupils were weak. This approach was feasible because the common practice in most primary schools in Brunei is that a particular mathematics teacher follows through his or her pupils from Primary 4 through to 6.

Objective of the Study

The objective of this subsection of the longitudinal study was to investigate the different types of common error patterns exhibited by Primary 5 pupils in the six components of fractions units tested. It further aimed to examine if this group would still exhibit the same error patterns when they moved to Primary 6 in the following year and after receiving further instruction on fractions. If there were errors, an important question was in which of the six components would the errors be more prevalent? The outcome and findings from the study will be used to inform both the teachers involved in the study as well as other mathematics teachers.

The first part of this longitudinal study involved four phases. It concentrated on

- the design of the diagnostic test according to literature and document review ;
- the implementation of the diagnostic test;
- pupils' interview sessions; and

- information sharing with the teachers.

The following research questions guided the study:

- What are the common error patterns exhibited by Primary 5 pupils when performing fraction problems?
- Which of the errors patterns are more prevalent?
- In which of the six components of fractions are the errors more prevalent?
- What are the implications for teaching and learning fractions?

This paper reports on the initial data obtained from the diagnostic pre-test, administered in June 2002.

Methodology

Fifteen classes of Primary 5 pupils in Brunei were given the diagnostic pre-test. Altogether, there were 396 pupils (214 males and 182 females). The pupils were from four government primary schools from the urban and semi-urban area of the Brunei-Muara district in Brunei Darussalam.

The researcher-constructed diagnostic pre-test consists of 28 items, consisting of 8 items on the understanding of fractions concepts and sequencing fractions; 6 items on addition and subtraction; 5 items on multiplication and division; 4 items on determining fraction of a quantity; 1 item on alternative form of fractions, and 4 items on solving mathematical word problems with fraction elements.

Pupils in the sample were taught the topic on fractions about one to two weeks before they sat for the diagnostic pre-test. In addition, they had been introduced to the initial concepts of fractions when they were in Primary 3 and they were taught more simple operations of fractions when they were in Primary 4. No revision was carried out prior to the diagnostic pre-test and the pupils were not informed in advance that they were to be given the test. The pupils worked individually and no consultation either from the teacher (in this case, myself) or classmates was permitted while the pupils were doing their test. The pupils were given one hour to complete the 28 items that covered the six components of fractions unit stipulated in the upper primary mathematics syllabus for Brunei Darussalam Primary schools. The six components are:

- Understanding of fractions concepts and sequencing fractions;
- Manipulating fractions symbols; addition and subtraction;
- Manipulating fraction symbols; multiplication and division;
- Determining fraction of a quantity;
- Alternative forms of fractions; and
- Solving mathematical word problems with fraction elements.

The questions and word-problems given in the diagnostic pre-test represented the types of questions and exercises that they normally did as class exercises, homework exercises and items that were normally found in their test and examination papers.

Results and Discussion

Two separate analyses were carried out in identifying the type of errors exhibited by the sample pupils. The first analysis was to identify the errors exhibited by the pupils in the first five components of fractions units tested, and the second analysis was to identify the type of errors exhibited by the pupils in the sixth component of the fraction units tested.

Five type of errors were identified in the first five components namely; Grouping error, basic fact error, defective algorithm, incorrect operation and careless error. The percentages of each type of errors identified and examples of errors are shown in Table 1.

Table 1

Percentages of Occurrences and Examples for Each Type of Error Among the Five Components of Fractions Exhibited by Primary 5 Pupils

Types of errors	Problem	Example of errors	% of occurrences
Grouping error	$1\frac{2}{3}+\frac{4}{7}$	$= 1\frac{14}{21} + \frac{12}{21} = (1\frac{26}{21})$ (s294, item 7)	7.51
Basic Fact Error	Express 4 kg as a fraction of 10 kg	$= \frac{4}{10} = \frac{1}{2}$ (s188, item 19)	46.29
Defective Algorithm	$\frac{\quad}{10} = \frac{1}{2} = \frac{8}{\quad}$	$\frac{5}{10} = \frac{1}{2} = (\frac{8}{4})$ (pre-test, s187, item 2b)	5.36
Incorrect Operation	$\frac{13}{15}+\frac{8}{15}$	$= (\frac{5}{15}) = \frac{1}{3}$ (s291, item 6)	4.29
Careless Error	Calculate $\frac{3}{8}$ x 64	$= \frac{3}{8}$ x 64 8 x 3 $= \frac{24}{1}$ (but wrote 21 as the answer) (s059, item 12)	2.05

From the table, one observes that the percentage of basic fact errors is larger than all other types of errors and this explains why the performance of the pupils on computational fractions items was very low. The expected highest score was 28. However, from the 396 sample pupils only 44 of them (8.3%) obtained 14 marks (50%) above, with the highest score obtained was 26 and the lowest 0.

Of the four questions on one-step word problems (the sixth component), five types of errors (after Newman, 1976) were identified. The errors were of a reading, comprehension, transformation, process and encoding nature. The percentage and examples for each type of errors committed in the diagnostic pre-test are shown in Table 2.

The table shows that Primary 5 pupils demonstrated with the highest percentage of process error and transformation error, followed by comprehension error, reading error and encoding error. The findings are similar to earlier study by (Mawang 2001), where, in her study with Primary 6 pupils in Brunei, the percentages for process skill and transformational errors were 29.00 % and 27.6% respectively.

The study further examined the percentages of each type of errors for the individual five components of fractions unit, mentioned above held by Primary 5 pupils. The results are presented in Table 3. In examining the percentages, the reader will notice that the percentages do not sum to 100. This is because the percentages of the correct and unattempted responses were excluded as they are irrelevant to the discussion.

Table 2

Percentages of Each Type of Errors and Examples of Errors for the Sixth Component (Word Problems) of Fractions Held by Primary 5 Pupils

Types of errors	Problem	Example of errors	% of occurrences
Reading error	A bag of sugar weighs $\frac{3}{4}$ kg and another bag weighs $1\frac{1}{2}$kg. What is the total weight of the two bags of sugar?	(Indicated by pupils not responding to the written items or just wrote some meaningless responses. During the interviews, pupils were not able to read the questions fluently.)	12.82
Comprehension error	A bottle holds $2\frac{1}{2}$ litres of orange juice. If $1\frac{2}{3}$ litres has been drunk, how many litres of orange juice are left in the bottle?	(Indicated by pupils could read the problems but did not know what the questions asked them to do, thus they just rewrote the given figures). $2\frac{1}{2}\quad 1\frac{2}{3}$ (s024, item 24b)	23.48
Transformation error	A piece of ribbon of length 4 metres is cut into 10 pieces of equal length. What is the length of each piece?	$4 \div 10 = \begin{array}{r} 2 \\ 4\overline{)10} \\ -8 \\ \hline 2 \end{array}$ $2\frac{2}{4} = 2\frac{1}{2}$ (s014, item 24d)	27.08
Process error	Ali bought 5 packets of chicken wings. Each packet weighed $1\frac{1}{2}$ kg. Find the total weight of the chicken wings he bought.	$5 - 1\frac{1}{2} = \frac{5}{1} - \frac{3}{2} = \frac{15}{2} = 5$ (pre-test, s051, item 24c)	27.4
Encoding error	A bottle holds $2\frac{1}{2}$ litres of orange juice. If $1\frac{2}{3}$ litres has been drunk, how many litres of orange juice are left in the bottle?	$2\frac{1}{2} - 1\frac{2}{3} = \frac{5}{2} - \frac{5}{3}$ $= \frac{15}{6} - \frac{10}{6} = \frac{5}{6}$ but finally wrote the answer as $\frac{25}{6}$ (pre-test, s 059, item 24b)	2.21

Table 3 shows that among the five identified areas of fraction unit, Primary 5 pupils exhibited an average percentage of errors between 0.69 to 3.84 for the five components. However, if one looks at the percentages for each type of errors for each component, just like in the overall result demonstrated earlier, the result further shows that most Primary 5 pupils in Brunei have problems with the basic facts of fractions even though they have been introduced to the concepts at P3 and receive further instruction when they are at P4 and P5. Percentages of basic fact errors are consistently higher than the other types of errors in all the five components of the fraction unit tested (3.42 % to17%). The first component of the test (Understanding fractions and sequencing fractions) was considered the easiest component, but this was where the percentage of basic fact errors was the highest. This confirms the results of earlier local studies such as those by Jabaidah (2002) and Shamsiah (2002). Understanding fractions and the ability to order and compare fractions are the prerequisites for subsequent learning of other components of fractions. However if the pupils were weak in this component, it was no surprise that they also performed badly in the subsequent components of the fractions unit tested. This is probably because that in order for pupils to be successful on a particular area in mathematics, they are more dependent on their mastery of the basic concepts and facts acquired earlier (Swedosh, 1996).

Table 3

Percentage of Each Type of Errors for the First Five Components of Fractions

Error Types Components	Grouping Error	Basic Fact Error	Defective Algorithm	Incorrect Operation	Careless Error	Av.% of errors
1	-	17	1.73	0.41	0.05	3.84
2	3.04	12.25	1.57	0.29	0.51	3.53
3	4.25	6.23	1.22	1.03	0.92	2.73
4	0.22	7.39	0.79	2.55	0.57	2.3
5	-	3.42	0.05	0.01	–	0.69

Similarly, the study examined the percentage of each type of errors for the four questions on the sixth component (Word problems) tested. The result is presented in Table 4.

Table 4

Types and Percentages of Errors for Each Question on Word Problems (Component 6)

Question	Reading Errors	Comprehension Errors	Transformation Errors	Process Errors	Encoding Errors
24a	9.09	19.44	7.83	45.96	5.81
24b	12.12	19.69	11.11	49.49	0.76
24c	15.4	25.25	39.39	14.14	1.26
24d	16.16	29.29	49.24	1.01	1.52

The percentages of the reading errors for all the four word problems were consistently low, showing that the pupils had not so much a problem in reading out the given examples, but that the percentages of the comprehension and transformation errors were increasing especially for Questions 24c and 24d. The percentages of the process error for the first two word problems were high. This implies that though the pupils could read and understand the two problems, they were not able to process them correctly. Since many pupils already processed the problems incorrectly, this explains why the percentages for the encoding errors were low. As for the third and fourth problems that required the pupils to perform multiplication and division of fractions, a number of pupils could not process the given information correctly and thus they exhibited transformation errors. Since transformation errors comprised a large percentage of the results, the percentages for the process and encoding errors were low for the third and fourth items for this area.

Conclusions

A considerable number of pupils confuse fraction concepts with whole number concepts. A possible reason for this phenomenon is that the pupils' prior learning of whole numbers and the early introduction of fraction may have had a negative influence on most pupils' understanding of fractions and their operations. Post et al, (1993) and Moss AND Case (1999) have demonstrated that children's whole number schemes can interfere with their efforts to learn fractions and this is what might have occurred with the pupils in this study. This is because by the time fraction instruction commences at Primary 3 level, children have considerable knowledge of whole numbers and how they work.

From the diagnostic pre-test result above, it appears that many Primary 5 pupils still have problems in doing fractions work, appearing to lack the basic fact knowledge. Not only have pupils lacked the knowledge but they also lack the mechanical skills to perform calculations. Evidence from the interviews show that some pupils still considered that their wrong answers were "correct" even after being probed in interviews. Only after being alerted and showed clearly where they went wrong did they accept the fact. For example, a number of pupils found it difficult to express why 0.85 and $\frac{1}{2}$ represented the same number. Perhaps a reason for this difficulty is that their knowledge of these fractions is so basic and fundamental. In a study by Hunting, Oppenheimer, Pearn & Nugent (1998), some Grade 6 pupils also encountered similar difficulty when they found it difficult to understand why _ and 0.5 were equivalent.

Apart from the possible inferences of whole number in fractions work, our experiences and observations as mathematics teachers and teacher educators for the last two decades suggests the two following factors as the possible causes:

1) Ineffective teachers' instructional activities: Ideally, fractions concepts should be introduced and taught using concrete materials and activities. However, teachers in Brunei have been observed to teach fractions using more procedural approach rather than conceptual approach. They overuse direct instruction which focus on content rather than deep understanding. Pupils were trained to learn the concepts by rote, just like when pupils were taught to learn multiplication table by rote, that is, without understanding the processes involved in the multiplication operations. As a result, the pupils could memorize at the time the concepts were introduced and taught, however, after a while, they tended to forget the earlier acquired concepts, and furthermore they were not able to relate them to the newly introduced concepts.

2) Language difficulties: The practice in the Brunei education system is from that all subjects are taught in Malay, the mother tongue, except for English language as a subject from Pre-school up to Primary three. From primary four onwards, the subjects Mathematics, Science, Geography, Art are taught in English, most Bruneian second or third language, in addition to English language as a subject. Other subjects like Malay language, History and Religious Study are taught in Malay. This implies that Mathematics which the pupils start learning as from Pre-school level is taught in Malay. However, when the pupils move to Primary 4, mathematics is taught in English. This switching of the medium of instruction could have an adverse effect on the pupils. As for the fractions unit, when it was introduced in Primary three, they were all in Malay but comes to Primary four onwards, the pupils have to learn them in English. Concepts, which were once learned in Malay, have to be relearned in English and this may cause difficulties to many pupils. Although Brunei is practising the Bilingual system of education, many pupils in Brunei were observed to be inefficient in English, in either speaking or writing (Liew, 1993).

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