High School Students' Interpretation of Tables and Graphs: Some Findings From Fiji

Sashi Sharma
University of Waikato
<sashi@waikato.ac.nz>

Concerns about students' difficulties in statistical reasoning led to a study that explored 14 to 16 year old Fijian students' ideas of statistics. Existing models developed for investigating students' thinking in statistics education were not completely satisfactory for describing these results, so Shaugnessy's model was adapted to explain the data . This paper presents how students made sense of information in tables and bar graphs. The results of the study confirm some findings of other researchers. The paper concludes with implications of the findings for mathematics teaching and research.

Over the past decade there has been a trend to include statistics at every level in the mathematics curriculum (Watson, 1992). *A National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991) suggests that Chance and Data should be one of the major content areas in mathematics. In New Zealand, a significant development has been the restructuring of the Form 7 Bursary Syllabus, where pure and applied mathematics have been replaced by mathematics with calculus and statistics (Begg, 1993). The development of alternative bursary-level courses has put more emphasis on probability and statistics and greater statistics content for lower secondary students. The Mathematics Curriculum (Ministry of Education, 1992) put emphasis on statistics and probability at all age levels from 5 to 18 years old.

In Fiji, selected topics in statistics are taught in schools from grades 8 to 13. The new mathematics prescription for classes 1 to 6 (Fijian Ministry of Education, Women, Culture, Science and Technology, 1994) also gives greater emphasis to statistics at these levels, probably because Fiji had followed the New Zealand curriculum.

These international initiatives have been undertaken without the benefit of research on the learning of statistics (Shaughnessy & Bergman, 1993), and there is a wide gap between the recommendations and what is taking place in classrooms. The Fijian Ministry of Education has just established a research unit, but as Begg (1993) notes there has been no major study there or in New Zealand to help teachers identify how children's statistics learning can be improved, and teaching in statistics has been mostly traditional with textbook orientation and organised by mathematicians rather than practising statisticians.

Shaughnessy (1992) reports that most of the research in probability and statistics has been done with elementary school children or with college students, resulting in an age gap in our knowledge about students' conceptions of probability and statistics at the secondary level. Shaughnessy adds that since most of this research has been done in a few western countries, there is a need to determine whether culture influences conceptions of probability and statistics, and whether biases such as judgemental heuristics and misconceptions are artifacts of western culture or whether they vary across cultures.

Research on Analysis and Interpretation of Graphs

Students are particularly weak in drawing inferences and predicting from data (Asp et al., 1994, Bright & Friel, 1995; Pereira-Mendoza & Mellor, 1991). Asp et al. (1994) described a preliminary study into primary and post-primary students' understanding of pictographs and bar graphs. They reported that students had fairly well-developed skills in reading, interpreting, and predicting from graphs, and that these increased with ability level and peer level, but the students still experienced difficulty related to prior knowledge, missing data, scale, and pattern. Bright and Friel (1995) studied ways that students in grades 6, 7, and 8 make sense of information in bar graphs exploring ideas of reading the data, combining and comparing graphs, and predicting. They report that although students had been exposed to many different bar graphs in and out of school, they were not highly successful with questions that required higher order thinking skills. The students tended to want to move quickly to manipulation of information.

Pereira-Mendoza and Mellor (1991) studied 248 grade 4 and 6 children. The students were questioned on 12 different graphs, covering/familiar topics such as the height of children. All graphs consisted of three questions: a literal question, an interpretation question, and a prediction question. Although there were very few problems with literal reading of graphs, there were major problems with the interpretation and prediction questions. For example, the mean success rate, for the prediction questions for grade 4 students was 16% and for grade 6 students was 18%. The analysis indicated two main sources of errors: data arrangement, and the fact that the information was not shown on the graph. The different arrangement of data resulted in many errors in prediction. For example, when given a graph involving height, many students predicted that the height of the ten-year-old child was more than that of a nineteen-year-old. Some even persisted with this interpretation by implying that, although this was not reasonable, it had to be true from the pattern. Some students tried to look for a pattern even for graphs with non-patterned data. This also occurred in cases where the data was not ordered in magnitude or when any attempt to search for a pattern made no conceptual sense. Students could also not give an answer because the information was not on the graph, e.g., when asked to predict data for 1990, they said that they could not since 1990 was not on the graph.

Concerns about the importance of statistics in everyday life and in schools, together with the lack of research in this area and students' difficulties in statistical reasoning, determined the focus of this study. The study was designed to investigate how form five Fijian-Indian students construct ideas about statistics.

Overview of the Study

The secondary school selected for the research was a typical Fijian Indian high school. The sample consisted of a class of 29 students, aged 14–16 years: 19 girls and 10 boys. The main study was preceded by a pilot study in a New Zealand secondary school. A small sample of 14 to 16 year- olds of differing abilities and ethnic backgrounds was interviewed before and while taking part in an instructional unit on statistics and probability taught by the class teacher. None of the students had had previous instruction on statistics. The whole class participated in the first phase of interviews prior to the instruction, and 14 students (representative of the larger group in terms of abilities and gender) during the instruction. Interviews were audiotaped and transcribed. Notes were made of student non-verbal behaviours observed during the 40–50 minute interviews. Paper, pencil, and a calculator were provided for the students if needed. Open ended interview questions and tasks were selected and adapted from those used by other researchers. The appropriateness of these interview tasks for the Fijian children was established by checking the tasks with the Ministry of Education Mathematics Syllabus (Ministry of Education, 1988).

An adaptation of Shaughnessy's (1992) model was used to analyse the data from the interviews to assess the students' initial knowledge and understanding in statistics and probability. The adaptation arose from a consideration of the data collected.

Results and Discussion

The main focus of this paper is the non-statistical responses (in which students made inappropriate connections with learning in other areas) and the partial-statistical responses (in which students applied rules and procedures inappropriately or forced patterns on data). Extracts from typical individual interviews are used for illustrative purposes.

Interview Responses About Interpreting Tables

The task comparing temperatures of Ba and Sigatoka (Item 1) was used to elicit the students' ideas about interpreting tables. The first question about whether Ba is warmer than Sigatoka was used to explore if students could read tables, and the second "What else do these figures reveal about the temperatures in Sigatoka and Ba?" attempted to find if students could make predictions from tables. Results are summarised in Table 1.

Item 1: Task comparing temperatures of Ba and Sigatoka

Temperatures (in degrees C) were taken from Sigatoka and Ba on six consecutive days.

(i) Look at the temperatures from both the towns and decide if Ba is warmer than Sigatoka. How do you know?

(ii) What else do these figures reveal about the temperatures in Sigatoka and Ba?

Day	1	2	3	4	5	6
Sigatoka	25	24	21	20	23	24
Ba	28	27	26	25	29	30

Table 1

Response Types for the Tasks Comparing Temperatures (N = 14)

Response type	Task (i): Reading tables	Task (ii) Interpreting tables	[Both tasks]
Non-response	–	2	–
Non-statistical	5	5	[5]
Partial-statistical	–	3	–
Statistical			[4]

While nine students could read tables, only four could both read and interpret tables. One possible explanation for these differences in reading and interpreting tables could be a lack of emphasis in classrooms on interpreting tables. Students who lack this experience would be more likely to use non-response or non-statistical categories.

Non-statistical Responses

The non-statistical category consisted of students who mostly related the data to their everyday experiences in non-statistical ways. Of the five students who gave non-statistical replies on the first task, four based their reasoning on their everyday experiences. The students said Ba was warmer than Sigatoka and when asked to explain their answer talked about the real weather conditions of Ba and Sigatoka. For example, Student 21 explained,:

> Each day the temperature for Ba is greater than the temperature for Sigatoka. And normally Sigatoka is called a valley. They are producing fruits; it rains there. My one uncle is there. He mainly plants Chinese cabbages; because of the rain it grows so well there.

Student 9 said Sigatoka was warmer than Ba because the temperatures for Sigatoka were lower than the temperatures for Ba, and even when questioned she did not change her explanation. It is possible that the student had language difficulties, confusing "warm" with "cold" although the researcher did try to explain the terms in Hindi by using an example of cold and warm water. The other four students continued to base their reasoning on everyday experiences. For example, Student 3 explained,

> Yes, because as I told before Sigatoka is a rainy place. It hardly rains in Ba.

Although this study provides evidence that reliance upon experience can result in biased, non-statistical estimates, in some cases this strategy may provide useful information for other purposes. For example, the student has drawn on relevant common sense information. The response raises further questions: Is there a weakness in the wording of this task in that it does not focus the student to draw on other relevant knowledge? Is the student aware of the differences in statistical reasoning compared with reasoning with certainty in other curriculum areas?

Partial-Statistical Responses

Students' responses that were classified as "partial-statistical" on the task involving interpretation of tables simply repeated the responses they had given for the first item. They looked at the data and made some type of visual comparison. For example, Student 17 said:

> Ba is warmer … Just by looking at the numbers, just to say that this is more than Sigatoka.

This student chose one of the temperatures in Ba and compared it with one of the days in Sigatoka, and did not realise that the question involved making predictions from the table. The response points to the dangers of making predictions on the basis of small and therefore possibly unrepresentative samples of experience. A possible way of using the table would be to take account of all the temperatures it contains. For example, the student could have calculated the mean temperatures in Ba and Sigatoka over the period.

Interview Responses About Reading and Predicting From Bar Graphs

The graph relating to the height of several Sharma children (Item 2) was used to examine students' understandings of bar graphs.

Item 2: Height of Sharma children
The following graph shows the height of four of the Sharma children, ages 4, 8, 13, and 19.

(i) How tall is the 4 year old? How do you know? How much shorter is the 4 year old than the 19 year old? How did you work that out?
(ii) A fifth child in the family is 10 years old. Can you tell how tall the 10-year- old is? Explain your answer.

(Graph not shown in this article)

The first task was designed to explore student ability in literal reading of bar graphs, requiring students to lift numbers from specific locations in the graph or compare two such numbers. In explaining their answers to this question, students had to simply point to a data point in the graph. In short, answers to literal reading questions are simple and can be unambiguously classified as either right or wrong. The second question explored student ability in answering questions requiring higher cognitive skills such as predicting. In contrast to the first question, this question aimed to elicit students' ideas about the meaning of the overall pattern of data in the graph. Results are summarised in Table 2.

Table 2
*Response Types for the Task Involving Height of Sharma Children (*N = *14)*

Response type	Number of students using it		
	Task (i) Reading graphs	Task (ii) Predicting graphs	[Both tasks]
Non-response	–	2	–
Non-statistical	–	–	–
Partial-statistical	–	7	–
Statistical	14	5	[5]

Unlike the task involving literal reading of the bar graph, two students' responses were classified in the non-response category for the prediction task. The students said they could not tell the height of the 10-year-old because this information was not given on the graph. Although the responses of all 14 students were classified as statistical on the first item, only five students did not attempt to impose a pattern or give a specific numerical answer on the second. These five realised that their answer could not be an absolute number but would have to be expressed in some provisional way. Like the interpretation of tables (Item 1), one possible explanation for these differences could be a lack of emphasis in classrooms on interpreting graphs. Since the students lacked experiences in interpreting graphs, their responses were more likely to be in the non-response and partial-statistical categories.

Partial-Statistical Responses

Half the students were classified as using partial-statistical approaches for the second item. When rules were applied inappropriately, non-existent patterns imposed or no patterns seen, or exact answer to the question given, the responses were categorised as partial-statistical.

The data revealed that Student 6 applied the rule for finding the mean in an inappropriate way. When asked to predict the height of a ten-year-old from the bar graph, the student used the add-them-all-and-divide algorithm. The student added all the heights given in the bar graph, divided by 5 and got 84 cm! Even further probing by the researcher did not have any positive effect on the student's reasoning.

Misinterpretations were caused by students forcing a pattern on the graph or not seeing the pattern. When asked to predict the age of the 10-year-old, three students tried to force a pattern on the bar graph. Student 14 justified this pattern in terms of a going up explanation. She said that it might be 130 cm because the first one is 100cm and the 8 year old is 120cm. The 10-year-old might be 130cm. The trend continues, 100, 120 and 130.

The other two students cited the absence of a pattern as the reason for their inability to predict. This occurred even in cases where any attempt to search for a pattern made no conceptual sense. The students believed that a pattern must exist and consequently their inability to find the pattern resulted in their failure to offer any prediction. For example, when asked to predict a ten-year-old's height, Student 20 said that he could not predict since there was no pattern on the *x*-axis. During the interview, the student continued to protect his flawed thinking rather than admit something was wrong.

Int: A fifth child in the family is 10 years old. Can you tell how tall the 10-year-old is?
S20: Could be 180 cm because here [meaning 4-year-old] they are increasing by 20 cm.
INT: So the 10-year-old is taller than the 19-year-old?
S20: Age 10 ... Oh I thought the fifth child. You can't tell from the graph.
INT: Why do you say that?
S20: Because it goes by age 4, age 8, age 13. If it was from age 4, age 5 and age 6 you can locate how they range.

It seems that the student believed that graphs have to show a more definitive pattern and he was unhappy about the arrangement of categories on the *x*-axis. The belief that graphs must have patterns seems to be related to other areas of the mathematics curriculum where recognition of patterns is stressed, as well as from specific experiences with graphs. For example, graphs in the teaching context or in media examples usually have a pattern.

Three students gave numerical answers. They did not realise that their answers could not be absolute numbers but would have to be expressed in some provisional way. Two students placed the 10-year-old on the *x*-axis half way between the 8-year-old and the 13-year-old and predicted the height as the corresponding point on the *y*-axis as 130cm. The other student talked in terms of the 10-year-old being the middle of the height of the 8-year-old and the 13-year-old, hence 120 + 140 and divide by 2.

It should be noted that the students were not alerted to the need to examine all the information presented and reflect on it before responding. Gal (1998) states that suggesting to students that a judgment is called for, rather than a precise mathematical response, will make students think more about data and not look straight away for some numbers to crunch.

Interpretation of graphs: a broader context. The finding that while many students can read tables and graphs, they have difficulty drawing inferences from graphs is consistent with the results reported by Bright and Friel (1995) and Gal (1998). These researchers found that while students had few difficulties with the literal reading of graphs, they were often unsuccessful in answering questions requiring higher order cognitive skills such as interpreting and predicting. The findings of this study add to the literature which reveals that while students can read tables and graphs, they experience difficulty in interpreting tables and graphs even after learning about these–although the main influence could be the lack of emphasis the teachers put on interpretation.

The belief that graphs must have patterns is consistent with the findings of Asp et al. (1994) and Pereira-Mendoza and Mellor (1991). Pereira-Mendoza and Mellor found that students have a tendency to impose patterns on data. They researched grade four and grade six students' understanding of the information conveyed by bar graphs. The researchers found that errors involving pattern arrangement of the data occurred in similar frequencies for both grades. The findings of the present study add to the literature that both use and misuse of prior knowledge can lead to errors and that students were persistent in their errors.

Implications for teaching and research. The results of the interviews show that while students could read tables and graphs, they were weak in drawing inferences and predicting from tables and graphs. Misinterpretations were caused by students forcing a pattern onto a graph, or not seeing a pattern and relating the situation to previous experiences (school and cultural). The findings of the study have several implications for teachers. First, it seems that interpretation of tables and graphs requires more emphasis and explicit teaching. At present teachers and text books place heavy emphasis on procedural skills in statistics, for example, calculating summary statistics and constructing graphs. More space and time needs to be allocated to developing students' ability to make sense of and communicate about information presented in tables and graphs.

Second, teachers should not assume that students who learn to process data in tables and graphs can transfer these skills to interpreting and predicting from data. Gal (1998) writes that the nature of needed interpretative skills will depend on the type of context in which they are needed: reporting and listening contexts. Gal adds that teachers need to focus on problems and skills that may be common to reporting and listening contexts.

Another implication relates to statistical investigations. In spite of the importance of relating classroom mathematics to the real world, the results of the present project indicate that students frequently fail to connect the mathematics they learn at school with situations in which it is needed. This suggests that real data should be used in statistics lessons. Since investigative work presents open-ended learning opportunities, more of this could be done with students. Rather than talking about the stages in a statistical investigation, students could be engaged in a statistical investigation. Starting from a real situation, they can pose a question, design a procedure for obtaining the data, analyse and display the data, and interpret the data to answer the original question. As a result of working through the full cycle, students may see data-handling techniques as useful skills that can help them answer questions.

Finally, the success of any curriculum innovation ultimately depends upon teachers. Teacher education programmes in Fiji do not require a course in statistics for education majors. There is a need to collect data from teachers at both the pre-service and in-service levels regarding their conceptions about and attitudes towards statistics. This information will help teacher educators design better in-service and pre-service experiences for statistics teachers. Most of the research on teachers has been done in a few developed countries such as the United States and the United Kingdom. These findings can not be generalised to Fijian teachers. It would be interesting to explore if the conceptions and attitudes of the Fijian teachers are similar to those of their European counterparts or whether the conceptions and attitudes vary across different cultures.

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