Re-visioning Curriculum: Shifting the Metaphor From Science to Jazz

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A plausibility argument is offered in support of the assertion that mathematics education is unduly dependent upon the forensic metaphor, and that the jazz metaphor is a useful and contrasting alternative. Five components of jazz playing are briefly outlined: structure, improvisation, playing outside, pursuit of the ideal, and 'ways of the hand'. The third of these, playing outside, is outlined more fully and applied to mathematics education via a discussion of the role of the mathematics curriculum as public knowledge policy.

The title of this conference theme group is *Re-visioning Curriculum*. The inference is that we are capable of changing our vision from one thing to another. This is exactly what I will argue. I will suggest that it is important that we reduce the mental formatting power of one lens that has become dominant in mathematics education. Such a reduction occurs when we learn to see through alternative lenses. The lens that has become dominant is what I call the forensic orientation, or metaphor. An alternative lens is provided by the jazz metaphor. This is the plausibility argument I am presenting.

Why jazz? There are five reasons, with the fifth being the most compelling. First, the jazz metaphor provides a contrast to the forensic metaphor. I will not argue this in detail, but I hope that the points I do make in support of this assertion will indicate that a more detailed case can be made. Second, I assert that jazz, ontology and ethics have a sufficient number of features in common that a study of the first will facilitate an understanding of the second and third. Ontology and ethics are, I have argued elsewhere (Neyland, 2001), two notions that deserve the attention of philosophers of mathematics education. By exploring the jazz orientation I am attempting to reveal something about ontology and ethics without explicitly making reference to the similarities between jazz and these somewhat abstract and daunting notions. Again, the details require a more lengthy treatment than is possible here.

Third, while I recognise that it is spurious to cite, as a justification, the fact that some teachers are interested in the topic, I do think it is relevant to report that there is such an interest.

Fourth, the jazz orientation has been explored by a small number of researchers in another discipline area. Management theorists have recently investigated the jazz combo as an example of an organisation that learns as it goes along. Education researchers have now begun to publish on the topic. Last year, *'Theory, practice and performance in teaching: professionalism, intuition, and jazz'* (Humphreys & Hyland, 2002) appeared in *Educational Studies* (I am grateful to one of my MERGA colleagues for drawing my attention to this paper during last year's conference). Where did my interest originate? I play jazz—at the moment in a quintet made up from staff from my university. I play, as they say, with more enthusiasm than skill; but hopefully with understanding.

Fifth, and most importantly. The above pales into insignificance when compared to the groundbreaking study of jazz undertaken by Sudnow (1978). Sudnow, a philosopher/sociologist and jazz musician, undertook a phenomenological study of his own processes of learning the art of improvisation. This, on its own, is no reason to call the study groundbreaking. One indication that his study is groundbreaking, at least in relation to the present discussion, is that his book, *Ways of the Hand*, typically appears, not in the music section of university libraries, but in the philosophy/theoretical sociology section. His study has received recognition, not because of what it says about jazz, but because of what it says, when used as a lens or metaphor, about philosophy; in particular, about knowledge, action and competent performance. His study provides a new lens through which others can look in order to think afresh about knowing. Sudnow's jazz metaphor, for instance, allows us to rethink the notion of expertise in a way that avoids the problems associated with what I refer to below as the 'cult of expertise'. So if I were looking for a precursor for the sort of research programme I am foreshadowing in this paper I could do no better than cite Sudnow's study.

I have referred to the forensic and jazz metaphors. What does metaphor mean in this context? Iris Murdoch argued, as have other scholars, that metaphors "are not merely peripheral decorations or even useful models, they are fundamental forms of our awareness of our condition" (Murdoch, 1997, p. 363). She asserts that certain kinds of notions cannot be discussed without resorting to metaphor, since they cannot be reduced to non-metaphorical components without loss of meaning. Mathematics education is such a notion. A metaphor, in the way it is discussed here, is an organ of perception; it is not a peripheral decoration that enhances expression.

Perhaps I should emphasise: we do not have a choice between a metaphorical and a non-metaphorical orientation. We have only the choice about the nature of the metaphors we use.

The Forensic Metaphor and the Cult of Expertise

What is the forensic metaphor? I've chosen this word because it refers to the detailed scientific investigation of something or someone—in this case a crime scene or the victims of crime—in order to find out what has happened. The work of the forensic scientist can be contrasted to that of health professionals, because the latter focus on ensuring the ongoing wellbeing of people and communities. (I recognise that ultimately that is what forensic scientists do, too.) No subject, argues Freudenthal (1978), is as exposed to ruin by the sorts of methods I am describing as 'forensic', as mathematics.

Science, being an open concept, is impossible to define. For this reason, Freudenthal (1978), in the chapter titled 'What is Science?' deliberately avoids a definition, or even a complete set of characteristics. I am concerned, here, with a particular subcategory of what we call science. It is a subcategory that I hope all mathematics educators will recognise immediately, even if it, too, eludes easy definition or a complete set of characteristics. It is the way of viewing mathematics education that results from the sustained and unrestrained application of the scientific model as a tool of analysis in the service of, or in response to, instrumental rationality as a mode of thought. It is much the same as the habits of thought that result when science tenaciously works hand in hand with, or in opposition to, what Postman calls the narrative of economic utility (Postman, 1995).

Applications of science need not lead to a forensic orientation. But many do. Even those that may produce results that call it into question. This is because, although the conclusions of such scientific studies reveal shortcomings—for instance, in the narrative of economic utility—the methods used serve to embed certain dispositions of thought that resemble those dominant in the metaphor. Other applications of science directly support the orientation being discussed. This is because the forensic orientation is fundamentally based on the application of science. Two such applications stand out. First, in mathematics education, there is a tendency, associated with what Freudenthal (1978, p. 93) calls “the most fashionable wisdom of the instruction industry”, towards atomising, measuring, partitioning, and the like—including what he describes as the grinding of mathematics down to “powered form” in order that it may be “administered by spoonfuls”. He goes on to write that, “Isolating, enumerating, exactly describing concepts and relations, growing them like cultures in vitro, and inoculating them by teaching—it is water to the mill of . . . pedagogues and general didacticians” (p. 97). Second, the administration of mathematics education is dominated by the scientific management system. Scientific management has, as part of its modus operandi, an insatiable need for precise and dependable information to aid rationalistic decision-making. Such information is provided by the scientific method.

One other thing needs to be noted in relation to the forensic metaphor. It is now evident that many teachers are experiencing, as a result of policies, procedures and modes of thought that resonate with this orientation, what we might call forensic fatigue. In fact these fatigued teachers are the lucky ones. There is evidence (Neyland, 2001) that some are experiencing something more insidious than fatigue. They are experiencing a loss of a sense of inner resourcefulness; a failure of spirit.

There is another phenomenon that goes hand in hand with the forensic orientation. It is the ‘cult of expertise’ (Neyland 2002). In a nutshell this cult arises as a result of a five step process. First, experts persuade us that it is good and proper to separate education theory and practice; thus creating the theory-practice gap. Second, they develop a ‘best’ theory. Third, they persuade us that they know the theory and we do not; and that we need this theory in order to be good teachers. Accordingly, and this is a key element, they create the expert-novice divide. Further, both we and they come to believe that they are experts and we are novices. Fourth, they turn their theory into a kind of technology or method. Finally they persuade administrators that they ought to be in charge so that they can arrange for teachers to operate their method via a raft of rules, protocols and other familiar devices—sometimes including audits. This, of course is done in the nicest possible way, and supported by professional training sessions, ample documentation, and the like. Sudnow’s ‘ways of the hand’ provides a counter-theory to that implicit within the cult of expertise because it does not postulate a theory-practice gap.

The Jazz Metaphor

In jazz, structure and improvisation combine, not as contemporaneous events, but as simultaneous facets of a single event, play. In jazz, structure and creativity mutually depend upon each other; they exist in a dialogical relationship. In this way the jazz combo learns seamlessly as playing progresses. It is able to do this because the players listen to and respond to each other during the playing. However, and importantly, although there is an essential co-dependence between these facets, improvisation is primary; it is prior to structure, in purpose. By this I mean that structure always needs to examine itself in terms of its capacity to support creative improvisation, and not the other way around. Jazz combos aim to achieve an optimally minimal structure that supports a maximal degree of creativity.

This optimal relationship between structure and creative improvisation may, at first glance, appear to be what scientific managers refer to as one of maximal efficiency. Thus it could be argued, in opposition to what I am claiming, that the structure of jazz is the most efficient means to an end (or target) which is improvisation. If this objection has validity then a problem is created for the jazz metaphor, because, on this point, at least, it would be failing to contrast with the sort of scientifically supported instrumental rationality that characterises the forensic metaphor. This is a difficulty of a similar order to that associated with the word expertise. No one wants inefficiency. But efficiency as a notion has also been colonised by scientific management. The word efficient, when used in relation to scientific management, means cost effective, or maximum yield for minimum resources. Its use is intimately connected with means-target thinking. This is not the sort of thinking that is at work in jazz. The minimal structure in jazz is not so much a rationalistic, as an aesthetic, concept. It is a matter of seeking to achieve an optimal balance, a stripping away of inessentials, a simplicity of form that supports the play. In addition, in contrast to the means-target mode of thought, which is essentially monological, the structure-improvisation mode is dialogical.

It is central to my purpose that the jazz metaphor can be applied to mathematics education. Which parts of mathematics education? It seems to me that the question of the relation of structure and creativity can be asked about at least four areas: the mathematics curriculum; the school scheme; the process of teaching and learning in the classroom; and the way mathematics itself is presented for learning purposes.

Characteristics of Jazz Playing

There are five characteristics of jazz playing that are of significance: (i) structure; (ii) improvisation and creativity; (iii) playing outside; (iv) pursuit of the ideal; and (v) 'ways of the hand'. A great deal could be said about each of these, even without applying them to mathematics teaching. Because space is limited I will limit myself to a brief outline of each, and then concentrate on the third, playing outside. Each can be applied to mathematics education. However, I will only indicate how playing outside can be thus applied; and with only a single example.

First, structure. Jazz has an optimally minimal structure that responsively supports creativity. Too much structure suppresses or stifles improvisation. Too little, or the wrong minimal structure, has a similarly deleterious effect. The combo has a structure—the various instruments have specific roles. The music has a structure—harmonic, modal and so on. A great deal could be said about these structures; but I need to move on. It hardly needs to be mentioned that a mathematics curriculum (written or otherwise), a school scheme, and mathematics itself, all have structures; and that learning involves changing structures.

Second, creative improvisation. Creative improvisation, the primary purpose of jazz, occurs both *within* and *outside* the structural frameworks. The structures provide certain constraints, and the improviser, using a mixture of inspiration and the results of a considerable amount of perspiration (practice), plays *within*, and sometimes *with*, these constraints. Now one could perhaps think: although creativity is absolutely fundamental to jazz, it is not fundamental to mathematics. This is a mistaken view, and one I discuss in the final section of this paper. One cannot understand mathematics without understanding creativity. That is, in its nature, mathematics is fundamentally a dialogue between structure and creativity. In addition, learning is a fundamentally creative process. And teaching, if it is to have much life and joy in it, must have an element of creative spontaneity.

I'll leave the third one, playing outside, until last.

Fourth, pursuit of the ideal. Jazz players have a very clear, but not necessarily fixed, idea about what is good jazz, and what is not. They will not be able to say, with much precision, what good jazz is. They certainly won't be able to define it. But they can recognise it when it happens. The notion of swing is a good illustration. Swing is an ideal. Combos want to swing. Combos try to swing. No one seems to be able to say what swing is. But you know it when you hear it. Swing is the standard. It is also ineffable. Without this shared sense of the ideal, jazz would not happen. This is for two reasons. First, the ideals help distinguish jazz from other art forms. Second, without ideals players cannot make the multitude of choices needed during play; when all options are accorded equivalence in status, choice becomes impossible. We will see in the next paragraph that it is a question of attentiveness to ideals, rather than consciousness of rules, that is at the heart of know-how. Rules cannot substitute for ideals because rules cannot cover all the eventualities that arise. Thus, applied to education, the jazz metaphor contrasts with the cult of expertise, which downplays ideals and valorises, in their place, rules and compliance procedures. Without ideals there is no development in an art form. Importantly, ideals are multiple and sometimes contradictory. Without the latter aspect, we would have something akin to utopianism; the situation where there is a small grouping of mutually compatible ideals. With jazz, as with education, there is a plurality of conflicting ideals, and, accordingly, agonising choices are required in their pursuit.

Fifth, 'ways of the hand'. This, as I noted earlier, is the title of Sudnow's book. And I asserted that this is another way in which jazz contrasts with the cult of expertise. What Sudnow found is this. When he became an accomplished improviser, his hands did the playing, on their own, as it were; his hands were not playing in response to commands from his mind or will. He would sit at the piano marvelling at the original improvised music his hands were playing. His hands played creatively, with subtle nuances and taste. He found the same phenomena occurred when he was typing; his fingers sometimes did the writing. Potters tell me that their hands shape the pot out of clay, and are not responding to directives from their minds. Sports people report the same phenomenon. In order to improvise, the improviser has to *throw herself* into the playing. There has to be a sense of abandonment to the play and dialogue. The improviser cannot think herself into improvisation; she throws herself into it, not knowing what will happen next. Thinking, or specific consciousness, becomes an inhibiting factor. When a juggler has four balls in play she cannot afford to think, she needs to abandon herself and become one with the act of juggling. This is not mindlessness. It is a particular quality of attentiveness; in the case of jazz, attentiveness to the beauty or power or expressiveness of the music that is being created.

The 'ways of the hand' notion paints a picture that is quite different from the theory-practice divide of the cult of expertise. Clearly experienced mathematics teachers have a great store of knowledge and understanding about teaching and learning. They have know-how. They act almost instinctively, sometimes making multiple and complex decisions in the space of a few moments. This knowledge is very different from the detached rule-bound knowledge promoted by the expert. Experts think in terms of learning rules and implementing them in practice, of applying protocols, of following procedures. The 'ways of the hand' is an alternative representation to the theory-practice divide, and a better one. In my experience the 'hands' representation also describes important aspects of the learning of mathematics, and certainly the know-how of the person who is fluent with mathematics. In fact, I suspect that the term 'fluent' is descriptive of just this quality. Creative teachers, using open ended programmes of work, throw themselves into the teaching/learning dialogue with their students, not knowing quite where things will end up.

But, you might ask, was Sudnow not a novice when he began to learn improvisation, and, afterwards, an expert when his hands did the playing? The answer is, no, if there is any suggestion in this question of a theory-practice divide. Eventually the improviser develops a fluidity of know-how that is characterised by her hands knowing how to play jazz and her mind astonished at what it is hearing. But before this fluidity and naturalness of action is achieved, when things were clumsy, or stilted, or self-conscious, the play was still of the same sort. Beginning jazz players are not doing something of a fundamentally different order from experienced jazz musicians; they do the same thing only less well. A beginning player can be attentive to beauty; she can feel the immensity, the richness, the attractiveness of even a single sound; and she can respond to it with abandon. A person doesn't fall in love by a concentration of will and the application of theory. One falls in love by changing one's focus of attention to the loveableness of the other; and love happens. Improvisation, even for beginners, flows from love of music, not from the application of theory. The way to improve is to develop one's attentiveness. Of course rules and protocols can help, but they must not interfere with the development of real know-how. One doesn't make oneself sleep by concentration and the application of intentional decision making. In fact this sort of thing makes it harder to fall asleep. This 'hyper-intention' is an obstacle. Teaching is similar. There is know-how all the way; even among beginners. It is just more proficient in some than in others.

Playing Outside

Coker (1978, p. 143) defines *playing outside* as "improvisation in which *few, if any*, of the selected notes are contained in the given chord or scale"; and *playing inside* as "improvisation in which *all* notes selected are contained within the given chord or scale". There is an intermediate position which corresponds to the most common situation when players improvise: usually in improvisation *the vast majority* of the important notes selected are contained within the given chord or scale.

For non-musicians, what does this mean? Most people know that tunes have a chordal structure that gives them a harmonic coherence. Chords are groups of three or more notes that are played simultaneously. In jazz the chords tend to be collections of four or five notes and often change twice each bar (that is, every 2 beats). Imagine that at a particular point in a tune the harmonic focus is the c-chord. While the c-chord is being played by the rest of the combo, if the jazz improviser plays only, or mostly, notes from the c-scale—that is, notes that come from the white keys on a piano—she is playing *inside*. If, however, she plays only black notes at this time (there are seven white notes and five black notes in every octave of the c-scale), she is playing *outside*, using what musicians know as the g-flat-major-pentatonic scale.

Thus, playing outside is the deliberate playing, during an improvisation, of *only those notes that clash* with what the rest of the combo is doing. Doing this is not easy because it involves playing only the wrong, never the right, notes. Normal improvisation, that is, playing in tune with the harmonic structure adopted by the rest of the band, is playing inside. Playing outside is playing *with the structure itself*. It is making the structure itself (not the elements of the structure) an object of play. It is using the structure to do exactly what it was set up not to do. If an improviser plays outside a small amount of time only, it sounds to listeners as though she is simply making errors. If she plays outside too much, the combo ceases to function and collapses in harmonic disarray. Not everyone is capable of playing outside. You have to learn to do it, and most importantly, you must first be able to play inside with accomplishment. Before you can play outside, say jazz musicians, you must first be able to play inside.

Why do it? If done properly, it creates a heightened feeling of tension in the music; a kind of expectant discomfort. Listeners find themselves wanting some sort of resolution to something harmonious. It leads to the structure being challenged. It is a kind of assertion that structure is secondary to creativity. If playing outside is not allowed, or is impossible, then structure is too dominant. Playing outside asks structure to re-examine itself. Sometimes new ways of thinking result from such experiments. Sometimes ideals are caused to be re-evaluated. Does anyone play outside in mathematics education? I suggest that the most accomplished teachers and students should be doing this at least a little. Playing outside is a sign of organisational health, and both organisational and personal learning.

The Mathematics Curriculum as Public Knowledge Policy

A national mathematics curriculum (whether we like it or not) is part of what I call public knowledge policy. It is a policy statement about the mathematical knowledge to which the public ought to be introduced. Like all policy it is open to scrutiny. Accordingly curriculum policy writers need to ensure, among other things, that the underlying philosophy of mathematics is defendable. More generally they need to be sure that the underlying philosophy of knowledge is defendable. Ten years ago, when the current mathematics curriculum was written for New Zealand, there was concern, particular among some academics, that the curriculum more generally was becoming relativistic and/or subjectivist. For instance, there was some disquiet expressed that some curriculum areas were concerned too much with 'making sense' and insufficiently with 'truth'. Since mathematics is neither relativistic nor subjectivist, it was important to ensure that such a misconception be avoided. There was a noisy public debate about the science curriculum on just this point.

I do not want to go into the rights and wrongs of all this for science. But I do want to talk about mathematics. How do mathematicians defend mathematics against the accusation that it is relativistic or subjective? We need a theory of truthfulness, or, if that is asking too much, because it implies the prior acceptance of an objectivist epistemology, a theory of justification. Of course, in this area, we are lucky. We have a mathematical theory of proof. Surely that is all that is needed! So long as the curriculum mentions deductive proof, and proof by induction, counter-example, contradiction, and so on, the public knowledge policy for mathematics will be epistemologically secure. Clearly, we could argue, mere pattern forming is insufficient. If we derive the formula for the nth triangular number by mere pattern forming, we are only 'making sense', not proving. Pattern forming is not enough, we correctly assert, and we have methods of proof that fit the bill.

Can we therefore rely on proof? No. Proof is not enough. Why? To use the jazz metaphor, proof is basically playing inside. But we also need to play outside. To see why we need to turn to the revolution in the philosophy of mathematics that occurred in 1976 when Imre Lakatos published *Proofs and Refutations*. Lakatos showed that mathematicians have often proved theorems to the satisfaction of other mathematicians, but that some of these theorems have later been found to be inadequate. Not because the proof techniques were in error, but because their imaginations were limited. Another way of saying this is to say that proofs are only as good as the structures within which they are logically consistent. What mathematicians failed to do, Lakatos showed us, was play outside those structures; in particular, to look for refutations. Lakatos showed that mathematical truth cannot be thought of as based on proofs; this is because mathematics is not structured in that way. Instead, mathematics is quasi-empirical. It is based on structures of imagination within which proofs are logically consistent and make sense. But it is also necessary to have a dialectic between proof and refutation. That is, a dialectic between playing inside and playing outside. Lakatos showed us that imagination and ideas are crucial to the nature of mathematics itself, not just to its teaching. Thus mathematics itself has a jazz-like quality. References within the curriculum to conjecture and refutation help ensure that the curriculum, as public knowledge policy, is epistemologically robust. It also happens that the dialectic of conjecture, proof and refutation is a wonderful way of teaching and learning mathematics. That is another good reason for including these ideas in a curriculum statement.

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Individualization of Knowledge Representation in Teacher Education in Mathematics

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The individualization of knowledge representation requires a knowledge base in which teachers can organize contents according to their personal needs. The objective in teachers education is to combine authoring activities with the embedding of existing contents. Student teachers have different mathematical and pedagogical experiences in mathematics education. These experiences determine their focus of interest and their authoring activities in the knowledge management system. They develop for example lesson plans in geometry embedded in the pedagogical and mathematical knowledge in the system. Individualized knowledge representation is a constructivistic approach to build up knowledge according to the progress in teachers education.

From Research Grows Research

The CD-ROM *Learning about Teaching* [LAT] (Mousley & Sullivan, 1996), introduced by Sullivan and Mousley at the PME conference in Lahti is an outstanding example of a learning environment that embeds questions about a lesson's design. Analysis of the lesson is facilitated with video recordings, transcripts, and graphs. Using these tools, student teachers can approach didactical (pedagogical) concepts by examining authentic mathematics classroom material, including the lesson plan, lesson activities, children showing problem solving processes, and outcomes of the lesson. These are sources for the student teachers' reflection, discussion and writing (see also, Honebein, Duffy, & Fishman, 1993).

At the end of 1998 Stein took this CD as a starting point for the internet based *Mathematik-Didaktik im Netz* [MaDiN], a multimedia facility for teacher education in mathematics and mathematics education. The production of didactical and mathematical content and the basic objectives of the MaDiN project were modelled on the LAT CD. A further basic objective of MaDiN was accessibility on the web, and another was that the development should attend to constructivist principles by allowing users of the resource to develop their own networks of knowledge.

Cooperation between MaDiN and the *Mathematics Education On the Web* [MEOW] in Edith Cowan University, Perth, began in 1999 (see Herrington, Herrington, Oliver 1999) .

Further, since 2001, four working groups from different German cities led by Weth (Erlangen), Weigand (Würzburg), Tietze (Braunschweig), and Stein (Münster) have joined the MaDiN project, focussing on different mathematical and didactical aspects, in primary, lower and upper secondary teacher education.

This demonstrates how one innovative resource, growing out of mathematics research, can lead to further developments. This paper addresses the nature of these developments and describes the structure of the knowledge base of the mathematical and didactical contents in MaDiN.

Knowledge Representation

Knowledge representation is one objective of MaDiN and of its development through the working groups. Knowledge representation is an aspect of knowledge management (Na

Ubon & Kimble, 2002) that describes the way contents are structured and linked, and it is a crucial element for the organization and individualization in a private workspace. The basic idea of the MaDiN-project is *context dependent knowledge representation.* The classification of information is achieved by themes, accessible from a desktop that serves as the user interface. The method for structuring themes has been outlined by Ernst and Stein (in press) and McAleese (1998).

The basic suppositions made about the didactical and mathematical knowledge in MaDiN are that (a) the known scope of German teacher education can be represented in a structured way, and (b) that this structure can be organized as a directed tree (see Figure 1 below). We refer to the elements of this tree as "nodes". In MaDiN, the nodes in the tree represent a *collection* of different types *of information* (HTML-pages, video, audio, animations, etc.) for a special theme. If we take, for example, the subject *"Problem Solving in Geometry",* this collection contains the following nodes:

- a *survey,* which gives a short description of the contents of the desktop;
- a *video* of student solving a Tangram problem in geometry;
- *activities* for student teachers to analyse the geometric problem solving activity of children (including analysis of difficulties that the children have);
- *internet links* to the subject *"Problem Solving in Geometry"*, for example to geometric problem solving activities for lower secondary schools;
- *theoretical* background information to the applications of *Problem Solving* in primary schools;
- additional references to *literature* on problem solving in geometry; and
- *news*, for example, relevant up-coming conferences.

A link between tree nodes in MaDiN is a link from one collection of information to another collection.

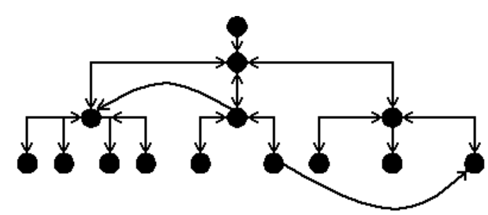


Figure 1. Directed tree.

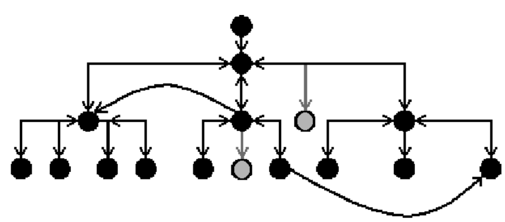


Figure 2. Individualized tree with grey nodes.

Following a *constructivistic approach* a learning process is individualized construction of knowledge and abilities. The structure of the tree should represent the process. Individualization of this tree for specific users or user groups includes uploading and editing of contents in the desktop and the embedding of new tree branches (the grey nodes shown in Figure 2) with existing desktop subjects.

We then needed an interface for this collection of different types of information. We chose a desktop as metaphor, as used by the MEOW-Project (Herrington, Herrington, & Oliver, 1999). Thus when accessing the resource, students see a desk (see Figure 3 below)

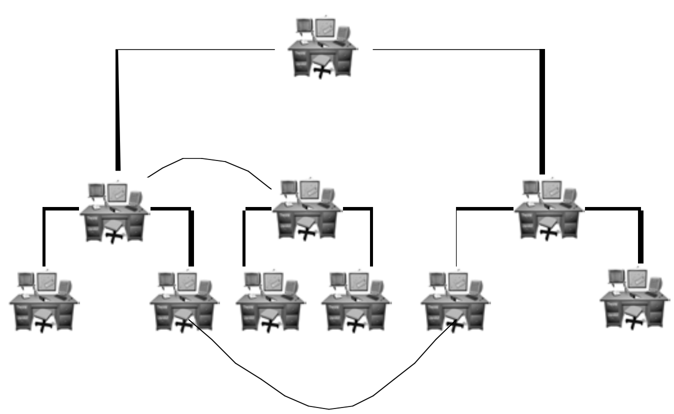
Problem Solving in Geometry

Figure 3. Desktop for “Problem Solving in Geometry".

Some drawers may be highlighted meaning that the desktop provides these elements of information on the subject *Problem Solving in Geometry*. Other information categories may be not highlighted because there is no information of that type available. Clicking on each of the labelled elements provides a different entry point to the information embedded in the website, so users can choose an individual approach and explore the subject *Problem Solving in Geometry*. Behind this interface, the material is arranged in a tree structure where we think of the nodes as further desktops, and some are linked. Thus when a user is navigating from one collection of information to another the drawers appear to be filled with hypertext, videos, links and references according to the subject the user is looking at.

Figure 4. Information in the desktop depends on the tree node.

This *context dependency* of the desktop has the objective that the user will only get the



information on the chosen subject, and only the specific media that are available on that topic

Teaching Experiences and Didactical Knowledge

When student teachers have completed their first teaching experiences they have only a limited knowledge about organizing and initiating learning processes for the children, so learning about teaching has to start from these limited preconditions. Because they teach different lessons in different classes, and have favorite teaching methods, there is a necessity to organize knowledge in a personalized way. Catering for these needs is a further principle of the research project.

Student teachers are encouraged to do online authoring in order to document their teaching experiences. This includes entering lesson plans and their reflections after the lessons into the resource base. Sharing such material can help other students in developing their lesson plans cooperatively on the web. This individualization of the knowledge representation and cooperative development of ideas makes it necessary to have the contents accessible to them as web authors on the internet. The student teaches create web pages and multimedia material for the information system, and this information also serves as a basis of discussion in seminars. The MaDiN system is designed in a way which demands (nearly) no technical knowledge for the generation of web based information, so that the didactical concept focuses on the *organization* of web based contents and the *structuring of mathematical and didactical knowledge*.

Beside the contents generated by student teachers, didactical and mathematical experts offer content for the knowledge base, so the knowledge base is continuing to develop.

In order to personalize knowledge representation, skills for authoring and embedding of contents is taught to student teachers. The following three items illustrate the process from the rec*eptive* to the *constructive* role of a teacher student (see also Simons, 1993).

Lectures (Introduction to Knowledge Representation). Lecturers use material like animations, videos, and HTML-pages in their presentations and these have been drawn from the MaDiN knowledge base. Here, students are in the *receptive* mode, but after the lecture the student teachers can access the information online and access additional information and see how the material fits into the wider organization in the information system.

Seminars (Analysis of Authentic Material). Here the mode is both *receptive* and *constructive*. In seminars, student teachers can analyse authentic classroom situations. Their analysis can be documented in their individualized knowledge representations. For example, they may work on problem solving activities with the Tangram in primary schools, and contribute solutions or other information about what happened. Beside the fact that this work of the students is presented in their own private workspace of MaDiN the student teachers have to integrate some of their contents into the existing information system of MaDiN. Thus the individualized information system of the student teacher combines personal material with material of the official MaDiN information system. For this combination of personal area and the official expert contents the student teacher has to explore the MaDiN information system for helpful connections (links) to the subject of the seminar. This includes further major *receptive work* with MaDiN.

Final Examination *(Knowledge Analysis and Structuring).* When student teachers in Germany take their final examinations they have to write a homework exam (duration 3–6 months). The student teachers' writing in the MaDiN project consists of *web-based contents* and a *theoretical text*. The web-based contents are their contributions to the MaDiN knowledge base about a didactical subject (for instance about "mathematically gifted children"). The theoretical text describes the learning principles they used in structuring mathematical and pedagogical knowledge in preparing it for use in the knowledge base. After assessment of the homework, their own web based product of good quality will be made accessible to other students via the internet as part of the MaDiN project. Even if the contents are not of very high quality it becomes part of a further knowledge development cycle in seminars and examination homework. This stage of their work is entirely constructive.

In-service teacher education (Participation in Knowledge Development). Here, student teachers are in a receptive-constructive mode, and their supervising teachers who work at school can benefit from the material developed at the university with students. The usage of the MaDiN materials in schools also provides teacher educators and student teachers with a feedback, and this has a further impact on the re-development and refining of the contents at the university.

Individualization of Knowledge Representation

According to Jacobson and Spiro (1995) and Niehaus (in press), individualization of web based knowledge representation means that:

- the users (student teachers, teachers, or academics) have private workspaces;
- each workspace contains only the chosen parts of a knowledge base that they see as important for their work;
- in the web based workspace, users can embed new contents according to their personal needs and add their own material that they produce for application in schools;
- users can grant access rights to documents in their private workspace; thus facilitating cooperative work in groups of student teachers and teachers.

While individualized web based contents (for example lesson plans or projects in geometry) are at first only visible for a single student teacher or teacher, we have found that these contents can be useful for others. If somebody grants access rights to their contents, other student teachers or teachers can embed these contents in their own private workspaces, and can add to them. This collaborative interaction (granting rights, embedding and authoring) is illustrated in the following figure. Further, granting rights to group of student teachers open up private contents for a collaborative development cycle by the group, and this information could then be opened for public scrutiny and use. Developments, here, need not be unidirectional (see Figure 5).

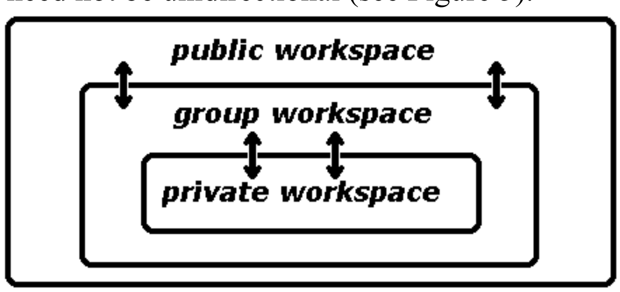


Figure 5. Cooperative work in individualized knowledge representations.

Keeping the constructive aspects of the didactic concept in mind, it is necessary that students can participate in the construction process of MaDiN knowledge without modifying the basic structure of the knowledge base. In the Figure 6 we can see black tree nodes and white tree nodes, symbolizing one single desktop. The black nodes symbolize the publicly accessible knowledge base. The white nodes symbolize a private modification.

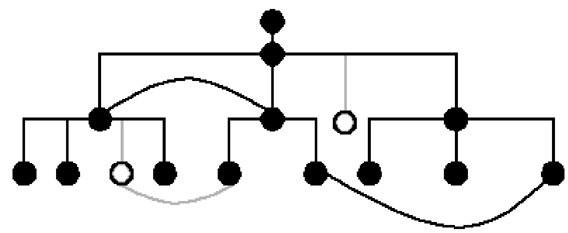


Figure 6. Public and individualized nodes in the tree.

The private white nodes in tree shown in Figure 6, could represent, for example, an evaluation of a geometric problem solving strategies of a particular student teacher in a specific primary school, using a problem from the section of the knowledge base that attends to Problem Solving in the primary-school geometry lessons. The student teacher (and supervising teacher and lecturer if given access) could see all of the black and white nodes. However, another student or a guest navigating through the mathematical and didactical knowledge base of MaDiN would not have access to the white nodes or be able to use the grey links because these would not be visible.

Conclusions

In the resource described above, individualized knowledge representations focus on the personal needs of a teacher for the organization of learning processes in mathematics. Building up this private network of resources and understandings is an objective in out teacher education program so that student teachers are able to share ideas and to develop and modify material for mathematics education collaboratively. In addition to the material produced by student teachers, the private workspaces also contain embedded contents from lectures and seminars.

MaDiN is now a didactical concept which supports teacher education from university to in-service training to teaching. This development follows a constructivistic approach in that it involved personal networking of ideas, but representations of these ideas are generally available for the use of others. The underlying idea of constructivism, together with the “object oriented” concept of multimedia development, presents a didactical approach which mixes the role of the author with the role of the reader. The student teachers involved have become not only receptors of information but also creators of an extensive and useful resource. In MaDiN system, *expert user, author* and a *didactical novice* share and embed contents in a knowledge base. Granting rights to the private workspace facilitates collaborative work and joint development of mathematical material in schools.

Evaluation, reorganization, creation, and embedding of contents represents a learning process in the private workspace as well as in the broader public arena. It is our contention that the skill to adapt the individualized knowledge representation is a key element for an integrated knowledge development from teachers education in universities to in-service training to the teaching profession.

In summary, this paper has reported on the development of an interactive knowledge base that grew out of research initially carried out in Australia. Elements included in the *Mathematik-Didaktik im Netz* [MaDiN] were developed in conjunction with other

Australian researchers and multimedia products. The paper illustrates the value of international co-operation in mathematics education research and development.

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