Difficulties Children Face When Learning to Count

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This paper explores the counting errors produced by 40 Grade 1 children from 16 Victorian schools prior to their commencement of a mathematics intervention program. Analyses of the errors highlighted several common difficulties and issues related to learning to count. It is anticipated that if teachers are on the lookout for these difficulties, then they will be able to identify children who are in danger of being "left behind' their peers, and may provide the type of experiences that will assist these children to construct a more powerful understanding of counting and the number word sequence.

Teachers know only too well the anguish of watching one or two children in their class struggle with learning mathematics. Indeed, despite teachers' best efforts, some children are left behind their peers, and never achieve success in learning mathematics at school. Success in learning mathematics is important for many aspects of everyday life, and for access to further learning and employment opportunities once children leave school. For this reason, school communities in Australia aim to improve learning outcomes for all students. Achieving this aim may depend, in part, on increasing the professional knowledge of teachers about learning and teaching mathematics, and on providing alternative experiences for those at risk of not learning mathematics successfully. Some children need a different kind of experience from the one traditionally encountered at school.

In 2000, twenty schools taking part in the Early Numeracy Research Project [ENRP] (Clarke, Gervasoni & Sullivan, 2000) introduced a mathematics intervention program, *Extending Mathematical Understanding* (Gervasoni, 2002), for the Grade 1 and/or Grade 2 children who were being left behind their peers in number learning. These school communities believed that it was important to assist these children as early as possible in their schooling, before they experienced failure. ENRP "Growth-point profiles" in Counting, Place Value, Addition and Subtraction and Multiplication and Division were used to identify the children who might benefit from this experience.

In order to highlight common difficulties or issues related to mathematical learning that would be useful for classroom teachers to be aware of, these children's responses to the assessment tasks undertaken prior to commencement of the intervention program were analysed. It is anticipated that if teachers are aware of the difficult points in children's mathematical learning, then they can be on the lookout for children who are having these difficulties, and provide experiences to assist these children construct more powerful understandings. Such an approach may lead to more successful learning for children. This paper focuses on the difficulties the 40 Grade 1 children experienced in Counting, and implications that arise for teaching children to count.

Key Growth-Points in Learning to Count

As part of the ENRP, a research-based framework of six growth-points (see Figure 1) was created to describe key "milestones" in the development of children's counting

knowledge. Similar to the work of Wright (1998), these growth points are concerned with the production of number word sequences. However, they include also children making the

1. Rote counting: Rote counts the number sequence to at least 20.
2. Counting collections: Confidently counts a collection of around 20 objects.
3. Counts forwards and backwards from various starting points between 1 and 100; knows numbers before and after a given number.
4. Counting from 0 by 2s, 5s, and 10s: Can count from 0 by 2s, 5s, and 10s to a given target.
5. Counting from X (X >0) by 2s, 5s, and 10s: Can count from x by 2s, 5s, and 10s to a target.
6. Extending and Applying: Can count from a non-zero starting point by any single digit number, and can apply counting skills in practical tasks.

Figure 1. ENRP counting growth points.

count-to-cardinal transition in word meaning described by Fuson (1992a) so that they are able to think about the number sequence to solve problems. The *count-to-cardinal* transition in word meaning describes the ability to link the last number word produced when counting a collection to the "manyness" or cardinal value of the collection. The ENRP counting growth points do not describe children's use of counting strategies in addition and subtraction situations. These strategies are described in the ENRP addition and subtraction growth points.

For some young children, the progression to *counting collections* and *counting forwards and backwards from various starting points* is prolonged or difficult. These growth points relate to two of the counting levels described by Fuson (1992b), the Unbreakable List Level, and the Breakable Chain Level. These levels describe the development that occurs in order for children to count collections, or count forwards and backwards by ones. The Unbreakable List Level involves the number word sequence being broken into individual words, which are used in counting by relating each number word to a perceptual item to be counted (Steffe, von Glasersfeld, Richards, & Cobb, 1983). Children begin to relate the last word counted to the cardinal meaning for the group of counted items (the cardinality principle). They can then use count-all strategies to add two numbers.

The Breakable Chain Level involves children being able to start saying the number word sequence from any number word. They eventually use this ability in combination with the *cardinal-to-count* transition in word meaning to add by a more efficient counting-on method, in which counting to determine the final sum begins with the first addend number word, instead of beginning the count from one. For example, when adding four more items to a known collection of eight items, a child switches from the 'manyness' meaning of eight, to thinking of eight as word in the number sequence, and counts on four more words to reach the total of twelve.

These two levels, as they relate to counting collections and counting forwards and backwards, are not only important for children's counting development, but also are important for the development of numerical problem-solving strategies. It is the progression to these growth-points, and Growth Point 3 in particular, that is difficult for young children 'left behind' in Counting (Gervasoni, 2002).

Identifying Children Left Behind in Counting

As part of the ENRP, all children took part in assessment interviews conducted by their teacher at the beginning and end of each year (March/November). The interviews were coded to determine the growth points each child reached in nine areas of mathematics, including Counting. The processes for ensuring the reliability of scoring and coding are outlined in Rowley and Horne (2000). Examining the growth point data makes it possible to determine the children who have not yet reached the median growth point in Counting, and enables the children being left behind to be identified. For example, Table 1 shows the percentage of 1505 Grade 1 children in ENRP trial schools who reached each of the Counting growth points in March 2000.

Table 1

Percentage of Grade 1 Trial School Children in 2000 Reaching Each Counting Growth Point

Counting Growth Points (March 2000)	Percentage of Children *(n=1505)*
0. Not yet apparent	5
1. Rote counting to 20	6
2. Counting collections of 20 items	56
3. Counting forward/backward by ones	15
4. Skip counting by 2, 5, 10 from 0	16
5. Skip counting by 2, 5, 10 from *x*	2

The distribution of children's counting ability across the growth points demonstrates a wide range in understanding, and highlights the challenge for teachers to cater for the range of abilities in classrooms. Eleven percent of Grade 1 children were not yet able to count a collection of 20 items (the median growth point), even after completing one year at school. These children were being left behind their peers, and possibly were faced with classroom experiences with which they could not adequately engage in order to learn successfully.

Table 2 shows the Counting growth point distribution for the 41 Grade 1 children from 16 ENRP trial schools in Victoria who participated in the *Extending Mathematical Understanding* (EMU) intervention program in 2000. These children were identified as being left behind their peers in several number domains, besides Counting. ENRP growth point profiles were used to identify these children (see Gervasoni, 2000).

Table 2

Percentage of Grade 1 EMU Participants in 2000 Reaching Each Counting Growth Point

Counting Growth Points (March 2000)	Percentage of Children (*n*=41*)*
0. Not yet apparent	22
1. Rote counting to 20	27
2. Counting collections of 20 items	51
3. Counting forward/backward by ones	0
4. Skip counting by 2, 5, 10 from 0	0

About half of these children were unable to count collections of 20 items, and no-one was able to count forward and backward from variable starting points (Growth Point 3).

The type of errors made by these children when counting may be helpful in illuminating for teachers the process of learning to count, and may highlight the types of challenges different children face when counting. Knowledge of these errors may assist

teachers to provide the type of experiences that will best assist children to construct more powerful knowledge about counting.

Counting Difficulties

In order to identify any common difficulties or issues that arose when these Gr 1 children were counting, the errors they produced during the *Extending Mathematical Understanding* Assessment Interview were analysed. The counting tasks in this assessment interview focused on number conservation, one-to-one correspondence, counting collections, and producing forward and backward number word sequences.

Number Conservation, One-To-One Correspondence, and Cardinal Value

Three tasks assessed children's number conservation, one-to-one correspondence and ability to determine the cardinal value of a collection of at least 20 items. Table 3 shows how many children were successful with these tasks.

Table 3

The Number of Children Who Were Successful With the Tasks Related to Number Conservation, One-To-One Correspondence and the Cardinality Principle

Conservation (6 items) *n*=39		One-to-one Correspondence (4 items) *n*=39		Cardinal Value (>20 items) *n*=40	
Yes	No	Yes	No	Yes	No
25	14	35	4	28	12

The *conservation* task required children to count a tower of six blocks, to break the tower into single blocks, place them in a container, and then state how many blocks were in the container without recounting. Fourteen children were unable to complete this task without recounting, suggesting that these children need experiences that focus attention on number conservation.

The first *one-to-one correspondence* task required children to look at four plates, and then get the number of spoons needed if one spoon was placed on each plate. Only four of the 39 children had difficulty with this task. However, difficulty with one-to-one correspondence was an issue also for some children when they counted collections of 20 items. Experiences that focus on understanding the principle of one-to-one correspondence would be beneficial for about five children.

The task that assessed children's ability to determine the *cardinal value* of a collection required them to take a handful of lima beans, estimate the number of beans in their hand, and then check their estimate. Twelve children experienced difficulty with this task. The errors were of two types. Five children were unable to demonstrate one-to-one correspondence, and the remaining children had difficulty producing the correct number sequence. One child made an error in the counting sequence below ten (skipped 8, saying "7_9"), two children made errors with 'teen' number word sequences (18_100, 14_20), three children had difficulty with decade transitions (19_??, 29_90, 39_??), and another child had difficulty with the number sequence in the thirties (33_35). Therefore, learning to produce accurately the number-word sequence is another issue when learning to count.

Producing the Number Word Sequence

The remaining Counting tasks required children to produce forward and backward number word (counting) sequences. One task required children to produce the forward sequence from one to 120. Only two of the forty children produced the number word sequence correctly. Eleven of the children could count to 100, but did not know the sequence beyond 108 or 109. Table 4 summarises the errors children made when counting from one to 120.

Although there were four main error types, children seldom produced identical error sequences. This highlights the challenge teachers have in recognising common counting difficulties. The most common error was bridging the decades, e.g., 39_50. Indeed, nearly half of the errors were of this type. This situation highlights that understanding the meaning and order of the decade number words is a considerable challenge for many children. Bridging 40 was the most problematic decade transition.

Table 4
Errors Produced by Grade 1 Children When Counting From 1 to 120

Error Types		Recorded Number word Sequence Errors (1 to 120)					Total	
Bridging Decades	Bridge 20	18_100	19_100				2	
	Bridge 30	28_30	29_50				2	
	Bridge 40	38_20	38_40	39_??	39_20	39_50 (x2)	7	16
			39_80					
	Bridge 50	48_50	49_60				2	
	Bridge 60	59_30	59_40	59_100			3	
Bridging 110		109_?? (x5)		108_200 (x2)		109_200 (x2)		11
		109_1000 (x2)						
With-in Decade	Teens	14_16	15_17	(14, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29,			3	
		23, 100)						
	Twenties	26_28					1	7
	Thirties	36_38					1	
	Sixties	67_70					1	
	Eighties	85_87					1	
Bridging the Century		99_??	100_??	98_90	100_102			4

The second most common error type related to producing the counting sequence beyond 108 or 109. Children were either unable to continue beyond this point, or suggested that the next number in the sequence was 200 or 1000.

Another error type related to difficulties within various decade sequences. Most of these errors were due to one number word being skipped (e.g., 36 _ 38), although one child had difficulty with both the "teen" sequence beyond fourteen and the decade transition to 30. The 'teen' sequence of number words was difficult for only three children, although two more children were not able to bridge the teen decade to reach 20. Bridging the century was difficult for a small group of children also.

Counting Forward From X

Counting Growth Point 3 relates to children counting forwards and backwards from various starting points; Fuson's Breakable Chain Level. None of these Grade 1 children had reached this growth point in March and only 14 of the children reached this growth-point by the end of Grade 1. Seven assessment tasks related to this growth point. Table 5 shows the errors that were noted by teachers when children counted forward from 43, 78 and 93.

Almost all the errors summarised in Table 5 relate to bridging the decades, the century, or 110. This suggests again that understanding the meaning and order of the decade and century number words is a challenge for many children. Only 21 children attempted to count from 93, and only one child counted correctly from 93 to 120. Of the errors recorded for this latter task, most related to bridging the century and the 109/110 transition.

Table 5
Errors Produced by Grade 1 Children When Counting Forwards From 43, 78 and 93

Error Types		Recorded Number word Sequence Errors (1 to 120)	Total	
Bridging Decades	Bridge 50	48_102 49_?? 49_20 49_40 49_100	5	
	Bridge 60	59_90	1	
	Bridge 80	78, 79, 76 (x2) 79, 70, 18, 70 79_70ten (x2) 79_?? 79_20 79_50	8	15
	Bridge 90	88_90	1	
Bridging 110		109_?? (x3) 109_130 109_200 (x2) 109_1000 (x2) unknown (x1)		9
Bridging Century		99, 99ten, 100 99_91 unknown error (x4)		6
With-in Decade		45_66 97_99		2
Bridging 120		119_???		1

Counting Back From X

Counting backward was more difficult for these children than counting forward. This was evident by the small number of children who were successful with the four assessment tasks, and by the large number of children who did not attempt some tasks. Only twenty of the 40 children attempted to count back from 22, nine children attempted to count back from 54, and five children attempted to count back from 94. Most children were able to count back from ten to zero, although eight children were unsuccessful. Beyond ten, children had great difficulty counting backwards. Only two children counted back successfully from 22, and no one was able to count back from 54 or 94. The sequence errors recorded by teachers are outlined in Table 6.

Table 6
Errors Produced by Grade 1 Children When Counting Back From 10, 22, 54 and 94

Error Types	Recorded Backward Number word Sequence Errors		Total	
Bridging Decades	*Bridge 20*	21, 20, ?? 21, 20, 90, 80, 70 (x3) 22, 21, 0 21, 20, 12, 10 22, 21, 11, 19, 98	7	11
	Bridge 50	53, 51, 52, 49, 48 52, 51, 50, 40	2	
	Bridge 70	70_10	1	
	Bridge 90	91_89	1	
With-in	*10_0*	6_2 6_4 3_1 8, 7_??	4	5

Decades	*Forties*	49, 48, 39		1	
Forward &	*10_0*	10, 8, 6, 7		1	
Mixed	*Twenties*	22, 23, 24	22, 21, 22, 23	2	5
Sequences	*Fifties*	54_55		1	
	Nineties	94_98		1	

There were three types of errors. Difficulty with bridging decades accounted for the majority of errors recorded for backward sequences beyond ten. Another error type related to mixed backward and forward sequences. The remaining errors were with-in decade errors, with most of these for sequences below ten.

Implications for Teaching Grade 1 Children

Analyses of the Counting errors made by Grade 1 children prior to their commencement of an intervention program highlighted three main issues. These issues and implications for teaching Counting are discussed below.

Conservation of Number and One-to-One Correspondence. A small number of children were yet to develop conservation of number or one-to-one correspondence, even after one year at school. It is assumed that Grade 1 children who have not yet constructed these understandings may have difficulty engaging in the number activities presented within the regular classroom program, and may gain little benefit from these experiences. Therefore, it is important for teachers to be on the look out for children who do not have number conservation or one-to-one correspondence so that they may provide experiences to assist children construct these understandings.

Bridging the Decades and 110. Another issue highlighted by the analyses was the difficulty many children had in producing forward and backward number word sequences. Most of the errors children made were of two types: difficulty bridging decades, and difficulty producing the number sequence beyond 109. In the latter case, children either ceased counting at 109, or predicted that the next number in the sequence was 200 or 1000. This situation suggests that some children do not understand the nature of the number word sequence and the patterns embedded in the sequence, particularly beyond 100.

Learning to produce the number sequence is a complex process. It takes years for children to learn conventions such as the order of the number words to twenty, the names and order of the decades, the cycling of one to nine through each decade, the sequence of number words beyond one hundred, and the sequence patterns related to hundreds and thousands. It is possible, though difficult, to learn the number word sequence as a rote memory exercise. However, the enormity of this task increases the likelihood of errors.

Alternatively, it is possible to learn to produce the number sequence by exploring patterns and relationships between number words in the sequence, and through learning how the number word sequence is constructed. Such knowledge means that the number word sequence does not need to be produced from memory, but can be produced at will by constructing the sequence based on an understanding of patterns and relationships. It may be this knowledge that children who make errors in producing number word sequences are yet to achieve. The challenge for teachers is to provide the type of experiences that will assist children to understand the nature of the number word sequence, and the patterns and relationships embedded in the sequence. For example, children can be asked to count while making a collection to represent each number word. The collection can be progressively

grouped in tens and hundreds to emphasise the number of hundreds, tens and ones represented by each number word. Children's attention can be focused on how each subsequent number word increases the collection by one, and how each new decade name increases the number of tens by one. Also, children can be shown how the decade names relate to the number of tens in the decade name, so that they can understand what the number-words mean. Such experiences may help children understand the order of the decades, the most common difficulties faced by these children, and may also help children to bridge 110 more successfully. When children make a collection of 109, they may be more likely to predict that the number after 109 is 110, rather than 200 or 1000.

Counting Backward. A major issue identified by the analyses was the difficulty experienced by most children when producing the backward number word sequence. This task was considerably more difficult than counting forward. Only two of the 40 children could count back from 22, and difficulty bridging the decades was again common. One explanation for this may be that children were trying to learn an entirely new number word sequence. If children learn to produce number word sequences by rote, then it is possible that they do not understand that counting backward involves saying, in reverse order, the same number sequence produced when counting forward. Hence, children may benefit from experiences that focus attention on this phenomenon. Further, counting a collection of items (grouped in tens and ones) that is repeatedly reduced by one may also assist children to understand how the number words relate to the reducing number in the collection.

Conclusion

This paper explored the counting errors produced by Grade 1 children who were about to commence an intervention program. Although a small number of the children were unable to demonstrate number conservation and one-to-one correspondence, most counting errors made by the children related to bridging decades (including 110) when producing the forward and backward number word sequences. It is possible that such errors occurred because the children did not recognise patterns and relationships between number words in the sequence, and the "manyness" concept represented by the number words. It is important for teachers to be on the lookout for children who have difficulty bridging the decades, the century and 110, and who have difficulty counting backwards, so that they may provide experiences to assist these children *understand* the number sequence and the way it is constructed. Number conservation and one-to-one correspondence are fundamental understandings for counting collections of items and associated problem solving. It is important also for teachers to identify children who are yet to reach these understandings, and then provide experiences to assist children construct this knowledge.

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