A Teacher-Researcher Perspective on CAS in Senior Secondary Mathematics

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The Victorian Curriculum and Assessment Authority (VCAA) Mathematical Methods Computer Algebra System (CAS) Units 1–4 pilot study 2001 – 2005, integrates the use of CAS in the curriculum, assessment and pedagogy of a senior secondary mathematics subject, with end of final year examinations. This paper reports on some preliminary reflections from a pilot teacher – researcher perspective and outlines directions for qualitative research into teacher and student responses to the use of CAS as part of a new doctoral study.

Mathematical Methods CAS Units 1–4 were accredited in February 2001 (www.vcaa.vic.edu.au/vce/studies/MATHS/caspilot.htm), and pilot implementation initially consisted of three volunteer schools, commencing with Units 1 and 2 in Year 11 in 2001 and Units 3 and 4, with corresponding examinations, in Year 12 in November 2002. The pilot program has been progressively expanded, with students from 16 schools enrolling in Units 1 and 2 from 2002, and a staggered introduction of student enrolment in Units 3 and 4 from 2003 and 2004 respectively. The initial stage of the pilot has proceeded in conjunction with a major Australian Research Council SPIRT grant for the CAS-CAT project 2000 - 2002, a partnership between the Department of Science and Mathematics Education at the University of Melbourne, the VCAA and three calculator companies, Casio, Hewlett-Packard and Texas Instruments (www.edfac.unimelb.edu.au/DSME/CAS-CAT). The VCAA has developed an extensive collection of teacher resource materials, including sample examination papers, supplementary questions and related advice, which, as well as the pilot examinations for 2002, and corresponding assessment reports, can be accessed from its website.

VCAA Mathematical Methods CAS Pilot Study 2001-2005

The VCAA pilot study (see Leigh-Lancaster, 2002) takes place in the context of international interest in the use of technology in mathematics and mathematics education generally, and more particularly CAS enabled curriculum, assessment and pedagogy. The use of CAS, in varying forms, is permitted or assumed in some examinations, or parts of examinations, in the United States (College Board, Advanced Placement Calculus), Denmark (Baccalaureat Mathematics) and France (Baccalauréat Général Mathématiques). The diploma review committee for the International Baccalaureate (IB) has approved a pilot course for mathematics that requires the use of computer algebra systems, which will be developed in parallel with the mathematics Higher Level course as part of the current IB review process. A distinctive aspect of the VCAA pilot is the congruence between the design of the curriculum, its areas of study and content; the outcomes for the course and key knowledge and skills and their assessment, with assumed access to CAS for both coursework and all parts of examinations.

A range of reasons has been articulated for the use of CAS: the possibility for improved teaching of traditional mathematical topics; opportunities for new selection and organisation of mathematical topics; access to important mathematical ideas that have previously been too difficult to teach effectively; as a vehicle for mathematical discovery; extending the range of examples that can be studied; as a programming environment ideally suited to mathematics; by emphasising the inter-relationships between different mathematical representations; as an aid to preparation and checking of instructional examples; by promoting a hierarchical approach to the development of concepts and algorithms (Carrs, 1984, pp. 162, 165). Also: long and complex calculations can be carried out by the technology, enabling students to concentrate on the conceptual aspects of mathematics; the technology provides immediate feedback so that students can independently monitor and verify their ideas; the need to express mathematical ideas in a form understood by the technology helps the students to clarify their mathematical thinking; the technology allows students to explore mathematics using different representations simultaneously; situations and problems can be modelled in more complex and realistic ways (Oldknow & Flower, 1996, pp. 43–46).

Various concerns have also been expressed about possible negative consequences: the extent to which the use of CAS may reduce students' knowledge and skills with conventional by hand or mental techniques; how students, including those who may be less mathematically inclined, will cope with a more conceptually demanding curriculum; a diminished role for teachers in terms of traditional pedagogy; whether appropriate cognisance has been given to the role of by hand approaches in the development of important mathematical concepts and skills (Oldknow & Flower, 1996, pp. 43–46).

The Victorian Mathematical Methods (CAS) Pilot Study

Pioneering work with CAS, and related research, was often based on existing course curriculum and assessment structures, with CAS use integrated as applicable, but with associated examinations conducted without access to CAS; thus teachers needed to balance developments in pedagogy stimulated by the use of CAS with assessment structures that precluded the use of CAS technology. The VCAA pilot aligns the use of CAS in curriculum and assessment and thus provides the opportunity for further exploration of associated pedagogical developments. These can impact significantly on teaching and learning practice, as has been noted in the VCAA pilot study (Garner, 2003; Leigh-Lancaster, 2003) and the CAS-CAT project (Flynn, Berenson & Stacey, 2002).

Context for Research

This paper describes a new area of intended research, and one for which qualitative methodologies are particularly suitable. The first author is one of the four teachers involved in teaching the first stage of the pilot study 2001 – 2002, and her continued involvement in the expanded pilot provides a sound basis for undertaking this research as a teacher-researcher.

Teaching With CAS: A Whole New Experience

Involvement in the VCAA pilot study and the CAS-CAT project was the catalyst for staff and students at Ballarat Grammar School to consider issues associated with the use of CAS in the classroom. Some aspects of change in the first author's approaches to teaching and learning have been described in various conference proceedings (see Garner, 2002, 2003). In the first phase of the pilot study it has been found that teaching and learning in the senior mathematics classroom changes substantially when students used CAS calculators on an everyday basis. The focus of the intended research will be to investigate the thesis that teaching with CAS necessarily entails significant new pedagogy rather than an extension of existing approaches. Support for this thesis arises from experiences of the first author as a consequence of her involvement in the pilot and research project; and the significant difference between this experience and her initial expectations.

In schools involved in the pilot and the project, it has been found that, after the initial hesitation in CAS calculator use, students used CAS whenever *they* thought it was suitable in various mathematical contexts: be it, class work, homework, school assessed coursework (SAC) tasks or tests and examinations. Tynan (2003, p. 258) explores the transition in students' attitudes, describing changes from *early students' responses* to *later students' responses* as students moved through Units 1–4 across Year 11 and 12. In summary he describes the early frustration of students making way for a clearer definition of problems; the change from inadequate use of brackets to effective, and even over-use, of brackets; from single steps in CAS computations to multiple steps; and from the use of calculator syntax in written solutions to more cohesive and mathematically accurate solutions. In the first author's school it was observed that students also developed an anthropomorphic relationship with the CAS calculator and praised or blamed it as a 'person-object' accordingly. One student wrote in his journal:

> During our time together, the CAS calculator and myself have held very much of a love/hate relationship-there are many things about the CAS which are extremely useful, yet other things which are very frustrating and misleading. (Garner, 2002, p. 390).

Early observations indicate two identifiable groups of students: mathematically able students who naturally integrate the CAS into their mathematical framework (see Pierce & Stacey, 2002); and students for whom the CAS helps them jump the hurdles that arise with formal symbolic mathematics, and enables the step into senior mathematics (see Garner, 2003). When students are using CAS, issues of locus of control and ownership of the technology arise, and while the use of CAS may make some students anxious or uncertain, it unexpectedly empowers others. Thus teacher and student values, beliefs and preferences, and the dynamic between these, need to be investigated with respect to new pedagogies.

The last two paragraphs have recounted some of the exciting changes for students. However this paper has as its focus, the teacher.

Significance

An important question is: to what extent, and in what ways, change in teaching approaches stimulated by the use of CAS is a function of the beliefs, values and preferences of the particular teacher? Some intriguing evidence is already emerging:

> Kendal and Stacey (1999) found that teachers will choose to highlight attributes of CAS which support their own beliefs and values about mathematics. Teachers who value routine procedures can find on a CAS a plethora of routine procedures to teach students; teachers who value insight can find many ways in which they can demonstrate links between ideas better than ever before. (Stacey, Asp & McCrae, 2000, p. 248).

The following extracts of interview of the first author, when reflecting on a sequence of lessons towards the end of 2002, highlights this.

> Interviewer: By teacher-led you mean *you* direct the students?
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> Teacher: Yes, yes. I think that's just who I am … So when you've got this technology its going to affect my teaching in a totally different way than its going to affect someone else because we've got different personalities and I think we'll see that across the project. But the fact that they (the

> students) voted that they were going to do it (a skills question) in three different ways, it surprised me but I was **absolutely** delighted. And I said to them. "This is t**errific**". Because what this means now is that teaching has moved from me telling you what to do into you making your own decisions that suits you … and the way you think about things. And that each was equally valid. (**Bold** indicates emphases given verbally during the interview)

At the end of the initial stage of the pilot and project, the four teachers discussed their experiences, and felt strongly that the success of a CAS-active curriculum sinks or swims with the teacher. While it may be argued that this is true of any significant curriculum development and its implementation, these teachers had previously been through such experiences in various contexts, and felt that the observation was particularly pertinent here. The following interview extract highlights a change in approach for both the teacher (first author) and students.

> Interviewer: So what criteria would you have been using at the beginning that you changed to now?
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> Teacher: I preferred by-hand solutions … Whether that's security, and I know I'll get it right … Security, safety, and familiarity in doing what I've always done … The students and I have moved to a feeling that the calculator gives you something that has got an intuitive feel to it, rather than just churning out a result that you use with no understanding.
>
> I: Is the CAS result triggering a mathematical *learning* situation?
>
> T: Yes, yes. And I didn't think it was going to. I thought it was going to be just "This is just an answer, and manipulate it, and don't understand". And the fear is all the time that you're going to lose your algebraic skills … I don't think that's the case because I think the answer means *so much more* when you've got the algebraic understanding underneath what's happening.

Methodology

The intended research will investigate changes in teaching and learning approaches, and related change in teacher and student beliefs, values and preferences and the culture of the classroom flowing on from the introduction of CAS in senior secondary mathematics curriculum. The research will draw strongly on qualitative methods with the first author as teacher-researcher, and involve autobiographical narrative triangulated with survey data, case study data and meta-analysis of the literature, as elaborated in, for example, Jaeger (1997) and Denzin and Lincoln (2000). The use of a new technology being advanced initially through the work of a small number of supportive individuals, whose experience is then drawn on in a broader context, is itself a time honoured tradition and associated problematic aspects of implementation and research are part and parcel of this sort of investigation. Laborde (2000) has reflected on a similar situation:

> The same kind of debate took place in the seventies about the four operations calculator, which would favour the student's laziness, as well as in the XIXth century in France when the metal quill replaced the goose quill. (Laborde, 2000)

Use of CAS in the classroom and its impact on by hand skills is expected to be a "catalyst for debate over the future goals for mathematical proficiency in the CAS age" (Flynn, Berenson & Stacey, 2002, p. 7).

With respect to the related question as to what empirical evidence supports the value of teaching with technology, a meta-analysis of the literature by Dunham (2000) notes the substantial discussion of issues in graphics calculator and CAS research. She clearly articulates a response to what, for many, is an important issue surrounding the introduction of any new technology into mathematics education:

> Students taught with technology, but tested without it, perform as well or better than students taught with more traditional methods. (Dunham, 2000)

This *comparative* perspective makes assumptions about the extent to which the goals of the curriculum are fixed, or at least common and agreed. Dunham's meta-analysis indicates that there is, in general, "no harm done - no loss of pen-and-paper skills when tested without technology, provided the skills were taught" (Dunham, 2000). Students in the first author's classes have performed as well or better than their peers after being taught with CAS technology, and tested without it, on important algebra and calculus content (Garner, 2003, pp. 91-93). Related data will be gathered with particular focus on students who are studying Mathematical Methods CAS Units 3 and 4 and concurrently studying Specialist Mathematics Units 3 and 4 using a graphics rather than CAS calculator.

Changes in teacher and student behaviour include: *student behaviour* - less passive learning; more conjecture and testing; more inquiry and independent investigation; less time on manipulation; more modelling; less note-taking and listening to the teacher; more discussion with peers; better communication; and more mature and fluent language. Corresponding changes in *teacher behaviour* include – less explaining; more higher-order questions, and, more consulting and monitoring. As the related discourse evolves, there is a clear need for new research especially at the stage where CAS technology is first introduced, to monitor and analyse subsequent changes in teaching practice, and the evolution of new stages and equilibria in this practice.

Teaching the Ends and Sides of a Topic

Some Preliminary Observations and Comments

The following observations and comments are preliminary and necessarily tentative in nature - their purpose is to highlight key aspects of the intended study and some of its dimensions. Preliminary results from three data sets are briefly described.

Autobiographical narrative. Teaching with CAS, appears to engender a new way of teaching. The first author has described her teaching as *teaching the ends and sides of a topic* rather than proceeding in a linear fashion (see Garner, 2003).

> If I'm going to start a topic, previously I would probably have started it by what you might call linearly. I would start from the beginning and go step by step to the end. Now I don't at all. If I'm going to start a topic, I very often start at the end. (Teacher, first author, 2002)

This experience is quite different from her early expectations:

> With the Year 11s I talked about instantaneous rate of change and then I looked at it on the graph. I looked at tangents. I traced the gradient. I looked at gradients in tables. I looked at gradients in the RUN menu and looked at gradients in the CAS menu. So I would have jumped to what might be perceived to be the end, to give an understanding of where were going. Its almost like you *don't need to do* the rules as such any more. It's interesting isn't it? The rules just develop as you go along. The *understanding* of the rules and *what you need* develop amidst it. I don't feel I need to go step by step any more. And I think the kids have a better understanding of it. (Garner, 2002, p. 398)

The pace and nature of change in teaching styles when teaching with CAS, is worth investigating to track the changes of a new way of teaching, rather than an adjunct that helps with mathematical discovery.

The dynamics of the classroom changes with the calculator viewscreen a central focus, and the multiple representations of numeric, graphical and symbolic forms more easily demonstrated. It has been observed that the students move more easily between these representations, thereby giving a deeper and broader understanding of the concepts:

> I don't any more teach the big introductory lesson with the notes and heading on top. That's gone. All changed in a year. (Teacher, first author, 2002)

Another key issue is locus of control in the classroom - teachers needed to be flexible while students were discovering, often more quickly than the teacher, new ways to tackle a problem:

> We were often surprised how quickly students learnt the syntax, and adapted to most of the idiosyncrasies of the calculator operations. Often students delighted in revealing to the class another discovery that they had made about a short cut or new method, or explain(ing) why a particular procedure might not work. (Tynan, 2003, p. 256)

Considering the scientific calculator using numerical representation, the graphics calculator extending to graphical representation, and the CAS calculator including symbolic representation as well, it could be said that this transition to CAS calculators will likely be more influential. This is because the transition is not just another part of a continuum, but that CAS incorporates all three representations and completes the continuum.

Case study. The following are some excerpts from interviews of the (first author) teacher:

> Interviewer: Would it make any difference which representation you're using?

> Teacher: No, I flip between them all the time. That's what's changed for me and what I mean about the holistic stuff. I would have been highly symbolic probably, naturally.

> T: There probably would have been lessons earlier in the year where I would have said "Lets actually look at this because this is going to be useful." But I find, if I teach a bit of technology, a bit about the calculator that is out of context, they're actually not interested. They're only interested in looking at it when a problem comes up as we go along. So … the thing about VARS to store the function … When I showed it to them they said, "Ho hum, yeah that's fine". But, during the application task, I could hear them all saying "Use VARS, its much better. Use VARS to store the whatever, then it'll be in the GRAPH." And I found out, marking last night that they've actually adopted it and used it extremely well.

Ball, Stacey and Leigh-Lancaster (2001) describe this aspect of CAS use: they expect teachers to find that students will not be particularly interested in the functions of the calculator until a specific need for that use arises. "Will students complete a unit on calculator use or learn about capabilities as they go along?" (Ball, Stacey & Leigh-Lancaster, 2001, p.269). Perhaps the use of CAS allows some students to jump hurdles that arise with formal symbolic mathematics.

> T: I asked him (a student) "Can you do maths better this year? He said to me, it was one of those crowning moments, "I can *do* maths. Last year I couldn't do it. I nearly didn't do Methods…This year I can do it". I said to him "Do you mean that you are using your calculator to do the maths instead of the maths. Is that what you are saying to me?" And he said "No, I understand what you're talking about and I am so excited. And I'm getting 8/10 for my SACs and I've never, ever been able to do maths before. And he's just a changed boy. It's not because he's using the calculator all the time. It's given him security to jump the fence to say "I can now do the maths"…it's sort of like it [the CAS] has *empowered* them to do maths when they thought they couldn't. That still keeps surprising me.

Laborde (2002) notes that:

> In our introduction we claimed that the process of integrating technology into mathematics teaching is a long and complex process - technology is not just an additional element in the system since it

> interacts with all components of the system, which are subject to change (Laborde, 2002, p.31) …and… (it) takes time for (teachers) to accept that learning might occur---without reference to paper and pencil environment … But it also takes time for them to accept that they might lose part of their control over what students do. (Laborde, 2002, p. 32)

Survey. A survey, containing 27 items, was given to students after the Year 12 exams and results were completed. Early analysis of their responses showed that the effective selection of by hand or CAS methods developed, depending on the situation, as the year progressed. As one student commented:

> It can't (usually) be used to solve problems that the user doesn't have an algebraic understanding of … the CAS may be used in the gaining of that understanding. (Garner, 2002, p. 391)

This comment is in line with the idea of "scaffolding students as they undertake unfamiliar processes" (Stacey, Ball, Asp, McCrae & Leigh-Lancaster, 2000, p. 62).

Conclusion

The *congruence* of CAS use in pedagogy, curriculum and assessment (see Leigh-Lancaster, 2000) leads to changes in teaching and learning in senior mathematics. This is certainly the experience of the first author: for this course, with this technology and for these students. Such changes do not occur in isolation, and will also likely impact on the use of calculators in earlier years, as well as informing the use of CAS technology at the secondary-tertiary interface. From the experience of the authors, this is certainly the case with CAS. The inclusion of CAS calculators in all assessment tasks in the VCE for this group of students changed how the students viewed their mathematics. It is important to see if the change in teaching and learning, which has been experienced by the first author as part of this study, can be generalised to other situations introducing the integrated use of the CAS calculator in the teaching and assessment of mathematics.

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