Windows Into Mathematics Teaching Through Data Maps

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Although perceptual inference facilitates reasoning, there has been scant attention to the use of icons in visual displays of qualitative data. This paper uses an iconic data map to provide a wholistic representation of two teachers' mathematics teaching and to establish similarities and differences in their practice. The map had five advantages for data display and analysis related to visual memory, strategic professional development support, the identification of anomalies, differences between espoused and enacted teaching beliefs, and the identification of further research questions. Thus, iconic data maps are a powerful tool in the display and analysis of qualitative data.

Information graphics is the broad field that addresses the use of visual displays including diagrams, maps, charts, tables, graphs, and icons in print and electronic forms for the management, communication, and analysis of information (Harris, 1996). While most of these types of information graphics are widely used in data display and analysis in mathematics education research, icons have received scant attention. An icon is "a sign or representation which stands for an object by virtue of a resemblance or analogy to it" (Delbridge & Bernard, 1982, p. 475). Icons offer particular advantages over text in the display of qualitative data. They communicate concepts rapidly and effectively because they look like what they represent (Lee, 1996). Additionally, they can be used to create rich but compact displays of information because they take up less space than the equivalent words (Lee, 1996). Iconic displays are rich because they can capitalise on the perceptual elements of position, length, angle, slope, area, volume, density, colour saturation, colour hue, texture, connection, containment, and shape (Cleveland & McGill, 1984) to convey quantitative, ordinal, and nominal information (Mackinlay, 1999). Thus, these displays have the potential to provide new insights about data because they can enhance reasoning through perceptual inference (Barwise & Etchemendy, 1991; Lindsay, 1995).

A critical test of the utility of icons in data display and analysis is their ability to provide insight into complex situations. The purpose of this paper is to explore the use of icons in the display of qualitative data, which depicts the complexity of classroom mathematics teaching, and the subsequent analysis of these data.

Visual Data Display and Analysis

Data displays that serve to provide a wholistic visual representation of a data set are henceforth referred to as "data maps". Earlier data maps were generally non-iconic (Diezmann, 1996, 1999). These maps were composed of geometric shapes but included a hexagon that depicted the "stopping" of a particular action. Lines and arrows linked the shapes and indicated interactions between the people, events or actions represented by the shapes. Applications of earlier (non-iconic) data maps included a comparison of students' strategy use (Diezmann, 1996) and the investigation of patterns and relationships between pairs of similarly performing students in interview situations (Diezmann, 1999). These maps capitalised on the advantages of visual displays in the analysis process, such as discerning patterns and relationships, noting relationships between variables (e.g., Miles & Huberman, 1994), and time-series analysis (e.g., Yin, 1994). However, despite the advantages of these earlier maps for reasoning, they failed to capitalise on perceptual inference due to the use of non-iconic shapes. A potential advance in the effectiveness of data maps is to replace the geometric shapes of the earlier maps with more easily recognisable icons that represent common classroom or interview events. For example, it is easier to "read" the icon on the left in Figure 1 as "an individual talking" rather than to recall that the circle to the right in Figure 1 means, "an individual talking". Thus, the use of icons in data maps should improve the quality of these maps for reasoning through the enhanced communication of meaning (Harris, 1996).

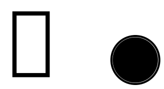


Figure 1. An individual talking.

Iconic data maps should be effective data displays because they can address the three conceptions of external representations. Although these conceptions are not readily apparent due to the ambiguity of the English term "representation", these conceptions are revealed through a German-English comparison of terms and meanings (von Glasersfeld, 1987). First, icons can be used to "depict" (darstellen) the data just as a sketch depicts an object. Second, links between the data represented can "mean" (bedeuten) a specific relationship. For example, the relative positioning of icons could indicate the relationship between the concepts represented by the icons. Third, the map itself, which consists of the integrated spatial arrangement of icons and relationships, "acts for" (vertreten) a particular context or situation, such as a classroom lesson. Thus, the potential of iconic data maps for display and analysis is embedded in their ability to simultaneously "depict", "mean" and "act for" qualitative data.

The purpose of data maps is to make explicit the relationships among data and facilitate perceptual inference. Hence, optimally, the spatial organisation of information on the map should be tailored to suit the research foci. For example, data maps have been constructed to represent the chronological order of critical events in an interview (Diezmann, 1996) and to compare two teachers' implementations of a lesson (see Figure 2).

The following section briefly addresses effective classroom teaching to provide a background for the iconic data map that follows (see Figure 2).

Practices in the Mathematics Classroom

Studying Classroom Teaching

Effective classroom teaching should provide rich mathematical learning opportunities for all students. According to Brown and colleagues (Brown, Askew, Rhodes, Denvir, Ranson, & William, 2001), who conducted a five-year study of mathematics in UK classrooms, there are four key teaching characteristics in effective mathematics lessons. These characteristics of Tasks, Talk, Tools (e.g., representations), and Relationships and Norms have been recognised by others as important in creating rich and supportive mathematical learning environments (e.g., Baroody & Coslick, 1998). Brown et al. developed a classroom observation instrument that identified the components of these characteristics (see Table 1). They also provided descriptions of these components at four levels of effectiveness. For example, they proposed that the four levels in the way teachers use "models" in mathematics teaching range from the lowest level where models are used superficially through two intermediary levels to the optimal level where the models are effective and appropriate for the task (see Table 2).

Table 1
Specific Teaching Characteristics and Their Components

Tasks
1. Mathematical challenge for all pupils
2. Integrity and significance of the mathematical tasks in the lesson
3. Engage interest in the mathematics of the lesson
Talk
1. Teacher talk that focuses on mathematical meanings and co-construction of knowledge
2. Teacher-pupil talk about mathematics
3. Pupil talk that focuses on reasoning and mathematical understanding
4. Management of talk to encourage pupils to talk about mathematics
Tools
1. Range of modes of expression including oral, visual, and kinaesthetic
2. Types of models used to represent mathematics ideas
Relationships and Norms
1. Community of learners comprising teacher and pupils
2. Empathy towards pupils' responses

(summarised from Brown et al., 2001, p. 14)

Table 2
Levels of Use of Types of Models

Nil	Model only provides window dressing.
Low	Model may lead to a unitary view of concepts.
Moderate	Range of models used but the teacher does not draw pupils' attention to the limitations of any particular model.
Optimal	Models used are appropriate and effective for tasks.

(Brown et al., 2001, p. 28)

Representing and Analysing Classroom Practice

To test the utility of iconic data maps for representing data and facilitating data analysis, a map has been constructed to represent two different teachers' implementations of the same lesson. The question to be investigated is: What similarities and differences in two Grade 3 teachers' implementation of a mathematics lesson are evident from the data map?

The Lesson

The selected lesson for analysis is from a larger study on mathematics teaching in Grade 3 classrooms (Diezmann, in press 2003; Diezmann, Watters, & English, 2001a, 2001b, 2002). Mrs I's and Ms U's implementations of the same lesson have been selected for representation and analysis. These teachers were selected because of their similar backgrounds and contexts. Both teachers taught in the same large outer metropolitan school; had in excess of ten years teaching experience; and taught comparative mixed-ability classes with 25 or 26 students aged between seven and eight years. The lesson selected for analysis was jointly planned by these teachers and incorporated a story-reading activity to introduce children to the concept of a mathematical investigation. The use of a story to provide the context for young children's mathematical thinking is widely endorsed (e.g., Whitin, 1994).

The teachers decided to use the "The Doorbell Rang" (Hutchins, 1986) to introduce the concept of investigations. This story commences with a small number of children sharing out a batch of 12 cookies. The doorbell then rings and more children arrive, and the cookies are subsequently re-shared to account for the new arrivals. This storyline of children arriving and the cookies being re-shared repeats until the doorbell rings again after 12 children have shared the 12 cookies. However, fortunately it is Grandma with another 12 cookies! This story provides the basis for investigating a range of mathematical concepts including division, multiples of 12, doubling, and inverse relationships between the number of children and the number of cookies. Both teachers agreed they would read the story and engage the students in mathematical thinking by having some students act out the story while it was read. These teachers also planned to use props, such as cookies and plates. Further details of this lesson are described elsewhere (Diezmann, in press 2003).

Iconic Data Maps

The iconic data map that follows (see Figure 2) was constructed to capitalise on the potential for perceptual inference through the incorporation of shapes, containment, position, and shading. The *shapes* are symbols from Microsoft WORD© that have some resemblance to the components of effective teaching (see Table 3). Thus, these shapes are icons. Each type of icon is *contained* in only one row on the data map with the *position* of the icon indicating its level of effectiveness. Rows are labelled with letters for ease of identification (see Figure 2). The further the icon is positioned towards the right on a row, the more effective the teaching component. Thus, an icon positioned towards the extreme left indicates that this component had minimal potential for learning, and conversely, an icon positioned towards the extreme right indicates that this component had optimal potential for learning. The numerals from “0” to “3” have been placed underneath the last row of icons in the map to indicate increasing levels of teaching effectiveness (see Figure 2). The identification of rows and columns by letters and numerals respectively, also provides for the identification of a particular cell within the map through grid references (e.g., 2A). The components of Ms I’s and Ms U’s teaching are distinguished on the map by *shading*. Icons are unshaded for Ms I and shaded for Ms U. Because much information about the lesson data sets is embedded in the map through the graphic elements of shape, containment, position, and shading, the iconic data map provides a perceptually rich but compact data display.

Table 3
Components of Effective Teaching and Corresponding Icons

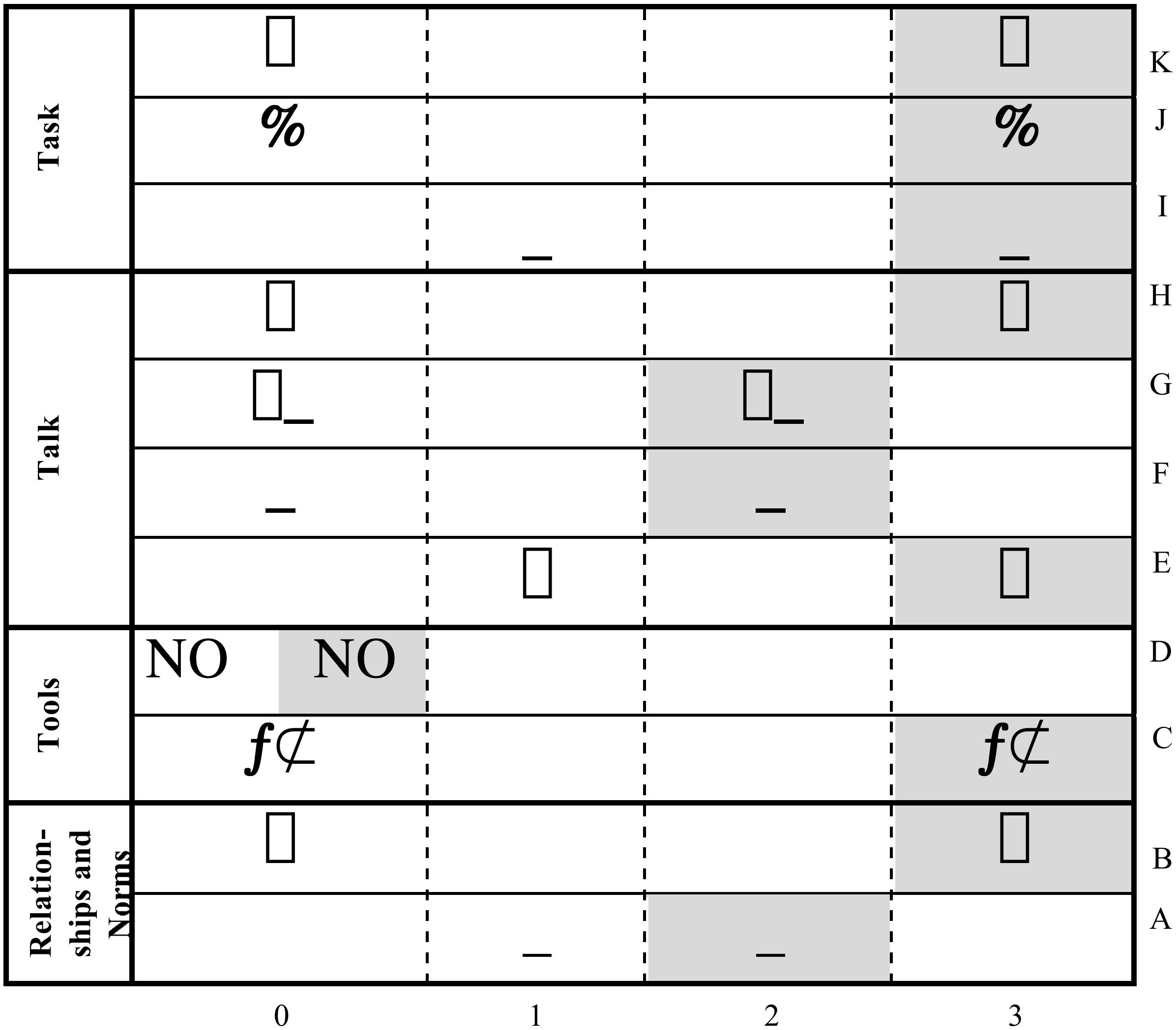
Teaching Characteristic	Components	Icons
Tasks	Challenge	□
	Significance	%
	Interest	
Talk	Teacher	□
	Teacher-pupil	□_
	Pupil	_
	Management	□
Tools	Range of modes	NO
	Models	ƒ⊄
Relationships and Norms	Community	□
	Empathy	_

Results and Discussion

The results and discussion focus solely on the use of an iconic data map for the representation and analysis of two teachers' implementation of a lesson. A discussion of the assessment of these teachers' performances on the various teaching components is beyond the scope of this paper but is reported elsewhere (Diezmann, in press 2003).

The Teachers' Map of the Lesson Implementations

A previous assessment of the two teachers' levels of effectiveness on the selected lesson on the various teaching components (Diezmann, in press 2003) was used to create the following iconic data map (see Figure 2). Ms I received eight ratings of 0 (i.e, the lowest level) and three ratings of 1 for the 11 teaching components. Thus, overall Ms I's teaching was assessed towards the lower end of the range of teaching effectiveness. In contrast, Ms U received seven ratings of 3 (i.e, the highest level), three ratings of two, and 1 rating of 0. This latter outlier rating is discussed later. Thus, in contrast to Ms I, Ms U's performance is generally towards the higher end of the range of teaching effectiveness. A comparison of the teachers' assessments on the various components reveals only two similarities in teaching effectiveness where the level assigned to the teachers was either identical or one level apart. In contrast, their responses were at least two levels apart on nine teaching components. These similarities and differences in teaching are discussed in turn.



Key
Ms I's assessments are unshaded; Ms U's assessments are shaded.
Rows A to K indicate specific components of teaching characteristics.
Columns 0 to 3 indicate increasing levels of teaching effectiveness.

Figure 2. Iconic data map of Ms I's and Ms U's lessons.

The teachers' implementations of the lessons were similar in two ways. First, neither teacher capitalised on a range of modes of tools (D0, NO) (e.g., oral, visual, kinaesthetic) during this lesson. However, this could be explained by the lack of a need for these tools in a lesson based on a story. Second, the teachers were assessed as similar in empathy towards pupils' responses (A-_) with Ms I's (A1) and Ms U's (A2) responses only differing by one level. Despite these similarities, the teachers' implementations of the lesson were more dissimilar than similar.

The two teachers differed markedly on nine teaching components with Ms U rated higher than Ms I on each of these components. There was a variation of two levels (0-2 or 1-3) on the four components of: management of talk (E-□), pupil talk (F-_), teacher pupil talk (G-□_), and interest (I-_). There was a maximum variation of the three levels (0-3) on the five components of: community of learners (B-□), models (C-ƒ⊄), teacher talk (H-□), significance (J-%), and challenge (K-□). This maximum variation in teaching components is noteworthy in three ways. First, on nearly half the teaching components assessed, Ms I's teaching was rated as ineffective, and Ms U's teaching was rated as optimal. Thus, students in Ms I's and Ms U's classes are experiencing substantially different learning environments. Second, there were scant opportunities for learning for mathematically gifted students in Ms I's classroom due to the lack of challenge (□), lack of mathematical significance (**%**), and limited use of mathematical models (**ƒ**⊄) (Diezmann & Watters, 2002). Third, because each teacher received the same ratings for these five teaching components, it seems likely that at least some of these components are interrelated. Brown et al (2001) identified an interrelationship between mathematical challenge (□) and mathematical significance (**%**) through the types of tasks that students undertake. Thus, providing strategic support to Ms I to improve the level of challenge within the lesson may also result in enhanced mathematical significance. However, further exploration is needed to determine other relationships between these five teaching components.

This analysis is limited due to space restrictions but it nevertheless provides some insight into the potential of an iconic data display to facilitate analysis.

Conclusion

The use of an iconic data map had five advantages in this study for data representation and analysis. First, the use of icons in the map acted as a memory support for the multitude of components that comprise classroom teaching. Second, the organisation of the icons on the map highlighted each teacher's strengths and weaknesses, and hence, provides some insight into strategic professional development support that might enhance their classroom teaching. For example, it might be useful for Ms I to focus on improving her teacher talk (H0-□), but Ms U is already using teacher talk very effectively (H3-□). Third, the map identified an anomaly in one teacher's performance. Ms U's assessment of a 0 for the use of a range of modes of tools (D0, NO) is inconsistent with her assessment on all other teaching components and warrants further investigation. In contrast, Ms I's assessment of 0 is consistent with her other assessments of teaching components. Fourth, the map revealed considerable overall differences in teachers' implementations of the lesson. This outcome was unanticipated because the teachers planned the lesson together. However, this outcome can be explained by the substantive differences that may exist between teachers' espoused beliefs and their enacted beliefs (Robinson, 1993). Finally, the interpretation of the data maps indicated a range of questions to be explored further. There are questions specific to the teachers, such as "What are the similarities and differences between these teachers' teaching components at different stages of the lesson, such as the Introduction?" This question could be investigated by preparing a chronological data map for each teacher and comparing their maps at various stages of the lessons. A further question specific to these teachers is, "Is this typical teaching behaviour for each teacher?" This question could be investigated by representing other lessons by the same teacher on data maps and comparing those maps with the map of this lesson. There are also more general questions that can be investigated, such as "Which types of mathematics lessons cue the effective use of particular teaching components?" This question could be investigated by representing similar types of lessons taught by a range of teachers on data maps and comparing these maps. The potential for iconic data maps to highlight new research questions and to provide the means to investigate these questions makes them a powerful tool that facilitates the interactive and recursive process of qualitative analysis (Miles & Huberman, 1994).

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