## Searching for Mathematical Ideas in Stone Walls

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This paper discusses the challenges posed by ethnomathematics as a field of study, and suggests ways of meeting these challenges. Using an on-going ethnomathematical study on the practice of stone walling among the Kankana-ey people of northern Philippines, it shows how an investigation of QRS conceptual systems may be useful in identifying unfamiliar mathematical ideas. It presents four indicators that may be used in uncovering mathematical systems in a cultural practice, and examines stone walling using these indicators.

The task of ethnomathematics as a field of study is to uncover mathematical ideas that are hidden within cultural concepts or practice. How to search for unfamiliar mathematical ideas and how to make sense of them in a culturally unfamiliar context are two interrelated problems in ethnomathematics.

The first problem suggests that the concepts we are looking for may not be recognisable or familiar in the conventional mathematical sense, while the second stresses the need to understand these concepts in their cultural context. These challenges are interrelated in the sense that the mathematical ideas or concepts are meaningless outside of their cultural milieu. Such ideas are produced and reproduced, or are changing, according to the material conditions and realities from which they are rooted.

Both problems point to the need for "external criteria" for admitting a certain concept or practice as mathematics (Alangui & Barton, 2002) or mathematical. The need for external criteria admits the possibility that the concept or practice that is the subject of an ethnomathematical study may originate from a culture that does not have the category mathematics. In this case, judging a cultural concept or practice using the categories of academic mathematics poses a dilemma to the ethnomathematician – it becomes a colonising enterprise, a case similar to what McConaghy (2000) refers to as "theoretical assimilationism" in indigenous education.

Making explicit these external criteria is a challenge to any ethnomathematical study. Alangui and Barton (2002) suggest one approach that may prove useful is investigating how people make sense of quantity, relationships and space (QRS), and how these are communicated and talked about in the culture. Such an approach avoids reference to the term mathematics. Studying the QRS system embedded in a cultural practice may help uncover the mathematical ideas in a cultural practice, while avoiding the pitfall of judging a practice using categories of academic mathematics. The study of QRS system originally proposed by Barton (1996, 1999) actually means investigating QRS *conceptual* system(s), not just technical ones. It is important to make this distinction because we want the mathematical ideas we seek in an ethnomathematical inquiry to be those that are part of a

<sup>1</sup> A technical system merely describes the practical and procedural aspects of say, a cultural practice. While this is in itself a useful endeavour, the investigation might miss out on the opportunity of going deeper in the understanding of the people's conceptions of quantity, relationships and space that may be embedded in their practice. In this case, the aim of the ethnomathematical study is glossed over.

"connected web of knowledge" (Hiebert, 1986). The study of QRS conceptual systems implies the need to look at such ideas or concepts within an (abstract) system. It suggests that the mathematical ideas we are investigating, initially unfamiliar to the western-trained mathematician, are the ones that reflect complex mental processes and human mind activities (Ebeid, 2001) that can be abstracted and freed from their specific contexts. Even though such ideas are constructed out of real-life problems, they become tied together by a web of knowledge that exhibits relationships either at the primary or reflective level (Hiebert, 1986).

It is important to remind us that it is the people or the practitioners, not the ethnomathematician, who are creating the webs of knowledge about their practice at the abstract level. They are the ones who make the leap from the technical to the conceptual level as reflected in the way they use and talk about their ideas. The ethnomathematician's role is to make the connection of such ideas to mathematics.

I contend that the search for QRS conceptual systems will allow ethnomathematician to investigate important questions such as how knowledge becomes a collective knowledge and how it is shared within the community. It also affords the ethnomathematician to ask what is useful knowledge, how new knowledges are generated and old ones transformed (McConaghy, 2000).

The question of whether "alternative mathematical systems exist" (Bishop, 1990) poses another challenge to ethnomathematicians. How do we identify the existence of a mathematical system in an unfamiliar context? In mathematics, we normally define a system based on sets, elements, and operations, and proceed in classifying and characterising the system and its properties. The problem is when we are to look for 'mathematical' systems in non-conventional terms.

In a mini-conference that gathered a group of ethnomathematicians<sup>2</sup> in Ubatuba, Brazil in August 2002 immediately after the Second Congress of Ethnomathematics, the following have been suggested as possible indicators in the search for a mathematical system in a cultural practice:

- 1. The practitioners are able to talk about the practice in the abstract. Being able to talk about the practice *ex situ* suggests that the practitioners have a firm grasp of the workings of the practice at the conceptual level. This also frees them from the physical and practical limitations of the practice in its actual setting, and allows them to theorise about the practice, make predictions, and talk about possibilities.
- 2. There is evidence of modelling. This is related to the first in the sense that modelling allows the practitioners to talk about their practice in the abstract and make predictions about their practice. We may not expect to find mathematical models in the conventional mathematical sense, but any evidence of modelling suggests an abstraction from the physical properties of the practice, which allows the practitioners to manipulate the relevant elements or aspects of their practice and imagine "what is and what could be."

<sup>&</sup>lt;sup>2</sup> This was composed of doctoral students from Brazil, Maldives, Portugal, United States and the Philippines, along with several experts in the field of ethnomathematics. The four-day conference featured extensive discussion and critiquing on the on going, as well as proposed work of these doctoral students of ethnomathematics.

- 3. The practice can adapt to change(s). This suggests a property that is inherent in (mathematical) systems and structures, which is that making new assumptions, or changing some assumptions about a system or structure may result to a completely new system/structure.
- 4. The practice can jump applications. This is related to the previous in the sense that allowing changes in the system and structure of the practice may result to new applications. This also suggests dynamism, which is a desirable feature for any system to have, whether mathematical or not.

The QRS conceptual system approach, and the system-indicators suggested above, are used as a framework in an on-going investigation of the practice of stone walling in two indigenous communities in the Cordillera region, in northern Philippines.

### Methodology

This research on the practice of stone walling was done among the Kankana-ey people in the Cordillera region of northern Philippines. Rice terracing is an important economic activity in the Cordillera and is a common undertaking in the region. The remote villages of Agawa and Gueday in Besao, Mt. Province were selected for the present study primarily for two reasons. One is the existence of extensive rice terracing activities in the area. Here, by extensive is meant rice terracing as a major economic activity (Florendo & Cardenas, 2001). The second is the employment of indigenous techniques of terracing, such as stone walling. One assumption of this research identified stone walling as a possible source of mathematical concepts and problems. Not all rice terracing communities in Cordillera region build stone walls.

Following the ethnographic approach, five methods of collecting data were employed. These were interviews, oral history, participant observation, archival search, and research journals. Two small group discussions were held to complement the individual interviews.

The formal structured interviews involved eleven (11) males and four (4) females. Of the eleven males, nine (9) were interviewed individually and the other two (2) were involved in the small group discussion (one of the nine interviewees was also in this small group discussion). In general, there were thirteen (13) individual interviews, and two small group discussions involving three (3) men and (3) women during the first field work.

Data gathering was done from December 2001 to July 2002. A second field work was done for two weeks in January 2003 for follow up and validation.

# The Practice of Stone Walling in Agawa and Gueday

The data about stone walling that were gathered during the field work may be categorised into three: (1) a description of the stone wall practice; (2) an account of how the practitioners and the people talk about the practice (as seen in the language used and in their discourse about the practice); and (3) a description of the practitioners. What follows is a brief presentation of the data gathered according to these three categories. A detailed description will not be reported in this paper but will be part of a doctoral thesis that is being written based on this research.

### Brief Description of the Practice

The people of Agawa and Gueday grow rice and vegetables by carving out rice paddies on the sides of mountains. The sloping nature of upland landscapes requires the use of soil and water conservation technology (Prill-Brett, 1985). Stone walls are built to hold the rice paddies, impound water, and in general, to prevent erosion. The stone walls in this study serve as retaining walls (or riprap) that are used to keep earth banks from eroding. For this reason, a good stone wall is highly desirable, one that is sturdy and stable—it lasts long and is not prone to erosion. In building walls for the rice paddies, beauty (*pintas*) or appearance (*buya*) is not important. Ensuring durability necessitates knowledge about pressure, force and friction. The way the foundation stones, the positioning of stones, and the backfill are put together suggests that all possible constraints (e.g., pressure from both soil and water) have been considered to minimise the possibility of erosion. The preferred shape of the wall (vertical for low stone walls, slightly concave batter for normal stone walls, or terraced walls for very high ones) also suggests knowledge about how height and inclination could contribute to the strength or weakness of the wall.

Here are some insights from the stone wallers of Agawa and Gueday:

Uray asi-asi nan rupa na basta puesto ti bato ket kusto ken abab na ket nabayo nga nalaing tapno haan nga maanod ti daga. (Even if the appearance of the stone wall is not so good as long as the stones are properly positioned and the backfill is compacted well so that the soil will not be washed out.)

Siguraduen nga agkagit ti bato, isu nga uray bassit nga bato ket magamit, haan lang kanayon nga dadakkel. Importante nga agkagit ti batbato ta nu agkuti ti tallo, magday met laeng ti kaaduan. (Make sure that each stone fits snugly with the other stones, so that even small ones may be used, not only big stones. It is important for the stones to fit properly, for if three stones start moving, the rest will eventually follow.)

Kailangan nga agkikinnagat ti batbato, wenno agi-innirot (The stones need to "bite into each other" or "tighten up like screws.")

An on-going challenge for this study is to understand how the people make sense of the concepts of "pressure, force and friction" not only in building stone walls, but also in other situations in the community. One helpful way is to investigate how they talk about these concepts, and how these are expressed in their value and belief systems.

Levi Strauss' idea of bricolage (Turkle & Papert, 1993) is an apt description of how farmers build or repair stone walls, where in the process, knowledge about stone walling is constructed. Bricolage is a style of work, and the bricoleur (in this case, the stone waller) engages himself with what Turkle and Papert (1993) refer to as a constant "arranging and rearranging, negotiating and renegotiating" with a set of materials that he uses in the construction (here, the materials are primarily stones).

It is important to mention that stone walling is employed not only in holding rice paddies along mountain sides, but also in supporting houses, roads and irrigation canals. Stone walling has also been widely adapted in urban centres as well as in the city. What the community calls modern-day stone walling involves the use of cement to keep the stones together. It should be noted that the stone walls in the villages do not make use of cement. They rely on the correct positioning of stones, proper back filling, and use of foundation stones in ensuring that the stone wall will hold.

## The Language of Stone Walls

One possible way to look for things, which are not familiar, is to study language. Alangui and Barton (2002) claim that "an examination of etymology and syntax may yield unexpected links between mathematical concepts, objects or actions." Studying how concepts that are more or less universally accepted as mathematical (e.g., number and orientation) are expressed in another language (e.g., use of metaphors) and determining whether they show different characteristics may lead to unfamiliar mathematical links.

Similarly, studying myths and legends, written archives, traditions and rituals, and well-developed symbols or monuments could prove to be mathematically rich, "for if there is important mathematical knowledge in a society it must be preserved in some way, and it is likely to be visible" (Alangui & Barton, 2002).

For example, stone wallers talk about a "rightful" stone that should occupy a particular space in the wall. To place a stone, one looks for the "best fit" – how the stone settles in relation to the other stones already in place. Although guided by certain rules, stone wallers claim there is a "feeling" that accompanies a correct placement of stone. They use the term "suni," which involves a continues repositioning of the stone until "it feels right." They say being able to discern the right position is an important skill that every stone waller should have.

There is no separate word for pressure and force. The local term for both is "pigsa", which they use to describe the pressure/force of water or the soil. There is no local term for friction. The idea of friction seems to be captured in the way they describe the positioning of stones, that is, when they talk about the stones as "biting into each other" or "tightening like screws." The idea is to maximise contact points between the stones to produce greater wall strength.

The importance of durability of the stone wall is used as a metaphor in describing certain situations. For example, the people compare the stone wall with the rearing of a child or the quality of a relationship between two people. Hence, a child with good values is like a stone wall with a strong foundation, while a lasting relationship is like a sturdy stone wall. It has permanence.

Some beliefs and stories seem to reinforce the need to build the wall in a concave batter. For example, they have the belief that "if the wife is pregnant, the man should not build a stone wall" or that the stone waller should not have sex while building stone walls for this might cause erosion of the wall. At the same time, stone wallers predict that a wall that resembles the shape of a pregnant woman, or one with a full stomach, will erode in due time. In this case, beliefs and stories seem to reinforce the desirability of a wall that is not "pregnant" (a concave shape is preferred over a convex one) suggesting its durability.

#### The Stone Wallers

Stone walling is a gendered knowledge. It is the men in the community who possess such knowledge. They are traditionally the stone wallers. How does the community view them? What other functions do they perform? How did they become stone wallers? The respondents say that every male member of the community should learn how to build stone walls. Indeed, it is desirable for every man to be able to build, or repair his stone wall when that time comes. However, the study shows that not every male farmer would have

this skill. The people suggest that the skill and knowledge of stone walling, although this can be learned, is something that a person is borne with. They claim that "you either have it or you don't." Although anybody can try to be a stone waller, there are those who are really good at it. They would point to a group of men who they think are the highly competent stone wallers in the village. They are the respected stone wallers in the community. I have the impression that even among the highly regarded stone wallers, some are more knowledgeable than others, which makes it a differentiated knowledge as well. For instance, some of these stone wallers are able to combine aesthetics with function. They prefer building durable stone walls that are also pleasing to look at.

There is no age requirement in learning the skill. One respondent learned it when he was twelve years old, another when he was eighteen, and still another when he was already 40 years of age. All of them learned it by trial-and-error, but under the guidance of an elder. I contend that trial-and-error as learning and teaching technique is an important aspect of bricolage. In this case, bricolage becomes not only a process of organizing work (e.g., stone walling), but also an educational strategy that is consistent with indigenous ways of learning and teaching, and of knowing and thinking.

There is an interesting parallelism between stone wallers and mathematicians in the way they are viewed by their respective communities (e.g., the idea of being borne with the skill and knowledge to become a good mathematician/stone waller). This parallelism is also seen in the status that is accorded a highly-competent mathematician/stone waller (a mathematician is generally considered as intelligent by the community, while a good stone waller is generally a respected member of the village, if not considered a wise person). The mathematician desires an elegant solution or proof, while the stone waller wishes to build a stone wall that is both durable and appealing to the eyes.

It is also important to note that the stone wallers can talk about stone walls and stone walling *ex situ*. There is planning involved in the practice (e.g., how to build a high wall given the contour of the land, what to do when there is water at the base of the wall, and so on). Stone wall construction problems are discussed not only in the actual site, but also in the comforts of their homes or the *dap-ay*, an indigenous socio-political structure that serves as a gathering place for the men in the village. In one interview with a stone waller, the respondent used pebbles to demonstrate the proper positioning of stones. Another stone waller, when interviewed in his house, used bottles of soft drinks to show the importance of placing stones as "headers".<sup>3</sup> These suggest that stone wallers sometimes make use of representations and models in talking about the practice, as well as thinking about the practice in the abstract.

Indigenous Knowledge of Stone Walling. In general, the practice of stone walling contains a whole range of knowledge from construction, to maintenance and repair. These categories of knowledge include classifying/defining (types of stones, whether small or big, round or irregularly shaped, hard or soft; kind of soil, whether hard or loose; quality of land, whether stable or sinking); explaining (why stones break; why stone walls erode); estimating (height of stone wall, area of rice paddy, number of stones to be used); decision-

<sup>&</sup>lt;sup>3</sup> This refers to a style of stone placement where thin flat stones are set up on their sides so that the flat area is placed more or less vertically.

making (positioning of stones; what stones to use for a particular space; inclination of stone wall; kind of backfill - pure soil/combination of soil and pebbles); and trouble-shooting (solving specific situation or problems; on the spot adjustments/decision-making - e.g., presence of water on the base of the wall). It also includes knowledge about other important things in the culture (beliefs and customary laws about stone walling; stories and rituals).

#### Conclusion

This ethnomathematical study is an attempt to meet the challenge posed by (Barton, 1999) of going beyond "surface mathematics" in understanding the connections between mathematics and culture. This research on stone walling practice may be divided into three aspects: theoretical, disciplinal and pedagogical. Each aspect has important implications in mathematics and mathematics education.

The theoretical aspect involves investigating how people develop models and theorise about the practice. In this case, stone wallers are able to make predictions about the strength and durability of a wall based on the way it is constructed (height, shape, inclination, use of stones, backfill). The language and the metaphors that they use (e.g., pregnant woman, stones biting into each other) may be linked to some mathematical concepts (such as concave batter and friction). The way practitioners make representations and predictions about their practice may provide mathematicians and mathematics educators new insights about how mathematical knowledge is generated or transformed across different cultures.

The disciplinal level involves investigating how the QRS system embedded in the practice parallel or resemble what is being done in specific fields of academic mathematics. In stone walling, the desire to maximise area and contact points between stones, minimise possibility of erosion while considering certain constraints such as contour of the mountain, slope, and availability of water may find resonance in linear programming (optimising an objective function given constraints). Such investigations may inform existing fields of mathematics with new mathematical problems and methods.

The pedagogical aspect involves understanding what knowledge and skills are important or valued inside the culture, and how such knowledge and skills are taught and learned, and communicated. This aspect should also include an understanding of how such knowledge stands in relation to dominant forms of (mathematical) knowledge, and how this form of knowledge "vie(s) for legitimacy" (McConaghy, 2000). In stone walling, there is a whole knowledge system that one needs to learn, where some may be mathematical in nature (the subject of the present study). Oral tradition and hands-on are primarily the ways this practice has been taught and learned. Stories, myths, customary laws and rituals have also served as media in the teaching of the importance of the practice, thus ensuring its continuity in the years to come. The important question of how individual knowledge becomes a shared knowledge seems to be answered by the use of rituals, stories, beliefs and other customary laws as these are collective, accessible, and effective ways of affirming traditions and knowledge within the community.

These unfamiliar (mathematical) ideas and the non-conventional ways of sharing and communicating them with other members of the community may encourage mathematicians and mathematics educators to reflect on their practice. At the same time, how such unfamiliar ideas and unconventional ways of learning need to be viewed in relation to academic mathematics, find out where they converge and diverge, and understand how they relate to each other.

The complexity of the knowledge involved in stone walling practice suggests an interrelationship of concepts that could only make sense within a system. It remains a challenge to make this mathematical system explicit and understandable within the context of the people's culture.

The four indicators discussed above may be used as a starting point in examining the existence of mathematical systems. While an initial investigation using these four indicators seems to suggest that the stone walling practice exhibits a mathematical system in a non-conventional sense, the characteristics of this system still needs further elaboration.

This paper has attempted to define the challenges posed by ethnomathematics as a field of study, and suggested ways of meeting such challenges. The QRS conceptual system approach and the use of the four system indicators for the existence of a mathematical system may prove a useful framework in any ethnomathematical study about a cultural practice. The present study on stone walling practice, which is but one component of a wider research on rice terracing agriculture that includes water distribution systems, shows exciting possibilities in the investigation of a cultural practice as a possible source of new mathematical concepts, problems and methods. Such studies may spring new surprises, eventually infusing mathematics and mathematics education with fresh vision and vigour.

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