The Potential of Open-Ended Mathematics Tasks for Overcoming Barriers to Learning

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Now that open-ended mathematics tasks have been used in many countries for some years, there is some debate about the implications of the use of such approaches for the learning of all students. This paper makes a contribution to that debate. While acknowledging both the advantages of open-ended approaches, and the potential of these to exacerbate the disadvantage of some students, it is argued that if open-ended approaches are accompanied by appropriate pedagogical supports, then disadvantaged students can also learn productively from their exploration of such tasks.

The Potential of Open-Ended Mathematical Tasks and Some Concerns

There is some debate about ways that open-ended tasks can be used effectively in mathematics classes. It seems that the issues encapsulated by this debate are at the heart of the contention described as *Math Wars* in the United States. The type of open-ended question that is the focus our discussion can be illustrated by an example. As presented in Sullivan (1999), a typical conventional closed mathematics question might be:

> A rectangle is 10 m long and 5 m wide. What is its perimeter and area?

A comparable open-ended task is:

> If the perimeter of a rectangle is 30 m, what might be its area?

The open-ended task differs from the closed task in some significant ways including that the open-ended task:

- more actively engages learners in thinking about the situation, and therefore enhances the potential for construction on new knowledge;
- is more accessible than the closed example, in that students can use what knowledge they have about perimeter to explore aspects of area, whereas the closed question requires the recall of specific area and perimeter formulae; and
- offers opportunities for extension of mathematical thinking, since students can explore a range of options as well as consider forms of generalised response.

Open-ended tasks are becoming more widely used in Australian schools. Clarke et al. (2002), for example, reported that one of the main outcomes of a large scale project focusing on professional development for numeracy teachers in the early years was that teachers markedly increased their use of open-ended tasks in their teaching.

Notwithstanding this increased use and the theoretical arguments in support, it is important to recognise that concerns have been expressed about both the nature of the mathematics being projected by such tasks, and the associated pedagogy. For instance, the above task was criticised by Wu (1994) and his views were liberally quoted on the *Mathematically Correct* website (see, for example, Becker & Jacobs, 1998). Wu's basic argument is that junior students could not find a complete mathematical solution so the task should not be used. This position was critiqued by Sullivan (1999), and it is accepted that tasks are only useful to the extent that the students can engage in appropriate

mathematical explorations, but readers are invited to explore the issue for themselves by posing the above closed and open tasks to a group of upper primary students and examining the quality of their explorations of perimeter and area.

There are, however, more serious concerns that relate to the accessibility of such questions for learners generally, and especially those who are less familiar with the purpose and processes of schooling. This is particularly relevant in Australia since it seems we are less successful than similar countries in addressing the needs of disadvantaged students. For example, a report of the *Programme for International Student Assessment 2000* (Lokan, Greenwood, & Cresswell, 2001) noted that while numeracy levels in Australia were comparable with similar OECD countries, socio-economic status was significantly related to achievement in literacy, numeracy, and science, and that "Australia has a long way to go compared with some other countries in compensating for socio economic disadvantage" (p. xv).

The idea that teaching strategies that are more focussed on the children's knowledge and strategies can privilege particular learners is widely recognised for the teaching of literacy (Delpit, 1988, 1995). This is also true for mathematics. It has been suggested that some pupils may be disadvantaged by problem solving, investigative, and open-ended approaches because the style of interaction may require appreciation of, and prior experience with, features such as the goals of schooling (Connell, Ashenden, Kessler, & Dowsett, 1982; Cooper & Dunne, 1998), the desired ways of thinking and interacting (Scarcella, 1992), the kinds of reasoning valued (Mercer, 1995), and semantic structures used (Bernstein, 1996; Brice-Heath, 1991). In other words, there may be some aspects of classroom processes associated with open-ended approaches that are intended to improve learning opportunities but, in practice, have the opposite effect. To give just one example, it is common for teachers to invite pupils to work with a partner on a task, yet seldom do teachers discuss with pupils the purpose of that collaboration, whether, for example, the purpose is for them to come to a consensus about a problem solving strategy, or to share out the work, or merely to pursue their own investigation discussing what they are doing from time to time. Pupils who are skilled at communication or who understand the purpose of schooling can use such strategies to their advantage. Other pupils may not appreciate the point of the strategy and so miss some learning opportunities.

Two important studies have raised specific concerns. Cooper and Dunne (1998) found that contextualising mathematics tasks created particular difficulties for low socio-economic status (SES) students, so much so that they performed significantly poorer than their middle-class peers whereas performance on decontextualised tasks was equivalent. Likewise, Lubienski (2000), monitoring the implementation of a curriculum program and materials based on open-ended contextualised problems, reported that target pupils who preferred the contextualised trial materials and found them easier all had high SES backgrounds, while most pupils who preferred closed, context free tasks were low SES. Many of the low SES pupils claimed to be worse off with contextualised problems, even though many were capable students, and none found the new materials easier. Such studies raise the concern that new practices in school mathematics might privilege some students but create unintended barriers to success for others.

Research by others has identified a variety of sources of potential alienation of students from lower socio economic groups. For instance, Anyon (1981) focused on the nature of the mathematics learning tasks posed; Mellin-Olsen (1981) proposed that

features of the social context influence learning goals and strategies adopted by pupils; Lerman (1998) attended to socio-economic background-related differences between classroom expectations and students' aspirations; and Zevenbergen and Lerman (2001) argued that the ability to decode unfamiliar problems corresponds closely with students' socio-economic backgrounds.

The key issue is whether it is possible to overcome such concerns. One important relevant study (Boaler, 2002) compared the teaching and outcomes in two schools. The focus of her study was on investigating the relationship between social class and achievement. The schools were chosen to represent similar socio economic mixes of students. In one school, the teachers based their teaching on open-ended strategies and in the other traditional text-based approaches were used. After working on an "open, project based mathematics curriculum" (p. 246) in mixed ability groups, the relationship between social class and achievement was much weaker after three years, whereas the correlation between social class and achievement was still high in the traditional school. Further, the students in the school adopting open-ended approaches "attained significantly higher grades on a range of assessments, including the national examination" (p. 246). Boaler argued that her project demonstrated the "particular teaching practices that need to be considered in mathematics classrooms and the effectiveness of teachers who are committed to equity and the goals of open-ended work" (p. 254). In other words, the open-ended approach to teaching mathematics proved effective in improving mathematics learning and overcoming disadvantage, but it took commitment from the teachers as well as the adoption of particular strategies. Our project is investigating further what might be these particular strategies.

Researching Barriers to Mathematics Learning

This project first identified and described aspects of implicit pedagogy (see Sullivan, Zevenbergen, & Mousley, 2002). Participants were asked to identify elements of pedagogy that may be problematic for implementation in their own particular contexts (e.g., rhetorical questions, double negatives, use of unfamiliar contexts, etc.). An outcome was the production of a manual that lists a range of strategies that teachers could use to make implicit pedagogies more explicit (Sullivan, Mousley, & Zevenbergen, 2002) and so address aspects of possible disadvantage of particular groups. The project then enlisted teachers from senior primary grades in four schools serving populations with significant proportion of students from lower socio-economic backgrounds. The aim was to explore the effect of adopting particular pedagogies, and especially making some aspects of the pedagogy explicit. The teachers in the project were involved in extended professional development seminars addressing issues such as open-ended questions in mathematics and the pedagogies that can be used to implement them. The teachers incorporated open-ended approaches into their planning and teaching, and support was offered for them in doing this. They were also asked to address the needs of individuals in their class.

A range of data were collected including 57 classroom observations followed by interactive discussions with the teachers, observations of target students, teacher surveys and completion of a planning instrument, and the eleven teachers involved were interviewed at the start and the end of the year. The data from two of the teachers and their interpretations as presented in Turner Harrison (in press) confirm that open-ended approaches are feasible and can address the needs of disadvantaged learners.

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