# Linking Researchers, Curricula and Teachers in Australasia—An Exploratory Case Study Based on the Teaching and Learning of Stochastics

John M. Truran

Kathleen M. Truran

University of Adelaide University of South Australia

#### **Abstract**

In both Australia and New Zealand since 1950 stochastics has become an increasingly important part of the mathematics curriculum at primary, secondary and tertiary levels. This paper looks at the ebb and flow of research over the teaching and learning stochastics in both countries, assesses its influence on pedagogic practices and argues that there has been both a mismatch and a failure of communication between the two.

## Introduction

It is impossible to view Australasian curriculum developments in isolation from those elsewhere. This brief paper examines the teaching of stochastics in the region against a background of worldwide trends. It is written for the MERGA Probability Special Interest Group to form a basis for a discussion which will refine and complete the factual information and the underlying theory.

As industry developed after the end of the 1939 - 1945 World War the need for a wider understanding of stochastics became increasingly apparent, and groups of industrialists and academics argued strongly for its incorporation into school curricula (OEEC, 1961). By then Piaget & Inhelder (1951) had published in French their seminal work in children's understanding of probability, not translated into English until much later (Piaget & Inhelder, 1975). Little basic work was done on statistics learning, but other important works on probability to appear at about this time were by Freudenthal (1974), who looked at mathematical aspects of teaching the topic, by Tversky & Kahneman (1974),

who proposed the heuristics of availability and representatives to explain much fallacious stochastic reasoning and by Fischbein (1975) who emphasised the development of intuition. None of these works has had much influence in Australasian schools.

By 1975 stochastics had, under the aegis of the 'new maths', already found a place in school curricula. The new courses of the late 1960s were influenced by decisions to decimalise currencies, by more comprehensive secondary school populations, by increased school retention rates, by changed philosophies of sound pedagogic practices, and by changed perceptions of what was involved in being mathematically literate. Probability formed part of these courses because it was a good example of 'a small axiomatic system that has remarkably extensive consequences' (CEEB 1959, p. 32), and was seen as a useful topic, particularly as application of set theory (SMP, 1965, p. 82). Independently, statistics came into the syllabus because of its obvious applications, and because it was found to be well received by the increasing numbers of less able secondary school students (UNESCO, 1969).

Douglas (1970) summarised the stochastics taught in Australian schools after these changes. While each system differed, probability formed part of most academic secondary courses and was included in the formal Year 12 examinations. Sometimes statistics was paired with probability, but more often it was a separate topic approached descriptively rather than inferentially. Statistics often found a small place in primary syllabuses but probability was rarely taught there until changes influenced by the National Curriculum

documents of the early 1990s (AEC, 1991, 1994; NZ Ministry of Education, 1992). In universities formal stochastics was widely taught to specialists from at least 1945. With the increase in university populations in the 1980s came an increased demand for service subjects like economics and business management which has led to many more courses in applicable statistics.

One outlier may serve to remind us that writings about stochastical thinking were being published long before these changes, but they did not impinge on educational decision making. Goodnow (1958) provided an extensive summary of people's reactions to chance processes from a psychological perspective. Her work presaged many later discussions in educational circles; had it been better known many a wheel need not have been re-invented.

It is against this overall background that we discuss Australasian research into the teaching of stochastics and its effect on schools.

#### The 1970s

When stochastics was introduced into secondary curricula, no underlying research base for an appropriate pedagogy existed in the English-speaking world. An international network concerned with secondary school teaching, and with which the Australians Hume and Douglas were involved, was pioneered by Råde (1970), network did not prosper. In any case the early Australian researchers were concerned, not with secondary schools, where there was an immediate practical need, but with primary school children. They examined whether young children could learn probabilistic concepts and all found that this was possible given appropriate teaching.

Jones (1974) investigated young children's probabilistic understanding in the U.S.A. and brought his findings back to Australia (Jones, 1977). He influenced Swinson's (1977) work on probability concepts among primary school children

and on curriculum development in junior secondary schools. The part on curriculum development was more disseminated (Swinson, 1978). Kempster (1982) investigated whether probability could be taught to upper primary children, and designed a test of probabilistic understanding in order to assess the effect of teaching the topic. These theses probably had little influence of pedagogic practice or on the decisions of curriculum developers. Rather the other way. Kempster made extensive use of the practical work being done in schools in Western Australia at that time.

(Douglas, 1970, p. 306) argued that stochastics was poorly taught at that time, but secondary school pedagogy was not an issue which attracted researchers.

#### The 1980s

Between 1979 and 1982 a stable worldwide network of researchers into stochastics learning was established. Teaching Statistics, first published in 1979, was the first international journal specifically concerned with schools. The National Council of Teachers of Mathematics (NCTM) in the U.S.A. published in 1981 its yearbook Teaching Statistics and Probability. Soon after, in 1982, the first International Conference on the Teaching of Statistics (ICOTS) was held at Sheffield, U.K. In the same year Green published results from his extensive survey of secondary school children which became a benchmark for research into probability. Many of the European workers in stochastics came to the International Conference on Mathematics Education Conference in Adelaide in 1984 and took part in a special interest group so it was possible for Australasians to link into this world wide network.

Few did so. Kissane contributed examples of ways of teaching inferential statistics to the NCTM yearbook and observed that 'statistics seems to have become the science of drawing pictures and doing sums!' (Kissane, 1981, p. 182). J. Truran (1985) published results of his

clinical interviews examining children's understanding of symmetry. No Australasian theses were presented during this period. A small amount of material on teaching stochastics appeared in teachers' journals (e.g., Watson (1980), Mills (1983), Tobin (1984) and Newell & MacFarlane (1985), but these were articles reporting practical ways of enriching teaching and were not based on systematic research. Watson coordinated a section in Australian Mathematics Teacher on 'Research for but stochastics was not Teaching' discussed.

Few Australasians attended ICOTS 2 in Canada in 1986 (Davidson & Swift, 1987); those who presented papers were concerned with teaching in universities and with statistics in industry. They showed almost no awareness of the growing research understanding of how students learn probabilistic ideas and had no firm links with the mathematics educators centred around the Mathematics Education Research Group of Australasia (MERGA)

#### The 1990s

All this changed rapidly when ICOTS 3 was held in Dunedin in 1990 (Vere-Jones, 1991). There were many Australasian contributions to the sections on Practical work and Competitions, Statistics in Her Education, Teaching and Content of University Courses in Probability and Statistics, Computers and Computing in Statistics Courses, Teaching Statistics in Non-Mathematics Courses, Teaching Statistics for Technical and Engineering Students and also for Business and Econometric Students. On the other hand there were no Australasian contributions to the joint sections on Psychology and Classroom Research, nor to the section on Assessment.

At about the same time stochastics began to be regularly discussed at MERGA conferences. J. Truran presented papers in 1990 and 1991. In 1992 papers were presented by Peard, J. Truran, K. Truran and Watson. Watson's (1992a) paper

summarised overseas research, commented on the paucity of Australian research, and proposed a plan of research into stochastic understanding based on a theoretical model which would help to improve children's understanding. She had acquired a major research grant to design an instrument to assess children's and teachers' understanding of stochastics Watson & Collis (1993).

By this time the National Curriculum documents mentioned above had ensured the place of stochastics in the primary curriculum for the immediate future. But this curriculum change was also uninfluenced by research.

More dissertational research was being . done, but it was not strongly linked to curriculum changes, though it was to some extent inspired by them. J. Truran (1992) analysed the thinking of both primary and secondary children within a Piagetian framework. Peard worked on an investigation of the influence of a home culture supportive of track gambling on upper secondary school children's understanding of probabilistic ideas. This work has recently been submitted for assessment. Toohey (1994) carried out an investigation into adolescents' perception of the concept of randomness, by firstly coming to terms with the varied use of the term 'random' and then by evaluating what his tests indicated about their understanding of the concept. Carr (1994) examined middle school students' understanding of the ideas of average and dispersion.

Some material began to be more widely disseminated. Some of this was more examples of practical classroom activities (e.g. McNaughton, 1991; Peard 1990); some concerned with the theory of probability (e.g. J. Truran, 1994b). Watson (1992a) continued her work into learning how to examine public statistics with an article which won an award from the Institute of Statisticians for the best article in *Teaching Statistics* in that year.

In Australia material was produced to help teachers introduce stochastics into the curriculum (Lovitt & Lowe, 1993a, 1993b; Finlay & Lowe, 1994). This material was developed from an applied perspective and was based on extensive trials of what worked well in the classroom. Official supplementary material was produced by Watson (1994) which was designed to develop teachers' understandings and which forms the basis for some in-service courses eligible for tertiary credit. While this collection of exemplary activities had a strong basis in some stochastics research, it made no claim to be comprehensive, and provided no links for the reader with the standard research literature. None of the advisory committee had a significant background of research into stochastics learning.

In 1994 ICOTS 4 was held in Marrakech and was attended by a large number of Australasians who on this occasion made a substantial contribution to the section on Research on Teaching and Learning Statistics and Probabilistic Concepts, as well as to the sections on Statistics for Social Scientists and the Training of Teachers. No opportunity was provided for a continuation of the discussions on Statistics in the Education of Women. The Proceedings were not comprehensive, but some papers were informally collated for circulation. These included one by Lipson (1994) who observed, in terms similar to Kissane's, that 'a key issue in statistics education is the calculation vs concept argument'. Plus ça change, plus c'est la même chose. Pfannkuch & Brown (1994) questioned whether a probabilistic approach to student learning of statistics could be a hindrance. Forbes (1994) questioned the traditional assessment of student's learning in statistics courses and suggested that statistics was so different from other courses that radically different and more equitable methods were required.

The effect of the 1990 ICOTS Conference on current research projects

has been clear. Many are based on the North Island of New Zealand. Some of these are Computers and Statistics (Naidu), Cooperative Learning and Statistics (Ingram), Teachers Ideas about Statistics (Edwards), Fijian Ideas about and a major Statistics (Sharma), investigation of Statistics in the New Zealand Mathematics Curriculum (Taylor, Bidulph and Carr). In Australia, Watson's test is nearing completion, Williams and Lipson are investigating hypothesis testing, Gordon the teaching of tertiary statistics. K. Truran and Way are investigating primary children's understanding, J. Truran is designing an test of probabilistic understanding, examining curriculum forces and reexamining the original work of Piaget & Inhelder. In the U.S.A. Jones & Thornton are furthering Jones' early work on young children's thinking.

There is insufficient space to outline recent contributions to research conferences but a MERGA Special Interest Group was formed in 1993 under the leadership of K. Truran and has communicated regularly by means of a simple newsletter which has a very high degree of member participation.

This paper started with one outlier and will finish with two. English (1993) has investigated children's solutions of combinatoric problems. Traditionally combinatorics has formed part of standard stochastics course, but her work has not yet been absorbed into the mainstream reported here. J. Truran's (1994a) studies of the history of stochastics teaching in Australia have received little attention, but have the potential to inform current decision-making practices.

### Conclusion

Several features are apparent from this survey. Most importantly, there has been an almost total mismatch between research and curriculum development in stochastics. Such research as has taken place has been, with the exception of Watson's work, idiosyncratic and not

directly involved with curriculum structures. This is not entirely the fault of researchers: one worker, usually studying on a part-time basis, must of necessity concentrate on a relatively small topic. Rather, it might be seen as a reflection of the low esteem in which research is held by those in the corridors of administrative power.

Dissemination of research has proved difficult. Theses are rarely read by nonlocals. Proceedings both of MERGA and ICOTS conferences have been poorly distributed among tertiary institutions and indexing bodies, and almost not at all among teachers or teachers' journals. It is significant that Toohey, the only classroom teacher who has done major research, has not been actively involved with MERGA, mainly on logistical grounds. Finally, there has been no codification of research findings constructed in a form which is accessible to teachers. Perhaps this might become a future project for MERGA.

The ICOTs conferences in particular draw together a very wide group of people concerned with statistics. Industrialists and bureaucrats mix, at least in the enrolment lists, with psychologists. tertiary teachers, mathematics educators and educational administrators, but not much with those who teach children. How to bring these disparate groups together in a way that can ensure real intellectual intercourse remains one of the our unsolved problems exacerbated by the tensions within schools about whether a pure or an applied approach is preferable.

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