Probability Education: Can primary children cope?

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Introduction

Research in the area of probability and statistics has taken two main approaches (Shaugnessey, 1992). The first has investigated learners' ideas (intuitions, beliefs, conceptions, fallacies) while the second has involved efforts to influence such ideas so that they reflect those of mathematicians.

With respect to learners' ideas, some interesting findings have emerged from the research. For example, Piaget and Inhelder (1975) claimed that up until age 10 years children were unable to consider all aspects of a given situation and were thus unable to understand probability but by age 12 years many children, without formal schooling in probability, understood the basic probability concepts. In contrast, Green (1979, 1982, 1983) found that most 11 to 16-year-old students in a 3000 sample British study lacked the verbal ability to talk about probability concepts, and that there was litle improvement of understanding with increasing age. Carpenter, Corbitt, Kepner, Lindquist, and Reys (1981) also found that students are unable to describe their probabilistic intuitions mathematically. Their research, however, suggested that the basic intuitions do develop with age. The work of Kapadia (1986) and Fischbein, Nello and Marina (1991) revealed that children are familiar with everyday probabilistic terms such as *likely, certain* and *impossible* but that their intuitions about these terms are not very precise. For instance, *certainty* was equated with a high likelihood, whilst *impossibility* was thought of in terms of personal experiences rather than logical events.

A number of research studies have revealed various misconceptions and fallacies about probability and statistics held by learners. For example, Kahneman, Slovic and Tversky (1972, 1974) observed that statistically naive learners are likely to use a *representativeness* strategy in which they ignore sample size and base conclusions on just a few outcomes or even a single outcome. Another strategy used is termed the *gambler's fallacy* ; this is based on the belief that, say, a run of four heads is most likely to be followed by a tail (rather than the logical stance that a head or tail is equally likely) since things of this nature even-up over a longer run. A further strategy is known as the *conjunction fallacy* and is used by people who rate certain types of conjunctives more likely to occur that the parent stem events. The *availability heuristic* is yet another example of a misconception in probabilistic thinking, this having to do with significance being attached to some limited data simply because the data has a bearing on something of personal significance. A misconceived strategy among younger children identified by Jones (1974) was basing choice on personal colour preference rather than actual colour area on spinners.

Attempts to change learners' intuitions through teaching have been made by several researchers. Fischbein, Pampat and Manzat (1970) reported extensive changes in intuitions and conceptions of probability over the course of instruction, with the most development occuring among 9 to 10-year-olds. Konold (1989, 1991) and Garfield and delMas (1989) have also developed various activities to try to shift misconceptions but their studies indicate that students' prior life experiences, including the use of many probabilistic terms in their everyday sense, results in considerable difficulty in trying to effect change.

Given the apparent problems experienced by younger primary school

children in dealing with probability in other than intuitive ways, it may be wondered why *Mathematics in the New Zealand Curriculum* (Ministry of Education, 1992) introduces this aspect of mathematics to New Zealand children from age five years. This contrasts with Australian schools where probability is not generally introduced until about age12 years. It seemed important to explore the viability with primary children of a number of the achievement objectives and recommended learning experiences listed in the New Zealand mathematics curriculum by undertaking some classroom-based research. This paper reports on some exploratory research undertaken for this purpose. The research approach is a combination of the two mentioned above; the children's probability strategies and ideas were examined in the course of teaching designed to develop them.

Method

The author and a mathematics teacher education colleague taught respectively a series of eight lessons over consecutive days in two classrooms in schools in the vicinity of Hamilton city, New Zealand. Lessons typically lasted from about 40 minutes to one hour. One school was an inner city school drawing upon a low socio-economic population and the other was a small rural school with a predominantly middle-class population. The children in the first school were aged mainly 9 to 11 years, while the children in the rural school ranged from age eight to 13 years.

Each of the two 'teachers' selected a number of objectives and learning experiences, listed by the curriculum as appropriate for the age level being taught (see Appendix 1), to test their viability with the children concerned. No special attempt was made by the 'teachers' to address any of the children's misconceptions that emerged, except in the normal course of constructivist-type teaching. The observers, however, deliberately attempted to uncover the way children thought and probe their ideas further.

Data were collected by several means. All lessons were videotaped by an assistant for later analysis; two colleagues acted as observers and concurrent interviewers during each lesson in the classrooms using audio taperecorders to assist them to probe and capture children's conversations, explanations and ideas; the lesson planning and children's responses to many activities were in written form and these were also kept for later analysis.

Results and Discussion

The data indicated that many children lacked sufficient number sense to engage with the activities in a probabilistic way (Taylor and Biddulph, 1994). The data also revealed that many of the children used strategies and held beliefs (based on prior experiences) which, in the perception of both the 'teachers' and observers, would tend to inhibit the children's development of probability ideas. This paper focuses on the nature of some of those strategies and beliefs.

Children's strategies

Four intellectual strategies illustrate how the children often approached the tasks.

1 Do whatever comes to mind

Although some children adopted a reasonably systematic approach to an activity, many did not. For instance, with respect to constructing model flags from four differently coloured cardboard bars, an observer asked Stacey how he knew where to put the coloured bars. 'Just moved them around.' [How many different flags do you have?] 'Seven.' [How did you know you've finished?] 'Cause I moved them around and there's no more.' Stacey claimed that others who found more combinations must have cheated.

2 Just guess

Some of what the children said were their guesses were fairly sound intuitions but others appeared to depend on vague associations with things they had heard

about. For example, the 'teacher' came across one boy who guessed that the first baby in a family would be a boy and a girl! He added, 'That's 5%.' [How did you work that out, that 5%?] 'I just guessed it.' [What does 5% mean?] 'It means like, if you've got $100 in the bank, it means that you get $500.' [How does it relate to the chance of getting a boy or girl?] 'Hardly anything.'

3 Do not pause to reflect

Despite their efforts, the 'teachers' found it difficult to get many children to reflect on the activities they were engaged in. This is illustrated by a girl's response to an activity involving visiting animals in a zoo on a tree-type diagram. She told the teacher she had to 'get the gorilla'. [What does that mean?] 'You gotta get the gorilla.' [Which animal did you visit the first time Katrina? Katrina, which animal did you visit first?] 'Tails, that one. Yes, it's gone another way. I'm visiting the gorilla.' Katrina was not to be deflected by the 'teacher' trying to get her consider which animals might receive more visits, and why that might be.

4 Regard tasks as separate activities

Again, despite the teachers' efforts to help the children see links between related activities, many children seemed unwilling to contemplate such links. The zoo task mentioned above was really the same as another involving pathways down a mountainside but no children saw any connection between them.

Children's experiences and beliefs

It appeared that the thinking of many children was dominated by past experiences and associated beliefs, and that this prevented them developing probability ideas, despite teacher challenges to do so. The following instances illustrate the power of children's experiences and beliefs in limiting their learning.

1 The influence of experiences

Thinking of examples of elements on a verbal scale of chance, Danielle claimed that, 'It is impossible for me to win lotto.' She knew that other people sometimes won money but was adamant that she had no chance at all, 'Because our family has never won anything.' In this case, Danielle's personal experience precluded her from accepting the logic that it might happen for her family too.

Based on experience, Sheryl wrote, 'It is certain I will go home from school today.' When asked by her teacher if she was sure that nothing could prevent this from happening, Sheryl replied that she always went straight home from school; no amount of reasoning could persuade her otherwise. Further, her certainty statement was accepted as a good one by all other children in the class bar two.

Other children clung just as tenaciously to their ideas and statements, especially when they were based on routine or special happenings in their lives. One girl for instance, in relation to the chance of a first baby being a boy or girl, insisted that the first baby could be twins. As she explained, 'My aunty has just had twins.' When one of the classes was asked to consider the chances of getting the salt shaker (from the pair of salt and pepper shakers) out of a dark cupboard at night, they suggested you could find it all right - by smelling it, or feeling for the number of holes or the label! On another occasion children were investigating the chances of a gorilla getting a banana from a selection of bananas and oranges. Many were convinced that the gorilla was sure to get mainly bananas because in their experience gorillas only like bananas; it was unlikely therefore that the gorilla would pick up an orange!

Sylvester was having trouble getting started during a group game of 'beetle'. He was sure that six was harder to throw than any other number. [Why is six so hard?] 'Because every time I throw a dice it doesn't land on six. It lands on other numbers like one, two, three or four.' Richard had a similar view. When asked by an observer how many times he was likely to get a six if the die

was tossed six times, he replied, 'Not much chance, 'cos I never get them when I want them.' This finding matches that of Graham (1992) who also reported that children thought a six was harder to throw than any other die number. Other reasons given by the children for this difficulty in throwing a six are mentioned in the next section. Children in this study went further and claimed that other numbers could be just as difficult. Like many other children, Richard was clear that, 'The numbers I want never come up.'

In one particular activity, each number was assigned a colour and the children asked to predict how many times each colour would result if the die were tossed 20 times. This posed a dilemma for Alys because six had been assigned blue, her favourite colour. She was convinced that six was almost impossible to throw but in the end estimated that blue would come up 10 times.To try to ensure that it did during her investigation she tried all sorts of contortionist methods of throwing the die.

The impression gained by the teachers and observers when they tried to challenge the children's experience-based views was that they were seen as outsiders who did not understand what went on in the children's lives. From an adult point of view the children seemed to adopt a fatalistic attitude toward the tasks and outcomes because they could not discount their powerful prior experiences.

2 The influence of beliefs

To return to the perceived difficulty of throwing a six, some children believed that the difficulty had to do with the way the die was thrown, or luck. Chris, for instance, found it difficult to get a six, he said, 'Because I was throwing it too big.' [What do you mean?] 'I don't mean that. I was throwing it wide; that's why I didn't get it.' Chris added that a one and a two would be easier to throw, but that 'Three's better. That's easiest because it's got an open space. Victoria's beliefs related to both method of throwing and luck. Her reason for not being able to get a six was that, 'I was throwing it wide, so I tried the other one (die).' [Is six always a hard number for you to throw?] 'Yep, because it's (an) unlucky number.' Other children's beliefs about throwing a six centred on the way one held one's mouth, and the amount of 'willing' it to happen that one could muster. Not one child related it to any probability idea and, when challenged about it, there was no thought that each toss was a separate occasion and there was a one-sixth chance of throwing a six, or any other number.

Tossing coins and using spinners were other contexts where children's beliefs influenced their activities and ideas. Many clearly held a representativeness view because they saw no need to make predictions for longer runs or to carry out longer runs. They were quite contented to come to conclusions on the basis of a few instances. Sometimes this stemmed from a belief that a coin generally landed a certain way up. This was not due to luck but depended on which side they believed had more weight and therefore dragged the coin to that side down. For example, Robert said he always went for tails as, 'Heads is heavier because it's got more stuff on it.' With respect to the New Zealand 50c coin Vanessa, it transpired, took the opposite view choosing heads. 'It always lands on heads (up) because it always lands on the other side, because it's got more drawing.' [Why is that again?] 'Because it has more weight on the other side.' [Tails has more weight on it, or heads?] 'Tails has got more weight on it because it has a ship. Yeah, it has.'

A similar belief was held by some children with respect to dice. Five and six were thought to be heavier, and the die would therefore land on them more often. There was a much greater likelihood of throwing one, two, three or four according to these children.

Some children believed that recording data is too much trouble to be worth it. This was clearly expressed by Serena to

her teacher. [The idea is to find out how many different flags you can make. You've only got one of each colour. You might want to write it down.] 'Nah, takes too much time.' Such a belief, together with the observation by both teachers and observers that many children could not record even brief data accurately, pose a real challenge for teachers hoping to engage the children seriously in some of the curriculum learning experiences intended to develop probability ideas.

A final example of children's beliefs influencing their ability to consider activities from a probability point of view involved a game in which the two players (children) always lost to the fairground operator. When asked if it was a fair game, many thought that it was. As Lisa said, it was fair 'Because the two people here always lost.' In other words the children believed that there was nothing unfair about the adult getting more (adults usually do, in their experience) and provided the two players were being treated equally, then the game was fair. In this example, the children's beliefs about fairness did not allow them to construct the 'equal chances' probability notion intended by the activity and curriculum.

Conclusion

Although exploratory in nature, the results of this study confirm a number of findings of other researchers. Most of the well documented fallacies and misconceptions were evident when the data was analysed, and there was no real evidence to suggest the understanding improved with age. It appears that understanding was idiosyncratic and not generalised between activities and investigations. The stronger the personal beliefs and experiences a child had, the more they were held on to and not open to challenge at this stage.

In some respects, the findings also go beyond those in the literature. The children'is ideas about fairness and hence the chances of winning were very different to what was expected. It is difficult to challenge mathematically when chidrens' idea of fairness is constructed from a different, yet logical perspective. In this study too, the need for accurate recording of data was dismissed by many children as being irrelevant. It could be argued that children are using the *availability heuristic,* but this finding has implications beyond that of developing understanding about probabillity to recording in mathematics generally.

The results raise questions about what might be realistically achieved in terms of probability education at the junior and middle primary school levels. This seems to be an issue that warrants further investigation.

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Appendix 1

Achievement objectives for probabability in The New Zealand National Curriculum are as follows:

Level 1	classify events from their experiences as certain, possible, or impossible
Level 2	compare familiar or imaginary, but related, events and order them on a scale from least likely to most likely
Level 3	use a systematic approach to count a set of possible outcomes;
	predict the liklihood of outcomes on the basis of a set of observations