The place of algebra in the secondary mathematics curriculum - Some historical reflections.

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Abstract:

Much attention in recent years has been directed towards an intensive study of the way that students learn elementary algebraic concepts and the best way that these concepts should be introduced into the secondary mathematics classroom. This paper will examine how and why algebra was introduced into the secondary school curriculum, especially in England in the Nineteenth Century. The period immediately preceeding the First World War will be particularly addressed since many reforms were advocated at this time in the teaching of algebra, but never taken up. The effect of these changes on the developing secondary algebra curriculum in Australia will also be examined.

In recent years, a whole lot of international research has occurred into the ways that elementary algebraic concepts should be introduced to early secondary age students to ensure better understanding eg. (Hart, 1981; Booth, 1984; Kieran and Wagner, 1989; Booth, 1995). The justified common compaint of these researchers is that students fail to understand many of these underlying algebraic structures. In order to cover up their lack of understanding, students resort to memorising rules and procedures and they eventually come to believe that this activity becomes the essence of algebra. Many teachers of secondary classes have now translated these research findings into their introduction of algebra and approach their task more slowly and deliberately and endeavour to encourage much more understanding of the algebraic processes. The use of concrete materials in the typical elementary algebra lesson now occurs in a number of schools.

However it is fascinating to examine historically the incorporation of algebra into the secondary school mathematics curriculum and to enquire what problems mathematics educators of the past experienced with its teaching and learning. Since the advent in the 1960's of the "new mathematics", in Western culture particularly, and the great upsurge in the amount of research into mathematics education that has occurred across the world since then, we are often inclined to dismiss former developments in the learning and teaching of this subject at the school level. This paper will attempt to grapple with some reforms which occurred internationally in the teaching of algebra, concentrating especially in the period immediately prior to the outbreak of the First World War.

The fact that the mathematics curriculum which existed in Australian schools till the 1960's was slavishly adopted from that prevailing in England has been well documented previously (Clements,1989; Clements, Grimison and Ellerton, 1989). In the Nineteeth century, and during the early part of the Twentieth, secondary education in England was principally the province of boys of prosperous parents selected on their high level of ability (Grimison, 1990). These boys attended Public Schools (called private schools in Australia), and until about 1840 studied mainly Greek, Latin and Euclid (usually the Greek or Latin translation). These Public Schools were independent of state control. The justification for the narrowness of the secondary curriculum was these subjects "trained the mind" or

afforded a mental discipline, and by such a process the mind of the child would be trained to perform the difficult tasks faced in later life. However, by about 1860, the demand from society for more "numerate" undergraduates for Oxford and Cambridge, as well as the requirement that more mathematics be part of the preparation of boys hoping to enter a military or naval college, led to nearly all English secondary schools incorporating more mathematics into their curriculum. (Price, 1983) The typical offerings then included predominantly Euclidean geometry, algebra, and some arithmetic, and a little mechanics, conics and elementary trigonometry. This widened mathematical curriculum was simply obtained by moving down the prevailing one at the time which existed in the first two years of university undergraduate mathematics. This allowed a corresponding expansion in the mathematical content of university courses (Howson, 1982).

This algebra curriculum fitted exactly into the "transfer of training" or "mental faculty" idea. It was characterised by being very abstract, demanding, devoid of immediate application and irrelevant for daily living. However in the mental effort required to manipulate algebraic expressions and solve equations, it was believed that mind training would occur. For the next fifty years, numerous algebra texts for secondary schools were published in England and these same texts were adopted in the various British colonies. Typically these texts were little more than a set of graded exercises in algebraic manipulation with a number of worked examples included. A number of "tricks" were incorporated in the anticipation that students would memorise the procedures and hence be able to reproduce the method in answering an examination question. These exercises were similar from reprint to reprint of the texts, and changed little up to about 1910. Most texts included questions from previously set examinations for Oxford and Cambridge School Examinations as well as Admission exams to Woolwich or Sandhurst. Three examples, two from the Junior Local Exam in 1885 (Year 8) and one from the Senior Local Exam in 1885 (Year 10) included in Hall and Knight's 1899 Algebraic Exercises will give an indication of what was required:

Junior Local Exam:

Prove the identities:

(a) $(x + y + z)^3 = (x + y - z)^3 + (-x + y + z)^3 + 24xyz$

(b) $\dfrac{1}{(1 - \frac{b}{a})(1 - \frac{c}{a})} + \dfrac{1}{(1 - \frac{a}{b})(1 - \frac{c}{b})} + \dfrac{1}{(1 - \frac{a}{c})(1 - \frac{b}{c})} = 1$

Senior Local Exam:

Show that if: $\F(x,a + 2b + c) = \dfrac{y}{2a + b - c} = \dfrac{z}{4a - 4b + c}$;

then: $\F(a,x + 2y + z) = \F(b,2x + y - z) = \dfrac{c}{4x - 4y + z}$

Another quaint text from the same period is that published in 1908 by Alfred Ikin, entitled, *Knotty Points in Algebra*. This book is intended for secondary mathematics students in algebra to be used as a supplement to the ordinary text :

> *To emphasise the points where students are likely to make mistakes, and to assist them to see the reasons for certain operations. The chief object is to train the reasoning powers of students. (Ikin, p. 1)*

Detailed working is shown of examples like:

Show that the product of $9 + \F(16,x - 1) + \dfrac{2}{x + 2} - \dfrac{3}{(x + 2)^2}$

and $1 - \frac{1}{x+3} - \frac{1}{x+1} - \frac{1}{(x+1)^2}$ is independent of x (Ikin, p. 44)

These attempts were typical but failed to teach algebra as little more than a set of the teachers' tricks which students memorised and hoped to reproduce in an examination. The algebraic content of the secondary school course tended to be theoretical and devoid of any graphical application or any other application for that matter. There was no attempt to integrate it with the teaching of other mathematical area - geometry, arithmetic, trigonometry or conics. In fact students often were allotted different masters for each of these branches of mathematics.

The Mathematical Association in England had been formed in 1897 following the earlier work of the Association for the Improvement of Geometrical Teaching. The first edition of the *Mathematical Gazette* appeared in 1894. The membership of that Association was made up predominantly with school masters from Public Schools, but also included a number of university mathematicians. By the beginning of the Twentieth Century, there was a growing concern amongst mathematicians in England about the way that mathematics was being taught. This concern was shared by many scientists and engineers who believed that the utilitarian aspect of mathematics should be emphasised more in schools. The major reforms in school mathematics took their initial burst at the meeting of the British Association for the Advancement of Science in Glasgow in 1901. Professor John Perry, Fellow of the Royal Society and Professor of Engineering at the Royal College of Sciences led the attack. Amongst other reforms, Perry advocated in school algebra, the freeing up of the extremely formal study of manipulation of expressions and an examination of functions and graphs using squared paper which could lead, if necessary, to the introduction of elementary calculus. (Price, 1983)

The British Association appointed a committee, chaired by Professor A.R.Forsyth, Sadlerian Professor of Mathematics at Cambridge, to investigate the teaching of mathematics in schools. Perry was appointed secretary. Following the 1902 Annual General meeting of the Mathematical Association, a Teaching Subcommittee was established which during the next 20 years proved so influential in shaping a mathematics curriculum for English secondary schools. It substantially reported on the teaching at the school level of arithmetic, algebra and geometry. The reports of this committee were extremely influential in setting the content and methodology employed in secondary school mathematics internationally, including each of the states of Australia. (Carslaw, 1914)

The membership included two Professors of Mathematics who had previously been members of the Association for the Improvement of Geometrical Teaching. By 1905 the Universities of Cambridge, Oxford and London had changed their matriculation examination rules in mathematics substantially and thus the newly appointed Board of Education's syllabus included the use of graphs and squared paper in school algebra. The algebra course was still very theoretical. Greater reforms were allowed in the teaching of geometry.

A number of new texts in algebra appeared after the first decade of the Twentieth Century reflecting some of the reforms which had been advocated earlier. Amongst these were Elementary Algebra which was written in 1912 by Charles Godfrey and Arthur Siddons. Both were schoolmasters. Godfrey was the young Headmaster of the Royal Naval College, Osborne, who had previously had experience in teaching mathematics at Winchester. Godfrey also co-authored a school text on

geometry, arithmetic and another on the use of four-figure logarithm tables. The latter sold over six million copies world-wide by 1980!!! (Howson, 1982).

Godfrey was prolific in his writings on mathematics education at the school level and was a man who in many ways was ahead of his time. His published numerous articles for teachers in the Mathematical Gazette and was clearly regarded as a leader in his field. Writing on school algebra in 1911, he said:

> *...Algebra is perhaps the mathematical subject which gives the smallest return for the amount of time spent on it.....the main preoccupation of teachers is to impart to their pupils a higher degree of dexterity in handling algebraic expressions.....the ultimate aim should be, not manipulation, but understanding and outlook. (Howson, 1982, p. 158)*

This comment is certainly quite ahead of its time and would be echoed by mathematics educators of today. England was not the only country in which great changes were occurring in the teaching of secondary mathematics. Changes were also occurring in U.S.A., and throughout Europe. An International Congress of Mathematicians had first met in 1896. At its fourth Congress in 1908 in Rome, a decision was taken to present detailed reports on the various syllabuses in secondary mathematics in various countries for the 1912 I.C.M. at Cambridge. Godfrey was largely responsible for the co-ordination of the British Report, which ran to two volumes. Some twenty countries were involved; Carslaw was responsible for the Australian report (Carslaw, 1914). Godfrey presented a detailed paper entitled " The Algebra Syllabus in Schools" as part of the first volume. It makes fascinating reading in 1995. So much of it still applies to many practices in the teaching and learning of school algeba.

Writing of the introduction of graph work into the school curriculum by 1905 he writes:

> *Such changes as did find their way into the algebra course were to a great extent beneficial, but did not appear to arise from any clearly defined principle. It is true that the introduction of "graphs" into algebra work was accomplished; but even in this case it is doubtful if those who followed the fashion were agreed as to the object of graph work in algebra; and in the absence of any accepted guiding principle, graph work led to fantastic and ill-balanced developments (Godfrey, 1912, p. 285).*

Or again,

> *As the teaching of English may be based upon the instinct to describe things seen, so the teaching of algebra may be based upon the instinct to propose and solve diverting problems. (Godfrey, 1912 p. 286).*
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> *Wihout proposing then to throw overboard the formal training theory - I shall assume that it will be no longer the touchstone in choosing topics for a mathematical course; that competent teachers of today will succeed in extracting as much formal discipline from a lesson on modern subject matter as did their predecessors from a lesson on subjects whose interest has now evaporated. (Godfrey, 1912, p. 295)*

J.W. Mercer described in detail the courses of instruction that were followed in mathematics at the two Royal Naval Colleges at Osborne and Dartmouth at the same international conference in 1912 (Mercer, J., 1912). Here ofcourse the curriculum developers were free of the constraints of the external examination system and were able to devise a curriculum that was modern in outlook and met the particular needs of the boys.

Algebra, in particular was pruned of its tedious formality or with :

> *....hosts of elegant devises for solving problems specially constructed for the purpose. Everyone should have a few simple rules and principles as part of his mental machinery. If these are in constant use, and are backed up by some common sense, quite difficult problems can be solved by very simple means. (Mercer, p. 193)*

The use of the function notation and appropriate graphical work was common as well as an introduction to the theory of logarithms and use of tables. Co-ordinate geometry is also treated, and no attempt is made to "take them through a course of Analytical Conics" (Mercer, p. 216)

Not everybody was impressed with the reforms in the teaching of secondary school algebra that had been advocated especially by Godfrey. In the same British Report presented at the 1912 International Conference, S. Barnard, who at the time was Assistant Mathematics Master at Rugby School, criticised Godfrey's ideas for reforming the teaching of school algebra and wrote the following:

> *Mr Godfrey refers contemptuously to those who regard mental discipline as the chief aim of mathematical education. Yet is difficult to assign any other reason for teaching mathematics to the ordinary boy.*
> *I am convinced that a dull boy could easily cover the ground in the proposed syllabus by the age of 19. Further, I know by experience that, by the the age of19 and with the usual allowance of time, the average boy can learn vastly more algebra and at the same time acquire a fair knowledge of geometry, trigonometry and mechanics (Barnard, S, 1912, p.337).*

The chief reason for teaching secondary school algebra which had been espoused in England since 1860 was under real threat in Godfrey's calls for reforms and the bastions of preserving the status quo were in a defensive and defiant mood. Many of Godfrey's ideas were running in advance of what could be reasonably expected of mathematics teachers who had only known the traditional justification for teaching algebra (Howson, 1982). Indeed, in England, and certainly in Australia, only lip service was paid to a major change in the teaching of high school algebra, and students continued to merely manipulale algebraic symbols in algebra classrooms without any real understanding of the underlying processes. Our secondary algebra syllabus had beed adopted straight from that prevailing in England. (Carslaw, 1914). An examination of secondary school algebra syllabuses in N.S.W. and Victoria in the period 1912 - 1965 together with the corresponding public examination papers confirms this view. Algebra continued to be seen by the vast majority of students as tedious, formal, irrelevant and extremely difficult to understand. Clearly this must have been accelerated by the increasing numbers of students who were able to avail themselves of some secondary school education in this period. In 1913 , in N.S.W. only 9% of pupils proceeded on to secondary school, but by 1960 the figure had risen to 98%.

However the syllabus writers did advocate that some time be devoted to graphical work and functions. Nonetheless, few teachers of algebra took notice of the suggestions for change, and continued to teach algebra emphasising a mechanical rather than a meaningful side. Speaking at a Presidential Address to the Mathematical Association on N.S.W. in 1952 , P. Anderson claimed:

> *It may be that that the revolution will not be completed for years, for experience seems to show that one of the most potent forces influences in the training of a teacher of mathematics*

is that exerted by his former teacher. There are still many schools where the significance of an algebraic symbol is skipped over and pupils are thrust into a maze of examples on such an artificial topic as "order of operations", and the key to thev way out of this maze is that mystic word BODMAS. Following this pupils are taught the mechanical processes of algebra. The only justification I have heard for such a procedure is that is what pupils can do and it gives them a sense of achievement. This would be justification enough if it did not apply equally well to jig-saw puzzles. If pupils cannot acquire a grasp of the meaningful use of algebraic symbols, then either they should stop learning algebra or we should stop teaching it. (Anderson, 1952)

Real changes did occur in the way that elementary algebra was to be introduced, but only in the last fifteen years following a great deal of international research in the area. It is nonetheless intriguing to note that in the decade immediately prior to the First World War a number of reforms were advocated which have only come to pass in recent days. Even now, there is still some opposition to the notion that elementary algebra can be taught to encourage students to understand the underlying assumptions.

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