Evaluating the Effectiveness of the Strategy *Draw a Diagram* as a Cognitive Tool for Problem Solving

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Abstract

Traditionally the effectiveness of the strategy draw a diagram as a problem solving tool has been assessed by using end product measures such as frequency and spontaneity of diagram use, performance scores, solution times, and the appropriateness of the diagram drawn. This paper argues that these measures can be unreliable and proposes that the dynamic use of the diagram should be monitored to ensure the validity of the assessment.

Background

Draw a diagram is a strategy that is often recommended in problem solving instruction (Kersch & McDonald, 1991), however the literature emanating from the current research base is divided on the effectiveness of the diagram as a problem solving tool (Simon, 1986). A diagram is defined as an abstract visual representation that exploits spatial layout in a meaningful way, enabling complex processes and structures to be represented wholistically (Winn, 1987). Thus diagrams are an external "window" to mental representation (Presmeg, 1986) allowing access to a student's knowledge through the connections that are constructed (Bereiter, 1991). Diagrams may be comprised of words and/or abstract pictures, for example matrices. The wholistic function of diagrams and their level of complexity, distinguishes diagrams from graphs and charts which represent simplistic relationships among a limited number of variables (Winn, 1987).

The Advantages of Diagram Use

In order for the strategy *draw a diagram* to be effective in problem solving, the diagram must facilitate the solution to a problem. There are five apparent advantages of drawing a diagram in problem solving. Firstly, diagrams act as an external sketch pad where interconnected pieces of information can be chunked together and thus relieve working memory (van Essen & Hamaker, 1990). Secondly, as diagrams portray the solver's connections between the components of a problem, diagrams are useful in determining a solver's understanding of the structure of a problem (Kersch & McDonald, 1991; Shigematsu & Sowder, 1994). Thirdly, diagrams enable information to be displayed in a wholistic manner, thus implicit information within a problem may become explicit to the solver on a diagram (English & Halford, 1995; Larkin & Simon, 1987). Fourthly, diagrams facilitate the reorganization of information (Larkin & Simon, 1987), hence new relationships may become apparent. Fifthly, diagrams provide a visual alternative to words (Mayer & Gallini, 1990). Each of the advantages outlined above is cognitive, therefore an assessment of the effectiveness of the use of the strategy *draw a diagram* needs to monitor cognition during problem solving.

Difficulties in Using Diagrams as a Cognitive Tool in Problem Solving

The criticism of diagram use in problem solving centres around the ineffectiveness of diagrams where students had difficulty deriving meaning from the diagram (Janvier, Girardon & Morand, 1993). Students' difficulty with diagrams

can be explained didactically and cognitively (Dreyfus & Eisenberg, 1990). In schools the presentation of academic knowledge is sequenced linearly thereby often omitting critical interrelationships and the links between specific information and a broad overview of the information. The continual predisposition towards an analytic presentation of information provides students with limited experience in using wholistic representations, such as diagrams, thereby affording visual thinking a lowly status in traditional classrooms (Lowe, 1987).

The difficulties that students may encounter in using diagrams as a problem solving tool highlight the importance of considering the diagram as a visual representation rather than a linguistic representation. There are substantive differences between visual and linguistic representations that impact on visual reasoning and distance it from linguistic reasoning (Barwise & Etchemendy, 1991).

Despite the cognitive computational advantages of diagram use (Larkin & Simon, 1987) students' reluctance to process visually (Dreyfus & Eisenberg, 1990) negates the advantages of diagram use. A compounding psychological factor for explaining students' difficulty with diagrams may be the differences between diagrammatic processing and analytic processing (Dreyfus & Eisenberg, 1990).

The viewpoints of the advocates and opponents of diagram use in problem solving, are not necessarily in opposition, but rather relate to the potential of the strategy *draw a diagram* and to students' application of this strategy. The advantages of diagram use specifically relate to reasoning with the problem data (Nunokawa, 1994a), however the effectiveness of the strategy *draw a diagram* in problem solving depends upon how the solver makes use of the strategy.

The Need for Research into the Use of the Diagram as a Cognitive Tool

The need for further research on the strategy *draw a diagram* has recently been advocated by Shigematsu and Sowder (1994): "For research, the teachers should try some action research...or a more controlled study on the effectiveness of using drawings in solving problems, with some teachers emphasizing drawings and others not" (p. 546). Although further research may provide insight into how the strategy *draw a diagram* is used in problem solving, it is also essential to establish the validity of the methods used to assess the effectiveness of using diagrams. Previous studies on the use of the strategy *draw a diagram,* can be categorized by how the effectiveness of diagram use was measured. There are two types of studies; studies which assess the diagram as an end product (e.g., Simon, 1986; van Essen & Hamaker, 1990), and studies which monitor the diagram drawing process (e.g., Nunokawa, 1994a).

Assessing the Diagram as an End Product

The assessment of the effectiveness of the strategy *draw a diagram* in problem solving has traditionally focussed on the end product, the diagram. For example, Carroll, Thomas, Miller and Friedman (1980) used performance scores and solution times, while Biron and Bednarz (1989) measured the spontaneity and frequency of diagram use. Simon (1986) assessed the appropriateness of the diagram by considering the degree of congruence between the solver's diagram and the problem's inherent structure. However each of these end product measures of diagram drawing isolate the product from the process.

In order to validate the use of congruence as an appropriate method for determining the strategy's effectiveness in problem solving, there are three assumptions that need to be considered which are questionable when utilizing product only assessment; *timing, impact,*

and *interpretation*. *Timing* refers to the specific time in the sequence of problem solving when the diagram was produced. The *timing* of the diagram drawing is critical in attributing causality of the problem solution to a diagram. In product only assessment of a diagram, the assumption is that the diagram was a precursor to the solution when the converse is also possible. The *impact* of a diagram is another issue on which product only assessment falters, for a diagram to be congruent with the solution, the diagram must reflect the solution process, however the diagram may only have been used in the initial stages of problem solving to help the solver understand the problem or to determine the solution process to follow, in which case the diagram may not necessarily be congruent with the solution. *Interpretation* is the final issue on which product only assessment is arguable. The diagrams that were used in the solution of a problem are working diagrams which may have been modified or even abandoned half done when the solution was apparent to the solver. These diagrams may only be intelligible to the solver, because what is presented on the diagram may be in "shorthand" with only the minimum detail included or an "abstraction" of the problem. Hence, any assessment of a diagram as "appropriate or inappropriate" is highly subjective and needs to account for the issues of *timing*, *impact* and *interpretation*. Therefore, the dynamic use of the diagram seems to be of particular importance when investigating the effectiveness of the strategy *draw a diagram* in problem solving, both to provide a framework for interpreting the diagram, and in order to determine how the diagram was used during the solution of the problem.

Monitoring the Diagram Drawing Process

Nunokawa (1994a) advocates monitoring the diagrams produced during the problem solving process to determine whether the solver's structure of the problem situation, which is evident from the diagrams that are drawn, becomes more aligned with the inherent structure of the problem. Novel problems provide support for the assumption of an initial difference between the student's perceived structure of the problem and the actual structure problem, because the solver has to develop a solution procedure for the problem (Nunokawa, 1994a). The importance of the changes that occur in a drawing during the problem solving process supports Nunokawa's (1994b) conjecture that the interaction between the diagram and the student's perception of the problem ultimately leads the student to identifying the problem structure. Nunokawa (1994a) concluded that the drawing can only be comprehended when the protocol is considered and the context of the drawing is understood. Hence the influence of a diagram on the problem solving process may be unrecognized if only the final product is evaluated and if this evaluation is isolated from the problem solving process.

In order to explore Nunokawa's (1994a) idea, a study of the dynamic use of the diagram by children solving novel problems was initiated. The aim was to determine the usefulness of monitoring the diagram drawing process, and to ascertain the reliability of using end product measures such as frequency and spontaneity of diagram use, solution time, performance scores and the appropriateness of the diagram drawn. This is a preliminary report on an individual student's use of diagrams in a problem solving task.

Method

Subjects

Fifty-five subjects completed a novel problem solving test. The students were then categorized into four groups based on their performance and frequency of diagram use (See Table 1). Fourteen of these subjects, who represented the

extreme cases in each category, were interviewed for an indepth analysis of their problem solving strategies.

Table 1: Categories of Students Interviewed

	Test Performance	*Frequency of Diagram Use*	*n*
Category A	*high*	*high*	*4*
Category B	*high*	*low*	*4*
Category C	*low*	*high*	*1*
Category D	*low*	*low*	*5*

Procedure

The subjects were individually presented with five items similar to those used for selection. They were video-taped during the problem solving process and their solution strategies probed on the completion of the problems. The video-tapes and work samples were kept for analysis. When presenting the problems the subjects were neither specifically instructed nor encouraged to draw diagrams and were given a range of resources that they could use e.g. paper, felt pens, unifix cubes, a ruler, a calculator.

Results and Discussion

The performance of one subject will be explored in this paper. The case discussed was purposely selected to support the assertion that end product measures of the diagram drawing process do not necessarily reflect the effective use of the strategy *draw a diagram* in problem solving.

Lara was initially identified as a high performer who frequently used diagrams in problem solving. She presented as a motivated, enthusiastic subject who diligently undertook the tasks. Her solution of the following task is reported because it illustrates the inconsistency that may occur between the assessment of end product measures and the assessment of the dynamic use of the diagram.

A sleepy koala wants to climb to the top of a gum tree that is 10 metres high. Each day the koala climbs up 5 metres, but while asleep slides back 4 metres. At this rate how many days will it take the koala to reach the top?

End Product Measures

Lara spontaneously drew a diagram of a tree with ten bars placed vertically beside the tree 13 seconds after beginning to read the problem, completing the task in 1 minute 37 seconds. Her rapid solution time, spontaneous use of a diagram and drawing of an appropriate diagram, would seem to indicate that she had understood the problem and had used the diagram effectively to solve the problem. Lara's initial solution of 10 days was incorrect, however during the interview Lara also expressed the correct answer of 6 days. Hence it is conceivable that Lara may have found the correct answer initially. Despite Lara's apparent competence with the strategy *draw a diagram* as indicated by end product measures, she was unable to use the diagram as an effective problem solving tool.

The Dynamic Use of the Diagram

Lara's interview revealed four areas of concern. *Firstly, Lara did not fully understand the problem, despite drawing an appropriate diagram for the problem.* During the interview Lara reused her initial diagram several times, drawing two further diagrams and finding answers of 4, 5 and 6 and 10 days by counting up 5 metres and back 4 metres on the diagram. Lara also calculated an answer of 10 days without the diagram, using the 1 metre difference between the climb of 5 metres and the slide back down of 4 metres in her reasoning.

L: It can't be 4 or 5 days. It's a 10 metre tree. 5 take 4 equals 1. If it (the koala) goes up 5 and slides down 4 (metres) it's going up 1 every day. ... That means you'd have to take (in

days) the same amount of metres as on the tree.

When asked to use the diagram of a tree to get the same answer, Lara counted up and back on the diagram, reaching the top after "5 days" and then made a comment which revealed an apparent lack of understanding of the problem. Her use of the diagram seemed to have caused her to rethink the problem.

L: I've got up to the top and I don't know whether I'm supposed to slide down when I've reached the top.

Secondly, although Lara's diagram was appropriate, she had difficulty with an element of the diagram, possibly due to a lack of experience with diagrams. Lara's difficulty with the diagram was that she was unsure whether 1 metre was ground level, or 1 metre up the tree.

L: The bottom of the tree is the first metre.

I: So is that the 1 metre mark? (pointing to the mark level with the bottom of the tree)

L: Yeh.

I: So at the very bottom of the tree it's 1 metre high.

L: No, no. That can't be right, can it? That means the first metre you'd go up 1 metre.

Thirdly, Lara was unsure how to use measures on the diagram, again possibly due to limited experience with diagrams. She sometimes counted the metre mark she was on as 1 metre before she had travelled 1 metre. She repeated this error when counting down. Using a combination of these methods Lara calculated answers of 4, 5 and 6 metres. When Lara used the incorrect method both counting up and counting back, she arrived at the correct answer of 6 days.

Fourthly, Lara's repeated use of the diagram did not enable her to correctly solve the problem. After several attempts to find the solution using diagrams, and then a calculator, a ruler and unifix cubes Lara was unable to decide on an answer. She appeared to base her final choice of an answer on the number of times an answer had occurred.

L: I'd probably put 5 first, then 10, then 6, then 4 (days). I suspect that one (pointing to 5).

I: Is there any reason?

L: Probably because I got it the most times.

Clearly Lara was not able to use the diagram as an effective tool for problem solving despite end product evidence suggesting the contrary.

Lara's interview provided three reasons to support the importance of monitoring the diagram drawing process. Firstly, the end product measures of spontaneity and frequency of diagram use, solution time and appropriateness of the diagram drawn were not reliable indicators of her problem solving performance. Secondly, her use of the diagram revealed an apparent lack of understanding of the problem. Thirdly, her use of the diagram indicated some confusion about how elements of the diagram should be used. Thus assessment of the dynamic use of the diagram encompasses not only the drawing of the diagram, but also the appropriate use of the diagram in the solution of a problem.

Conclusion

Although end product measures are easy to score, Lara's case illustrates that they may not be reliable indicators of the effective use of the strategy *draw a diagram*. In contrast, monitoring the dynamic use of the diagram has two advantages; firstly, tracking the use of diagrams in problem solving can provide an insight into how the strategy *draw a diagram* has been used and secondly, the assumptions of *timing*, *impact* and *interpretation* can be accommodated. Hence, future research exploring the effectiveness of the strategy *draw a*

diagram should consider the validity of the assessment method.

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