Error Patterns in Mental Computation in Years 3 - 9

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The importance of mental computation compared to written computation has been well established (McIntosh, 1990; Hope, 1986; Reys and Reys, 1986). However, investigations of children's error patterns have tended to focus more on the written algorithms. With greater emphasis being placed on mental computation in contemporary curricula it is essential that teachers be well aware of the common error patterns since these can reveal underlying misconceptions.

An extensive survey of children's mental computation skills in years 3, 5, 7 and 9 was undertaken in the Perth Metropolitan area as part of an international study involving Australia, Japan and the USA (McIntosh, Bana & Farrell, 1995; Reys & Reys, 1993). This paper deals with perceived error patterns arising from the Perth study.

The Study

The Perth study included the administration of mental computation tests to two classes in each of years 3, 5, and 7 in each of three primary schools, and to six classes at year 9 level in a high school that the majority of the primary students tested would move into for year 8. The tests were developed jointly by mathematics educators in the three countries involved. The sample sizes were 163, 163, 163 and 152 in years 3, 5, 7 and 9 respectively.

All items were non-contextual to eliminate any problem solving component and many items were used across two or three, and even four year levels to provide information on students' development of mental computation skills. For the purposes of this paper a selection of such items was made to investigate error patterns across the various year levels, to identify possible misconceptions and to suggest some remedial action.

Results

The results presented below are for a selected sample of the mental computation test (MCT) items which satisfied three criteria as follows. They span two or more of the years tested; they collectively represent most of the key MCT topics; and they demonstrate interesting error patterns.

Table 1 shows that students in all the years 3, 5, 7 and 9 found the item 79 + 26 markedly easier than the related subtraction item 105 - 26. This is consistent with other studies (Bana & Korbosky, 1995). Some students responded that 79 + 26 was 95, probably due to a failure to 'bridge' ten. The most common error in 105 - 26 was 81. It seems likely that this was attained from 10 - 2 for the tens and the common reversal error of 6 - 5 for the ones. Another common error, especially in the two lower years was that students added the two numbers instead of subtracting. However, year 3 students found these items difficult since more than 50 percent did not attempt them.

Table 1 *Results of Items 79 + 26 and 105 - 26 in Years, 3, 5, 7 and 9 **

Responses	*Year 3 (n = 163)*	*Year 5 (n = 163)*	*Year 7 (n = 163)*	*Year 9 (n = 152)*
Item: 79 + 26				
Correct response	16	66	81	89
No response	57	10	2	0
95	2	6	2	2
96	4	2	0	0
115	1	2	1	1
Other responses	20	14	14	8
Item: 105 - 26				
Correct response	5	42	69	84
No response	52	19	7	1
74	1	2	2	1
81	8	7	4	4
84	2	6	1	0
89	1	2	6	1
131	4	3	1	1
Other responses	27	19	10	8

* Percentages

Table 2 shows three common errors for 3500 ÷ 35. The percentages of students responding with 1000 were 8, 6 and 4 in years 3, 5 and 7 respectively. Two or three percent had an answer of 10; again showing a lack of understanding of the order of magnitude of numbers. Those who responded with 350 may have had similar misconceptions but then recorded the product of the two factors 10 and 35. The significant percentages for 'no response' shows that many students cannot apply basic facts and/or lack numeration concepts.

Table 2 *Results of Item 3500 ÷ 35 in Years 5, 7 and 9*

Responses	*Year 5 (n = 163)*	*Year 7 (n = 163)*	*Year 9 (n = 152)*
Correct response	29	65	82
No response	40	10	4
10	2	2	3
350	7	6	3
1000	8	6	4
Other responses	14	11	4

Table 3 shows that the most common error for 38 x 50 is 190; again one related to order of magnitude. This persisted into year 9. Another common error in year 5 was 150 - probably obtained from 3 x 5 = 15 and 8 x 0 = 0. This error disappeared by year 9. The response of 1500 is probably a related error. The two errors 158 and 1580 are similar in that they probably derive from the error of 8 x 0 = 8, but they almost disappear by year 9. Four percent of year 5 students added the two numbers, but no students in years 7 or 9 did this.

Table 3` Results of Item 38 x 50 in Years 5, 7 and 9

Responses	*Year 5 (n = 163)*	*Year 7 (n = 163)*	*Year 9 (n = 152)*
Correct response	7	31	57
No response	45	33	14
88	4	0	0
150	8	2	0
158	4	2	1
190	4	8	7
1500	2	2	3
1580	4	3	1
Other responses	22	19	17

Table 4 Results of Items 60 x 70 and 4200 ÷ 60 in Years 5, 7 and 9

Responses	*Year 5 (n = 163)*	*Year 7 (n = 163)*	*Year 9 (n = 152)*
Item: 60 x 70			
Correct response	30	73	80
No response	15	2	1
130	4	1	0
420	38	12	15
Other responses	13	12	4
Item: 4200 ÷ 60			
Correct response	20	56	82
No response	37	15	7
7	1	4	0
700	17	12	7
7000	5	5	0
Other responses	20	8	4

For the item 60 x 70 the percentages of respondents with 420 were 38, 12 and 15 percent in years 5, 7 and 9 respectively, as can be seen in Table 4. It seems that even students at higher year levels do not estimate to check the validity of their results. For example, 'Are there 60 seventies in 420?'. Yet in the related division item 4200 ÷ 60 very few responded with 7. The most prevalent error here was 700 and this error also persisted into year 9. Another order of magnitude error was the response of 7000. It appears likely that students use a rule of taking off zeros before calculating, but then add them incorrectly afterwards These errors again indicate that students do not seem to use estimates to check results.

For the item $^1/_2 + ^1/_4$ Table 5 shows that 19 percent of year 5 students gave $^2/_6$ as a response but this error almost disappeared by year 9. It seems that these students see fractions as consisting of separate numbers so they add numerators and add denominators. The other two identified errors, $^1/_6$ and $^2/_4$, are probably similar in nature since they could be obtained by adding either the numerators or the denominators. Errors for the related subtraction item are of a similar type, with one ($^4/_6$) showing subtraction interpreted as addition. Note that 18 percent of year 5 students responded with $^2/_2$; most probably as a result of subtracting numerators and denominators.

Table 5 *Results of Items $^1/_2 + ^1/_4$ and $^3/_4 - ^1/_2$ in Years 5, 7 and 9*

Responses	*Year 5 (n = 163)*	*Year 7 (n = 163)*	*Year 9 (n = 152)*
Item: $^1/_2 + ^1/_4$			
Correct response	37	72	86
No response	11	6	1
$^1/_6$	8	0	1
$^2/_6$	19	5	1
$^2/_4$	3	2	2
Other responses	22	15	9
Item: $^3/_4 - ^1/_2$			
Correct response	38	76	-
No response	10	7	-
$^2/_2$	18	4	-
$^2/_4$	4	4	-
$^4/_6$	5	1	-
Other responses	25	8	-

Students also have difficulty with decimals, as shown in Table 6. The percentages who gave the response 0.80 to 0.5 + 0.75 were 49 and 8 in years 5 and 7 respectively. It seems that students simply ignored place values of the decimals. It is significant that not one student responded with 0.8. There is no ready explanation for the answer of 2 from 13 percent of year 7 students.

Table 6 *Results of Item 0.5 + 0.75 in Years 5 and 7*

Responses	*Year5 (n = 163)*	*Year 7 (n = 163)*
Correct response	13	58
No response	21	3
0.80	49	8
1.1	0	3
2	1	13
Other responses	16	15

Table 7 *Results of Items $^1/_{10}$ of 45, 0.1 x 45 and 10% of 45 in Years 7 and 9*

Responses	*Year 7 (n = 163)*	*Year 9 (n = 152)*
Item: $^1/_{10}$ of 45		
Correct response	54	82
No response	17	7
15	4	0
45	6	1
Other responses	19	10
Item: 0.1 x 45		
Correct response	48	66
No response	8	4
0.45 or .45	13	18
45	11	4
450	2	1
Other responses	18	7
Item 10% of 45		
Correct response	52	85
No response	16	5
4	5	0
5	6	2
15	2	0
35	4	0
45	2	1
Other responses	13	7

Table 7 shows results for three related items using fractions, decimals and percentages. There is little difference in performances between the fraction and percentage formats, but decimals are markedly lower. Here the error that really stands out is 0.45 or .45 with totals of 24 percent and 22 percent in years 7 and 9 respectively. It seems that many students simply treat decimals as if they are whole numbers, and this error is still very common in year 9.

Discussion

The most common error in subtraction of whole numbers is the use of the 'reversal' in cases where there are insufficient ones to subtract. This is consistent with other research. The second most common error here is the failure to 'bridge' tens so that the response is 10 more than it should be. A similar problem occurs in addition where bridging ten is necessary. A further error which is all too prevalent is the use of the addition operation where subtraction is required. These errors generally decrease over the year levels. It is suggested that remedies lie in a more intensive treatment of the numeration system, exploration of commutativity, and the use of estimation with all calculations.

It is interesting to note that students perform much better on an addition item than on its subtractive equivalent. The same may be said for multiplication items and their division inverses, but this difference seems to disappear by year 9. Students need more experiences that specifically explore operations and their opposites or inverses.

Students in years 5 and 7 seem to treat a fraction as a pair of numbers rather than one number. Much more needs to be done to further develop concepts about fractions before requiring students to operate on them. As with fractions, students also need more experiences with decimals and percentages before operating on them. The relationships between fraction, decimal and percentage forms need greater emphasis to ensure that the equivalences are well understood.

Errors in order of magnitude are very common in multiplication and division with whole numbers and decimals. In all mental computation, as in number work generally, it is crucial that students have a sound understanding of the numbers they are operating with and that they use estimation both to provide a useful check on their solutions and to develop a better number sense.

Thus the error patterns identified here provide useful guidance for classroom teachers. In addition, they indicate a number of possible research directions. Firstly the student interviews and case studies would be useful to confirm the interpretations made and to identify any error types that are not clear. Secondly, intervention strategies could be designed and checked out for their effectiveness in reducing errors and building up number sense.

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