PRODIGY: A SIMULATION SYSTEM FOR DIAGNOSIS AND REMEDIATION WITHIN THE DOMAIN OF COMMON FRACTIONS.

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This paper describes the on-going research and development activities of the PRODIGY project. The major aim of this project is to develop intelligent computer-based simulation systems which facilitate the development of high level diagnostic and remediation skills in the domain of common fractions. The development and the functioning of the present generation of PRODIGY simulation systems (PRODIGY1) is first described. The methods used in and results from the formative evaluations of the PRODIGY1 generation of simulation systems then are described. Some of the modifications which had their genesis in the results from the formative evaluations are briefly discussed. Three serious limitations of the PRODIGY1 simulations are identified and discussed. The paper concludes with a description of the architecture of next generation of PRODIGY simulations together with a discussion about how it is hypothesized that they will overcome the serious limitations of the PRODIGY1 simulations.

Research conducted in schools clearly indicates that most teachers either do not diagnose to obtain a clear view of their pupils' understandings and misconceptions or limit their attention to the product of their children's work rather than focussing on the processes and strategies employed by their children (Bennett, 1987). Because of this, many students are being presented with inappropriate lessons. The learning of mathematics thus is for many students a frustrating and unsuccessful experience. Unfortunately, little effective work is being done at present to ameliorate this lack of diagnostic skills in teachers.

In this paper, I describe the development and evaluation of the PRODIGY simulation system currently being done at the Queensland University of Technology's Centre for Mathematics and Science Education (CMSE). The major aim of this system is to provide a learning environment in which novice teachers may acquire expertise in the diagnosis and remediation of mathematical learning problems. As they are acquiring this expertise, it is envisaged that the novice teachers will develop higher levels of lesson structure knowledge (Leinhardt & Smith, 1986) than that achieved by current teacher education activities. It is also envisaged that the interactions with the system will facilitate the development of pedagogical content knowledge (Shulman, 1986) by helping novice teachers to better understand how particular topics, principles, strategies, and the like in mathematics are comprehended or typically misconstrued, are learned and likely to be forgotten.

PRESENT VERSION (PRODIGY1)

PRODIGY1 (Nason, 1991, 1993) is a simulation environment that lets teachers develop and practice skills in tutoring students who are having difficulty with the addition of fractions. With this system, the tutor first assesses the simulated student's knowledge by giving it some addition of fraction problems (e.g., 1/2 + 1/3) and observing how the answers to the given problems are generated. In order to gain further insights into how the simulated student generates answers to the problems, the tutor can ask the simulated student to explain how it got its answer for each problem. The simulated student responds to this request by presenting a written, sequential explanation of its procedures. When the tutor has decided what the student's misconceptions are, he can plan and administer a sequence of instructional activities. The simulated student may or may not overcome its misconceptions when it is given this instruction, so the tutor must again assess the student's knowledge. This cycle of diagnosis and remediation continues until the tutor is satisfied that the simulated student knows how to add fractions.

The PRODIGY simulation is based upon a set of real children. The simulation of each child was constructed in the following way:

1) In interview 1, each child was asked to carry out some addition of fractions items. As the subject carried out the computations, (s)he was encouraged to think aloud. Each session was videotaped. Following the problem solving session, a stimulated recall session was conducted with the aid of the videotape recording.
2) An initial production system model of the child's computational knowledge and an initial semantic network model of the child's intuitive and conceptual knowledge was generated from a detailed protocol analysis of the videotape transcript and the subject' stimulated recall.
3) As there was some ambiguities to be overcome before psychologically valid model could be generated, each child was recalled for further sessions in which other addition of fractions and fraction concept items were administered. The items administered depended on the information required by the investigator to complete the models. After each session, the production system and semantic network model were revised in the light of the new data produced. This observation-analysis-modification procedure was repeated until a stabilized model was produced. The primary purpose of this procedure was to construct a model for each child based on converging experimental evidence.

One of PRODIGY's three case studies is "Toni". Toni's bug is that whenever the two denonimators are different, it adds them to get the answer's denominator; then it cross multiplies the numerators and the denominators and adds these two products to generate the numerator of the answer (See Figure 1 below). Thus, if Toni is asked to add two fractions with different denominators, she always produces an incorrect answer. If they are unit fractions, it always produces an answer equivalent to one whole (e.g., 1/5 + 1/6 => 11/11). The diagnosis of this bug is complicated by Toni's tendency to perform the operations of generating denominator answer and numerator answer in random order. It is also further complicated by two other factors: Toni can correctly add fractions with common denominators and Toni's explanations of how an answer is generated are very convoluted and full of mathematical jargon. However, by observing Toni attempts at different types of addition of fraction problems and by carefully juxtaposing what they have observed against Toni's explanations, novice teachers soon can "get inside Toni's head" and discover the reasons for her errors. While doing this, they are simultaneously developing two crucial diagnostic skills: how to identify and ask the right questions and how to listen and observe while students are performing mathematical computations.

$$\frac{3}{4} + \frac{4}{5} = \frac{31}{9}$$

15 16
9

Figure 1 Toni

An expert teacher would recognize that Toni's misconceptions are deeply seated: Toni in fact does not understand that a fraction is a number and many other basic notions about fractions. The remediation thus would need to start at initiating level explorations of the basic notion of a fraction and then proceed through a sequence of initiating, abstracting and schematizing activities (Ashlock et al, 1980) on comparison of fractions, estimation with fractions, equivalence of fractions, and generating common denominators before Toni would be ready to learn how to add fractions with different denominators. Many novice teachers do not seem to realize this. Their remediations tend to focus on Toni's procedures not the underlying misconceptions. Their initial suggested remediations thus tend to have no effect at all on Toni's addition of fractions' procedures. Their remediations only have an overt effect on Toni's procedures when they address its basic misconceptions about fractions and have introduced correct procedural rules into Toni's repertoire of algorithmic processes. The important lesson novice teachers learn from this is that teachers need to focus on underlying misconceptions before attempting to remediate algorithmic procedures.

FORMATIVE EVALUATION OF PRODIGY1

While PRODIGY1 has been undergoing development, it has been formatively evaluated. Evaluation data has been collected from two different sources: teacher education students and expert educators.

Five groups of three undergraduate and two groups of two postgraduate teacher education students have been observed as they interact with the simulation system. These observations have focussed on their protocols and strategies for diagnosis, the speed of their diagnoses, the quality of their diagnoses and their suggested remediation programmes, and their ease in the use of the computer interface and its menus. As the groups proceed through the three case studies, particular emphasis has been placed on identifying qualitative changes that are occurring in the teacher education students' diagnostic strategies and in their general perceptions about the nature of and underlying causes of mathematical error patterns.

The evaluation data collected from the observations has been supplemented by data from post-simulation session interviews in which the groups were encouraged to discuss what they felt they had learnt from the simulation session, what new questions about diagnosis and remediation had emerged during the session and any difficulties they had with using the system. They also were invited to make suggestions about how they felt the system may be a more effective learning tool.

Expert educators also have been involved in the formative evaluation of the system. Experts in the field of multimedia education such as Prof. Joe Henderson (Dartmouth Interactive Media Laboratory) have provided comments about the computer-user interface and the effectiveness of the simulation aspects of the system; experts in the field of artificial intelligence in education such as Drs Stellan Ohlsson and Kurt VanLehn (LRDC, University of Pittsburgh) have provided comments about the cognitive modelling aspects of the system and mathematics education experts in Australia, Europe and the USA have provided comments about the mathematics education aspects of the system.

The formative evaluation data from the teacher education students and the expert educators have been used to identify how the system can be modified so that it may become a more effective teacher education tool. Many modifications thus have been made to the system during the past two years. For example, evaluation feedback from the teacher education students and from Prof. Joe Henderson indicated that the system would be much more effective if the users could gain access to the underlying thought processes of each case study and in particular the type of language the case study would use to describe what it had just done. The system thus was modified; users are now able to print out an explanation of how the case study generated its answer to the example it has just completed. Evaluation feedback from mathematics educators has resulted in greater emphasis being placed on the development of conceptual knowledge during the remediation phases of the simulation. Thus, in it present version, the error patterns of none of the three cases studies will be successfully overcome unless the users focus much of the remediation programme on overcoming the underlying misconceptions.

The formative evaluations have identified a number of limitations of the PRODIGY1 system which cannot be ameliorated by simple modifications to the PRODIGY1 system. Probably the most significant of these limitations is that PRODIGY1 only enables the user to directly evaluate the "virtual" students' knowledge of the addition of common fractions algorithmic knowledge. Although the information generated by the simulations may enable the users to make inferences about a case study's underlying conceptual knowledge, PRODIGY1 does not allow the user to directly evaluate the validity of these inferences. Another limitation is that it does not allow the user to assess the case study's understanding of these fraction concepts and processes at the concrete/oral and pictorial/oral levels of representation. One can thus only hypothesize the case study's knowledge at these non-written-symbolic levels. This information, however, is very important when one is planning and implementing a remediation programme. A third limitation of PRODIGY1 is that it does not enable the user to diagnose, begin initial instruction and then evaluate the effects of this initial instruction. In its present form, most of the remediation programme has to be completed before its effects become apparent while diagnosing. Because of these limitations, a second generation of PRODIGY simulations is currently being developed.

PRODIGY2: THE NEXT GENERATION

In the second generation of PRODIGY currently being developed, it is envisionaged that novice teachers will be able to investigate each case study's ability: to concretely/pictorially represent fractions; to represent fractions with oral and written representations; to translate from one form of representation to another; to compare fractions; to estimate the sum of two fractions; to generate equivalent fractions; and to generate common denominators. It also will enable the users to ascertain whether the case study has the notions that a fraction can represent being part of a whole or partitive division.

In order to include these new features in the next version of PRODIGY, a new production system model is currently being developed. The development of this new model is proceeding in two phases. In Phase 1, a semantic network of approximately 80 fraction notions has been produced. During the production of this semantic network, constant reference was made back to the semantic models generated earlier in this investigation. The information contained in this semantic network and the PRODIGY1 procedural knowledge production system is now being incorporated into a "hand-run" production system network. In Phase 2, this network will be translated into a computer-based production system which contains two parallel and complementary subsystems: a qualitatitive subsystem in which the number sense and estimation notions and procedures are located; and a quantitative subsystem in which the system's algorithmic procedures are located.

CONCLUSION

The PRODIGY-Mk2 architecture has implications for both education and A.I. Its architecture will easily be able to be adapted for simulating diagnosis and remediation of other mathematics topics such as operations with decimals and whole numbers and also many algebra topics. It thus has the potential to significantly affect the quality of mathematics education from early elementary to junior college levels. Its significance for A.I. is that this model integrates conceptual knowledge with procedural knowledge more directly than most previous A.I. simulation models. It also provides us with a means for exploring the learning relationship between both these types of knowledge. The major limitations of this simulation model seems to be that it can only simulate number and algebraic concepts and processes and not many important measurement and geometrical concepts and processes.

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