LEARNING MATHEMATICS AT UNIVERSITY LEVEL: INITIAL CONCEPTIONS OF MATHEMATICS.

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This paper reports preliminary results from an ongoing investigation to identify the conceptions of mathematics held by beginning first year university students and their orientations to their previous study of mathematics. A questionnaire was administered to approximately three hundred students during their first week at university. The questionnaire contained five open-ended questions designed to elicit students' own conceptions of mathematics and their orientations to studying it. Two of these are discussed in this paper. Phenomenographic techniques were used to analyse responses and identify qualitatively different categories of description.
In-depth interviews of a subsample of twelve students revealed details of the range of conceptions of mathematics and the related qualitative differences in approaches to learning mathematics after several weeks of university mathematics.
Analysis of responses revealed that, although a wide range of beliefs was elicited, the majority of students view mathematics as a necessary set of rules and procedures to be learned by rote that are unrelated to other aspects of their lives. The survey results also indicate a relationship between conceptions of mathematics and approaches to studying mathematics at university level. There was no evidence of gender differences in either conception of mathematics or approach to learning.
This paper will report on the evidence of relationships between conceptions of mathematics, approaches to learning and course results. These preliminary results raise questions about the impact of prior experience on approaches to learning mathematics at university level and the quality of learning outcomes.

Mathematics in school is the foundation for later learning at the university level. Research at the school level indicates that the learners' previous experiences influence the quality of their approaches to learning, attitudes to and outcomes in learning mathematics (Clarke, 1985; Clarke, Fraser and Wallbridge, in press; Crawford 1983, 1986, 1990a, 1990b; Resnick, 1987). There are indications that students' conceptions of mathematics and their orientations to study affect the quality of cognitive activity and of learning outcomes (Crawford, 1986, 1989, 1990b, 1992a, 1992b). Also, the ways in which learners interpret the context of their mathematical learning and hence the ways they relate to mathematical activities influence their mathematical thinking (Crawford 1990a; Cobb, Yackel & Wood, 1992; Lave 1988; Steffe & Cobb, 1988; Solomon 1989). Furthermore, Biggs and Collis (1982) have shown that the characteristics of learning activity as expressed by students' verbal reports about learning reflect differences in thinking in ways which are related to their learning outcomes.

There is also a growing consensus among researchers in the field of student learning in higher education (Marton, Hounsell, & Entwhistle, 1984; Ramsden, 1991) that university students learn from their experiences. It would be expected that students would enter university with a range of conceptions of mathematics and orientations to mathematics learning derived from their school experiences. There is also evidence that the quality of these experiences, from the students' perspectives, substantially relates to the ways in which they approach learning at university level and the quality of learning outcomes (Crawford, 1993; Prosser & Millar, 1989; Trigwell & Prosser, 1991a, 1991b; Volet & Lawrence, 1989).

This study has focussed on three research questions:

1. What are the nature and range of beginning university students' conceptions of mathematics and how are these distributed?
2. What are the nature and distributions of orientations to learning mathematics for beginning university students?
3. Are there relationships between students' initial conceptions of mathematics, their orientations to learning and their achievements at university level?

METHOD OF RESEARCH

A survey questionnaire on student conceptions of mathematics and mathematics learning was administered to approximately 300 first year mathematics students at the University of Sydney. The questionnaire consisted of five open ended questions that were designed to elicit students' own conceptions of mathematics and their approaches to studying it. In-depth structured interviews were also carried out with twelve selected students in order to clarify their written statements and to enhance interpretations of the questionnaire responses. The interviews were taped and transcribed.

A phenomenographic approach (Marton, 1988) was taken to analyse the students' responses. The first stage in this analysis was to identify a set of ordered categories of description to the open ended questions. The categories were hierarchical in the sense that each category in the sequence included all elements of all preceeding categories. Although the research team were influenced by knowledge of recent theories of learning and research in the field of mathematics education, every effort was made to ensure that the categories emerged as the result of a qualitative analysis of the data. This process actively involved all members of the research team in the following procedure:

1. Twenty one representative responses were selected.
2. Five researchers individually identified an initial set of categories.
3. Researchers met and compared categories and explored the relationships between them.
4. The research team agreed to a draft set of categories.
5. Each researcher took the same subset of questionnaire responses and classified them according to the draft categories.
6. The individual classifications were compared and a final set of clear statements of each category was agreed upon.
7. One researcher then classified all three hundred responses.
8. Any responses that were difficult to classify were later discussed by all five researchers.
9. Later, responses involving a mismatch between conceptions of mathematics and approach to learning mathematics were further reviewed by two researchers.

The students' responses to the questionnaire were then analysed to explore the relationships between conceptions of mathematics, approaches to learning mathematics and attainment in first year mathematics at university.

RESULTS.

The analysis of the questionaire responses yielded an ordered set of categories for each open ended question. The categories that emerged from the analysis of students' responses about the nature of mathematics (Q. 2) is shown in Table 1.

Table1: Categories of responses for Question 2.
Question 2
Think about the maths you've done so far. What do you think mathematics is?

Category[1]	Representative quote from student survey.
A. Maths is numbers, rules and formulae.	*Maths is the study of numbers, and the application of various methods of changing numbers.*
B. Maths is numbers, rules and formulae which can be applied to solve problems.	*Mathematics is the study of numbers and their applications in other subjects and the physical world.*

1 To economise space the statement of category B has not been repeated in the statement of the subsequent categories but its inclusion is assumed.

C. Maths is a complex logical system; a way of thinking.	*Mathematics is the study of logic. Numbers and symbols are used to study life in a systematic perspective and requires the mind to think in a logical and often precise manner.*
D. Maths is a complex logical system which can be used to solve complex problems.	*Maths is an abstract reasoning process which can be utilised to explore and solve problems.*
E. Maths is a complex logical system which can be used to solve complex problems and provides new insights which develops our understanding of the world.	*Techniques for thinking about observable, physical phenomena in a quantitative way and also for thinking more abstractly with little or no relation to the directly observable universe.*

The categories derived from students' responses to the question about the nature of mathematics (Q.2) revealed a clear shift in conception between the first two categories and later elements of the ordered set. Responses in categories A and B generally described mathematics as a fragmented collection of rules and procedures whereas responses in categories C, D and E present views of mathematics as a more cohesive theory. The original questionnaire was designed so that students had two opportunities to describe their orientations to the study of mathematics. Consequently when analysing the data questions 3 and 4a were treated as one. The categories of description and representative student quotes are given in Table 2.

Table 2: Categories of description for the responses to Questions 3/4a.

Question 3/4a
3. Think about some maths you understood really well. How did you go about studying that? (It may help you to compare how you studied this with something you feel you didn't fully understand.)
4a How do you usually go about learning some maths?

Categories[2]	Representative quote from student survey.
A. Learning by rote memorisation, with an intention to reproduce knowledge and procedures.	*I liked calculus because I could remember formulas which is how I used to study. I would rote learn all the formulas and summarise all my theoretical notes.* o
B. Learning by doing lots of examples, with an intention to reproduce knowledge and procedures. (Drill and practice)	*The way I go about studying for mathematics is by doing a lot of questions and examples. Firstly I would study the notes and learn formulas, then I put all of that to use by doing heaps of exercises.* o
C. Learning by doing lots of examples with an intention of gaining a relational understanding of the theory and concepts.	*To understand a topic well it was important to gain an understanding of the basic concepts involved, backed up by some problem solving on the topic. However, concepts which were not fully comprehended could become well understood through extra work on related questions, i.e. It is essential to do a wide range of questions on a topic to fully understand it.*

2 Note, each category again includes all preceeding categories.

D. Learning by doing difficult problems, with an intention of gaining a relational understanding of the entire theory, and seeing its relationship with existing knowledge.	*After listening to explanation of how a particular maths works the most essential features a repetition to develop speed (this usually consists of boring menial tasks) and an equal component of very difficult problems which require a great deal of thought to explore that area and its various properties and their consequences.*
E. Learning with the intention of gaining a relational understanding of the theory and looking for situations where the theory will apply.	*Read the relevant theory and try to get on the same "wavelength" as the person who actually discovered it. Before I attempt any problems I try to think where you can use the concept: i.e. what the concept was invented for. Then I attempt problems (on my own).*

Students' approaches to studying mathematics showed a similar shift to that discussed above. Their responses indicated a shift in intention from reproduction of knowledge and procedures, in categories A and B, towards a relational understanding of the theory and the concepts, in categories C, D and E, of increasing complexity. The Gothenburg school (Marton, 1988) describes this type of shift as being from a surface to a deep approach. Categories A and B are examples of a surface approach and categories C, D and E are examples of a deep approach

Quantitative analyses
In this section we will describe the distribution of results across categories for both the conceptions and approaches and the relationship between conceptions and approaches. As well, we will show how the conceptions and approaches relate to student performance. Table 3 shows the distribution across categories of conceptions and approaches. In this sample, there was no evidence of gender effect.

Table 3: Distribution of Prior Conceptions and Approaches

Conception/Approach	N	% of responses
Conception		
A. Maths as numbers, rules and formulae	62	26
B. Maths as numbers etc. with applications to problems	124	51
C. Maths as way of thinking	32	13
D. Maths as way of thinking for complex problem solving	18	7
E. Maths provides new insights used for understanding the world	6	3
Missing Data (no response)	52	
Total	294	
Approach		
A. Learning by rote memorisation	17	6
B. Learning by doing examples (drill and practice)	215	76
C. Learning by developing relational understanding by doing examples	30	11
D. Learning by extending understanding with difficult problems	15	5
E. Learning by extending understanding to a broader context	6	2
Missing Data (no response)	11	
Total	294	

Table 3 shows that 77% of conception statements were classified in the first two (fragmented) categories, while 23% were classified in the last three (cohesive) categories. Eighty two percent of the approach statements were classified in one of the two "surface" categories, while only 17% were classified in one of the three "deep" categories.

Table 4 shows a cross-tabulation of the relationship between prior conception and prior approach.

Table 4: Relationship between Prior Conception and Prior Approach among responses

Conceptions *Approach*

		Surface		Deep	Row totals
Fragmented		179 (91%)		17 (9%)	196
Cohesive		4 (10%)		36 (90%)	40
Column totals		183		53	236

Chi-square=126, $p<0.001$

Table 4 indicates a relationship between conception and approach. It shows that 91% of the students who had a fragmented view of high school mathematics had adopted a surface approach to the study of high school mathematics, while only 10% of those who had a cohesive conception had adopted a surface approach.

Table 5 does not indicate a relationship between both prior conceptions and prior approaches to study and achievement in the first semester. However, after a year of university study, the results

Table 5: Relationship between Prior Conception and Prior Approach and University Achievement

Conceptions/Approaches Mathematics 1

		Semester 1 *Mean SD		Semester 2 *Mean SD
Conceptions				
Fragmented		125 28		123 21
Cohesive		134 25		134 23
T Test: T=		1.53 not significant		2.16 $p<0.05$
Approaches				
Surface		125 28		122 29
Deep		134 24		136 21
T Test: T=		1.89 not significant		2.66 $p<0.01$

* Exam result marked out of 200.

indicate that students with a cohesive conception of mathematics and/or able to adopt a deep approach to study tended to achieve at a higher level.

CONCLUSION

The results show a range of beginning university students' conceptions of mathematics. A large proportion of the sample of successful school leavers studied, conceive of mathematics as numbers, rules and formulae which can be applied to solve problems. In our sample, these views of mathematics are associated with surface approaches to studying mathematics. There are indications that a more cohesive conception of mathematics and/or a deep approach to studying mathematics are positively and cumulatively associated with achievement at university level. The observations on conceptions of mathematics and approaches to learning mathematics at university level raise questions about the ways in which mathematical thinking is nurtured and assessed at university level. These are issues being explored in an ongoing study.

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