Pedagogy by my Standards: A Teacher's Views on Two Process Standards

Jaguthsing Dindyal
National Institute of Education, Singapore
<jdindyal@nie.edu.sg>

This study focused on how a high school mathematics teacher in the United States interpreted and used the NCTM standards: *communication* and *reasoning and proof* in class. The Algebra I class of a grade 10 teacher was observed for one week and after each observation he was interviewed. This teacher valued mental computation, drill and practice, and the use of basic facts and rules. The study showed that he used his own standards based on the wisdom of his experience.

The reform movement marked by the National Council of Teachers of Mathematics (NCTM) standards (NCTM, 1989, 2000) placed a significant emphasis on processes besides on specific content strands. For many teachers, adopting the new standards implied a considerable change in their pedagogical styles. Pedagogy based on the standards has been termed standards-like pedagogy which Muckerheide (2001) described as having the following characteristics: (1) It makes use of student knowledge to inform instruction and it recognizes mathematical knowledge to be constructed from within the student but grounded in the social context of the classroom. To this end, a standards-like pedagogy encourages classroom discourses, non-routine problem solving, student autonomy, and conceptual understanding; (2) It considers the present and future quantitative needs of the child. To this end, a standards-like pedagogy recognizes and seeks to impart in the child a belief that mathematics is vibrant and growing; (3) It considers the quantitative needs of the student in the twenty-first century and so increasingly makes use of sophisticated technologies in instruction; and (4) It emphasizes the need for continual and diverse student assessments of what the student understands and how the student thinks.

However, teachers have their own knowledge, skills and values that shape their teaching in the classroom. No two teachers employ exactly the same strategies for teaching. It is important to find out how teachers actually interpret and use the standards in their daily lessons. Accordingly, this study focused on the strategies used by an experienced high school teacher to attain two of the process standards highlighted in the NCTM (2000) standards document: *communication* and *reasoning and proof*. Focusing on only two of the standards limits the size of the study but still gives an overall perspective of how the teacher uses and interprets the standards. The specific research questions were: How does the teacher interpret and use the two process standards: *communication* and *reasoning and proof* in class? Which strategies does the teacher use to attain those process standards?

Communication

Communication is not necessarily verbal only; it can take various forms (see Pirie, 1998). Communication needs to be nurtured in the classroom and the teacher has an important role in monitoring how communication takes place in the classroom. Students

can gain a lot from the way communication takes place in the classroom. The NCTM document (NCTM, 2000) places a very high stake of the process of communication in the classroom.

> Through communication, ideas become objects of reflection, refinement, discussion and amendment. The communication process also helps build meaning and permanence for ideas and makes them public. When students are challenged to think and reason about mathematics and to communicate the results of their thinking to others orally or in writing, they learn to be clear and convincing. Listening to others give students opportunities to develop their own understandings. (p. 60)

The NCTM document highlights three major roles for the classroom teacher as far as the communication standard is concerned: (1) Establishing norms within a classroom learning community that support the learning of all students; (2) selecting worthwhile mathematical tasks that allow significant communication to occur; and (3) guiding classroom discussion on basis of what is learned by monitoring students' learning (p. 270).

Reasoning and Proof

Reasoning and proof also play an important role in mathematics learning. The emphasis on the reasoning and proof standard will depend on the importance a teacher attaches to this standard. The goal of emphasizing reasoning in the teaching of mathematics is to empower students to reach conclusions and justify statements on their own rather than rely solely on the authority of a teacher or a textbook (NCTM, 1991). The NCTM (2000) document asserts that mathematical reasoning and proof offer powerful ways of developing and expressing insights about a wide range of phenomena and that the instructional programs for the reasoning and proof standard enable all students from pre-kindergarten through grade 12 to recognize reasoning and proof as fundamental aspects of mathematics; to make and investigate mathematical conjectures; to develop and evaluate mathematical arguments and proofs; and to select and use various types of reasoning and methods of proofs. Reasoning and proof should be a consistent part of students' mathematical experience. Even though the idea of a proof can be thought of as an advanced form of mathematical thinking yet this thinking can be developed in some logical way at all levels.

Methodology

In this study, a grade 10 mathematics teacher in a Midwestern high school in the United States was selected. The teacher called Mr. M. Green (pseudonym) in this study had been teaching mathematics for the past 16 years at High School level in the same school since his graduation. Mr. Green is highly experienced having taught most of the courses at High School level except calculus courses. Mr. Green's Algebra I class had 25 students of whom 11 were girls. All students in the class had studied geometry in their freshman year of high school. The class was observed five times during a one-week period on five consecutive days, from Monday to Friday. This ensured that certain classroom routines would be easily identified in a series of consecutive lessons. Each class was of one-hour duration. During the week in which this study was carried out Mr. Green actually taught the topic *factorization* and *solution of polynomial equations*. On the Friday of the week, he gave a test. After each class he was interviewed for about 20 minutes. Each of the five interviews were audiotape-recorded and then transcribed. Classroom observation focused on how the teacher used the two process standards of communication and proof in the classroom. Regarding the communication standard, the observation focused on how he communicated

with the students, how he guided them on tasks, how he asked questions, elicited responses, gave hints and cues, encouraged group work, gave feedback on their written work and allowed them to critically appraise each others work. Regarding the reasoning and proof standard, the classroom observation focused particularly on how he explored the reasoning behind particular solutions that the students came up with, their understandings of particular topics, how he looked for justifications in their work, how he allowed them to form conjectures and explore those conjectures and how he justified the steps that he himself used in the classroom in particular topics. While the grade 10 class of Mr. Green was observed, the students from the class were themselves not actually part of the study, as the research focused on the teacher.

The Interviews

Mr. Green has been teaching mathematics for the past 16 years during which time he has witnessed many changes taking place in the mathematics he taught at school. He is certainly aware of the NCTM standards, but he uses them differently.

> MG: I have used them. I do not rely on the standards to create my daily lessons. We did however over the past couple of years do an extensive overhaul of the math curriculum. Then we used the standards extensively in our decisions of what courses and how we want to shape our courses or what topics we would include or downplay which courses… In a day-to-day sort of thing the standards are not primarily important in my planning but in the overall scheme of how we lay out our courses and how we select content topics for the courses they play a very big role.

So, for Mr. Green the standards do not play a big role in his day-to-day planning; rather, it has an important role in the overall departmental planning for the mathematics courses. I asked him about the standards that he felt was most important at the level he teaches. He had a prompt answer to this and clearly has strong reservations against the standards.

> MG: See, I am still very old type, still old-fashioned in my belief. I want to get my students to do mathematics and get along with the program. That is not good for being old-fashioned but I still believe that we need more drill and practice. We need more skill development...Right now my goal is to make sure that the students leave the class with a very strong background in the skills… but my gut feeling is that if they can't do the basic skills then I think we are doing them a disservice.

Mr. Green says he is old-fashioned and he believes in drill and practice, a fact confirmed by the observations of his teaching in the class. He sees a threat in the standards in the form of slowing down his progress for he wants to get ahead with the program and also a threat in the use of technology, for he values mental practice. The mastery of the basic rules is so important for Mr. Green. He feels that his students cannot move ahead if they have not mastered the rules at a given level. He does not find the standards as relevant to his daily practice as he sees many shortcomings in them, as claimed below.

> MG: I would have liked them to be more specific. Because they are written on a national level … because they are written in academia, sometimes they are very vague and sometimes they are very obvious. When the first standards came out, they had a section on here is the items we think that you should emphasize and here is the items we think you should de-emphasize. They took that out of the new standards because too many people took that literally and I think that's the kind of thing that was useful…I wish they would be more specific, I wish they would be more teacher-friendly. I haven't spent a lot of time on the new standards …

I asked Mr. Green if he had used the standards to teach what he would have done differently. He responded that he would have used more problem solving and technology.

This gives a background of Mr. Green and his beliefs about the standards and their use in the teaching of mathematics. It is important to consider this background as they influence his views on communication and reasoning and proof. Regarding communication, he adds:

> My communication with them, I feel very comfortable with. Their ability to communicate with me mathematically is something that we have been working very hard on improving. Today their ability to coming forward and at least to show their work is a great step for them and now the next step for me is trying to able to get them to explain their own work. And again if the students are not comfortable with the skills then it is harder for them to communicate. That is an area where I want my students to improve to communicate mathematically both in written and verbal forms.

Mr. Green acknowledges the importance of communication in the classroom. His students are not very good at it but he says that they are improving. He also mentions that he uses some group work and discussions. What is significant about that is the fact that Mr. Green again ties the communication aspect to the idea of mathematics being a skill-based discipline: "They need to develop the skills before they are able to discuss and pursue those themselves." He values drill and practice above all, so he does not see anybody in his class being good at communication if that person has no mastery of the skills. The strategies he uses to improve their communication skills has more to do with the improvement of their verbal communication. He mentions in the interview that his students have difficulties in explaining their own work which is an area he is working on. Regarding shortcomings in the students' communication skills, he says:

> I still think their ability to communicate mathematics in a formal sense. I think they can do it in a general sense and they can do it in an informal sense but it is a difficult skill to get them to communicate mathematics in a formal sense. … paying attention to the details.

He mentions in the interviews that his students are not very good at using symbols but they are fairly good at using graphs, charts and diagrams, which is probably because they find these everywhere in magazines, papers etc. Regarding the students' written work, Mr. Green wants them to show their work, but he is very specific about the answer.

> MG: I try to get them to identify the problem they are working with. To show some sort of work and make sure that they clearly label and identify their work …and final answer... I also stress that they answer the problem the way in which it was asked. That if a problem contains fractions they won't give me a decimal answer and if the problem was asked in sentence form then your answer should be in sentence form.

We can note the emphasis of Mr. Green on the nature of the final answer. He does not speak about the importance of showing the steps and justification of those steps or that the steps might be more important than the final answer. Given his emphasis on the rules and correct answer this is somewhat expected. When asked about whether he allows the students to comment on each other's work, he says that he does encourage them to do so, particularly to discuss and compare their answers with those of their friends sitting around them. The students that Mr. Green now has in his class have studied geometry the year before and as such they have a background on the idea of proof in mathematics. I asked him about the reasoning and proof skills of his students.

> MG: Their reasoning is improving. The group of students that I am working with happen to be a little bit better because we have taught to them geometry before algebra.

Mr. Green is clearly aware of the importance of reasoning and proof in the curriculum and he acknowledges the importance of having geometry before algebra in the school. This decision is based on the fact the overall planning for the courses at the school is based on

the standards. I asked him about the strengths and weaknesses in the reasoning skills of his students.

> MG: I think one of their biggest weaknesses right now is their ability to see a problem through. They become so programmed that they want to have it at the snap of their fingers. They are not willing to spend the time required … to make a mistake and then try again… Then that desire to stick with a problem and that mental toughness to work through to the end of a problem is something we need to build back in our students.

He claims that: "They are not too bad." about the students justifying their steps. This is somewhat contradictory to the fact that the students have difficulty in showing their steps in their work, which he earlier mentioned about their communication skills. It is interesting to note what he says about what he does to improve the reasoning skills of his students.

> MG: In my algebra class I try to be skill-based… but their reasoning skills… boy… I think one of the biggest things is to try to push some of the mental calculations, because you can start to do some of that in your mind and you'll a bit more comfortable taking it to the next level in trying to think through a problem.

Once again, Mr. Green again connects the idea of reasoning to mental calculations, drill and practice and learning of rules about basic facts, which play a very important role in his philosophy of teaching mathematics. This is certainly related to what he values the most in mathematics.

> MG: I value my ability to mentally calculate… it allows me to problem solve at my feet…. I try to turn it into a game where other people might struggle with …it becomes a challenge a game…I am proud that … that's why I try to push a lot of my kids to that mental calculation.

Classroom Observation

In the first class that I observed Mr. Green had all the steps for factorization of polynomials written down on the board. The students had to copy those and work on some exercises that he had copied on the board. Mr. Green kept on emphasizing the rules even when the students were on task - "Remember four terms tell us to look at factorization by grouping" and "Difference of two squares give conjugate binomials (this plus that)(this minus that)", also "Look for a common factor … monomials cannot be factored… factor completely". His questions to the class were short factual questions. They did not emphasize any higher order thinking on the part of the students. No group work was assigned, nor did the students work in pairs. The students did not ask any questions. Two students were finally sent to the board, but they simply wrote down the answers. The written work of the students was not checked and the teacher had no way of knowing about the students' difficulties. There were no conjectures and checks of solutions.

In the second class, the teacher went around checking the work of the students. He made a note of those who had and those who had not done their homework. He looked simply at what the students had scribbled and not really at the solutions. He then wrote the answers to those questions in which some students said they had problems. Without considering the answers of those students who might have done it well, he simply wrote down the answers on the board. Again, the solutions were not checked. He wrote a few more exercises on factorization on the board and asked four students to solve them on the board. Without even considering the answers of the other students in class he mentioned, "I believe those four answers to be correct". He then proceeded to the zero property and explained how to solve polynomial equations. He discussed two solutions on the board and

wrote four problems to be solved on the board. At this point one cannot be sure whether the students had a good grasp of 'solving an equation'. There were no questions to the class. He told the students to look for what they had been doing in class for the past two weeks as a hint to solving the equations. He then wrote two further problems on the board in which the terms did not equate to zero. He then quickly proceeded to solve the problems without asking any student in the class for the solutions. He set problems 1-3 and the *evens* from 14-30 as homework questions from the prescribed textbook. Up to this point there were still no questions to the teacher from the class. The students were not given the opportunity to discuss solutions in the class or to clarify a point or clear up misunderstandings. The reasoning skills of students were not specifically attended to as the teaching was closed and rigid.

The third class was very much similar to the previous two classes that I had observed. The class started with a check of the homework and more problems to solve. Mr. Green has the habit of asking "No questions?" and generally the students do not respond. He again emphasized the use of rules in this class. For example for the problem $(x+8)(x+1)= -12$. He quickly said "It is not equal to zero so use the FOIL property on the left.", implying that the students expand the brackets. He gave a worksheet to the students and asked them to work in groups. He then left the class for about 15 minutes. There were practically no questions from the students to the teacher. The written work of the students merely contained the answers. The classes seemed to have a rigid format and the students were skilfully led by the teacher to follow that format. Students' responses were not highlighted.

The fourth class was no different in format. Again there was a homework check and then Mr. Green read out the solutions to the homework problems. He moved on to the word problem on the worksheet and assumed that it had to be explained. He did not ask if there were any correct solutions for that problem. He briefly explained about a parabola being symmetrical and is a representation of a quadratic, which was done very quickly. It was difficult for the students to grasp the main idea. He then circulated two sheets, one was a sheet with multiple-choice items that he gave as extra credit and the other was a regular worksheet to get them prepared for the test the next day. Most of the students simply ignored the extra credit work. The students still did not ask questions to the teacher.

The fifth class was devoted entirely to the test. The students had to work individually on the test. The items included factorization and solving of equations problems. There were a few word problems.

Discussion

Mr. Green is no doubt a very experienced teacher of mathematics. He has his own views about the NCTM standards, which he says he does not use in his day-to-day planning. However, he has at heart the achievement of his students in mathematics. He mentions the use of technology as a weakness in his teaching but otherwise he is confident that he is doing a good job.

Mr. Green views communication differently from what the standards suggest for instruction in mathematics at this level. The standards (NCTM, 2000) suggests that teachers can use oral and written communication in mathematics to give students opportunities to: (1) think through problems; (2) formulate explanations; (3) try out new vocabulary or notation; (4) experiment with forms of argumentation; (5) justify conjectures; (6) critique justifications; and (7) reflect on their own understandings and on the ideas of others. Mr. Green generally did not give students enough time to think through

the problems and work on them. All students had to finish at the same time and work at the same pace in his class. The correction of homework was done very quickly with little opportunity to students for formulating explanations. The questions to the class were factual and did not really challenge the students' thinking. Given that the class was very structured, students' autonomy was limited.

There were no new vocabulary terms or notations introduced in the classes that I visited. The students did not experiment with any form of argumentation, justification of conjectures or critiques of justifications. Martino and Maher (1999) have claimed that movement from a classroom that is teacher-centred to one that is student-centred suggests a critical and central role for the classroom teacher. The classroom of Mr. Green is still very teacher-centred. There is predominance of teacher talk in the class. The students merely reacted to the teacher's requests. They practically never asked questions or sought further explanations. The tasks that he set were pretty closed, like working on selected problems from a worksheet.

Mr. Green values reasoning and proof but his students do seem to be having some difficulties on this area. The students did not seem to feel comfortable to discuss, question or listen to other students in their class. "Teachers should expect their students to seek, formulate, and critique explanations so that classes become communities of inquiry" (NCTM, 2000, p. 346). The students were very passive and so they missed out on the opportunity to get at the very heart of the reasoning and proof standard. Mr. Green's classroom practice is strongly tied to his beliefs about the nature of mathematics, mathematics teaching and the learning of mathematics. He strongly believes that mathematics is a skill-based discipline and so drill and practice is very important in teaching mathematics. Accordingly, learning mathematics involves grasping the basic facts and rules. Thus, Mr. Green has an instrumentalist view of teaching. Thompson (1992) defines this view as one in which the content is organized according to a hierarchy of skills and concepts; it is presented sequentially to the whole class, to small groups, or to an individual, following pre-assessment of students' mastery of prerequisite skills. Mr. Green has an absolutist philosophy of mathematics, which Ernest (1998) defines as a view that mathematical truth is absolutely valid and thus infallible. From this view, clearly Mr. Green is emphasizing instrumental understanding of mathematics (Skemp, 1987). The reasons why he does that can be related to his deep-rooted beliefs about what is mathematics and how it should be learned. Regarding a standards-like pedagogy, as suggested by Muckerheide (2001), Mr. Green clearly does not demonstrate such a pedagogical approach in his teaching. Student autonomy and classroom discourse in which students are involved is limited in his classroom. Classroom observations show that he mostly uses routine problems and students show a lack of conceptual understanding. As for technology, the calculator is used for routine calculations but not as a tool developing some higher skills. As mentioned previously answers are not checked. The assessment tasks that he uses are very traditional and lack diversity. As such, his teaching cannot be categorized as being standards-like.

Conclusion

While the NCTM standards are rooted in a constructivist philosophy of teaching and learning mathematics, Mr. Green's overall philosophy has a very absolutist perspective. Is Mr. Green doing the right thing? This is a value judgment that will be difficult to pass on the work of Mr. Green as the standards are also value-based (see Hiebert, 1999). He is

committed to the welfare and progress of his "kids" but he is positive that his methods are better than those proposed by the NCTM. He admits that his own philosophy of teaching is deeply rooted in his own beliefs about the nature of mathematics and the way the subject has to be taught and should be learned.

Why does Mr. Green choose to ignore the NCTM standards? He says they are too broad and not specific enough and also that they make students become too technology dependent. Mr. Green cannot be forced to change his teaching style but he can certainly be convinced that there is value in what the NCTM standards proposes for mathematics instruction in the school. As long as the core beliefs of teachers like Mr. Green do not change, there is little hope that the reform mathematics curriculum will be successfully implemented. Hiebert (1999) argued that one of the reasons why many alternative programs are not implemented effectively in schools is because it is difficult to change the way we teach. He added that just as students, teachers require an opportunity to learn, but that most teachers have very few such opportunities.

This study is based on snapshots from a particular teacher's classroom and his general philosophy of teaching and learning mathematics. I cannot claim that I have captured every aspect of the classroom practice of Mr. Green nor do I claim that this study can be generalized to other settings. However, there are some implications for practice at large. Also, given the constraints of time to complete the study, it focused only on two of the process standards, namely communication and reasoning and proof. However, there is a general indication of how the teacher attains the other three process standards. Hiebert (1999) concluded that we know now that we can design curriculum and pedagogy to help students meet the ambitious learning goals outlined by the NCTM standards and the issue is whether we value these goals enough to invest in opportunities for teachers to learn to teach in the ways they require to teach.

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