The Role of Cognitive Conflict in Developing Students' Understanding of Chance Measurement

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In individual interviews, twenty students in each of grades 3, 6, and 9 responded to a task involving chance measurement, then viewed video recordings of other students' conflicting responses and decided which response they preferred. Seven students improved their levels of reasoning and seven agreed with higher-level prompts without expressing reasoning. Only two students agreed at some point with lower-level prompts, and both reverted to the level of their original response in conclusion. Educational implications are noted.

Considerable research has investigated student understanding of probability at various ages (e.g., Green, 1983; Watson, Collis, & Moritz, 1997), some involving longitudinal information (e.g., Green, 1991; Watson & Moritz, 1998). Little of this research, however, has investigated what happens when students are exposed to conflicting ideas. This study extends the survey-based research of Watson et al. (1997) and Watson and Moritz (1998) by exploring how students respond (a) in interviews and (b) to conflicting ideas of others.

Conflicting Cognitions

Cognitive conflict has been discussed in education circles for quite some time. Piagetian theory describes the integration of new experiences into old conceptual categories as *assimilation*, whereas an experience that conflicts with one's concepts may prompt one to revise these concepts to *accommodate* the experience. As well for student learning to occur, there must be dissatisfaction with the student's existing conceptions, and a new competing conception must be intelligible and appear initially plausible (Strike & Posner, 1992).

Group-work is often advocated as an appropriate setting for learning by exposure to other students' ideas that are conflicting, intelligible, and without the authority status of a teacher. To emphasise conflict, some mathematics educators (e.g., Swedosh & Clark, 1998) have recommended that students should first clearly express their own ideas, and then be exposed to cases where these conceptions fail. To promote intelligibility of new ideas, theorists, such as Vygotsky, have emphasised social aspects of learning, suggesting that students are ideally grouped with others who have slightly higher-level ideas that are intelligible; the range of intelligible ideas is described as the *Zone of Proximal Development* (Goos, 2000). Some researchers have explored the resolution of student uncertainty, acknowledging different authorities to which students can appeal in reaching a resolution (e.g., prior experience, empirical data, a knowledgeable person, or a text) (Clarke & Helme, 1997) and different meta-cognitive activities that assess whether the uncertainty has been resolved satisfactorily where no clear authority is available (Goos, 1998). Considerable research in statistics education has asked students to state their ideas, but little has then provided students with conflicting views of others and asked them to decide their preferred response, ensuring consistency in aspects of "the other" (e.g., familiarity with the person, gender, and age) and "the other's idea" (e.g., terminology).

Previous Research on Students' Strategies for Comparing Chances

The task reported here involved comparing the chances of a particular outcome in two situations, as shown in Figure 1. Similar tasks used by researchers have been found to give considerable discrimination of students' understanding of chance measurement. Green (1983), for example, found consistent usage of four strategies as the basis for a choice of box: (a) number of marbles, (b) number of blue marbles, (c) difference of numbers of blue and red marbles, and (d) ratio of blue and red marbles. Green (1991) reported on a longitudinal study of 305 students responding to ten similar questions involving various ratios. For tasks where the ratios were the same but not 1:1, recognition that two boxes with equal ratios gave the same chance improved over a four year interval, from about 10% of upper primary school students to 30% in the first three years of high school. For tasks where the ratios were 1:1, recognition improved over the four year interval from about 25% of upper primary school students to 60%.

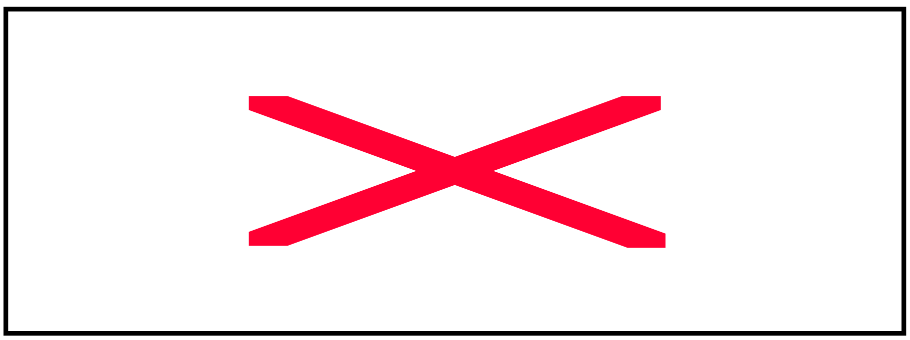

Figure 1. Chance measurement task (from Watson, Collis, & Moritz, 1997; Watson & Moritz, 1998).

Watson et al. (1997) surveyed more than 1000 students in grades 3, 6, and 9 using the item in Figure 1. Some responses were eliminated from analyses, for selecting a multiple-choice option with no reason or for repeating one of the options as the reason. They categorised remaining responses into seven levels according to the *Structure of Observed Learning Outcomes* taxonomy (Biggs & Collis, 1982). At the *Ikonic level* (IK) were responses that indicated non-mathematical beliefs, for example, "(=) Red and blue are both my best colours". The remaining six levels were described in two cycles of Unistructural-Multistructural-Relational (U-M-R) response levels. The first cycle involved use of the actual numbers found in the boxes.

- U_1 Responses indicate basic uncertainty or possible outcomes, but include no measurement aspect. An example is "(=) you could get red or blue".
- M_1 Responses qualify chance, indicating which outcome is more likely; a simple numerical comparison may occur. Unresolved conflict may occur between the disjoint aspects. Examples include "(B) you would have more marbles to pick from", "(A) less red", and "(B) you have more of a chance with 40 than 4".
- R_1 Responses involve all relevant numbers as absolute values rather than as proportions. Responses may indicate a qualitative judgement, such as "(=) there are more reds in both boxes", or a numerical judgement using subtraction, such as "(A) there are only 2 more reds in A and 20 more in B".

A second U-M-R cycle involved the use of proportional reasoning to establish the equality of the chances for the boxes, with increasing degrees of justification. Examples include at the U_2 level “(=) they both have the same chance” and “(=) they are filled alike”, at the M_2 level “(=) there are just ten times the same amount of each in Box B”, and at the R_2 level “(=) there are 40% blue in each box” and “(=) the factor of both boxes are blue over red which is equal to Box A = 4/6 = 2/3 and Box B = 40/60 = 2/3”.

Watson et al. (1997) found that, of responses analysed, the M_1 level was common for students in grade 3 (69%) and grade 6 (41%). Responses in the second U-M-R cycle, involving proportional reasoning, increased with grade, from 4% of third-grade students, to 27% of sixth-grade students, and 56% of ninth-grade students. In a longitudinal follow-up study, Watson and Moritz (1998) found many students re-surveyed two- or four-years subsequently improved their levels of reasoning, providing further evidence that the response levels identified constituted a useful model to describe cognitive development over the school years. Moritz (1998) observed a tenth-grade student express conflicting ideas at different levels in an interview setting, and described a method of using video of conflicting ideas of other students in a structured interview to prompt cognitive conflict.

The Current Study

This study aimed to explore how students respond to the task in Figure 1 in an interview setting compared to those administered the task in written surveys (Watson et al., 1997; Watson & Moritz, 1998). The U_2, M_2, and R_2 levels were combined into a single level (P_2), because all of these responses acknowledge the *proportions* are the same for both boxes. The level of justification given for a proportional argument is of less interest in this study as prompting with other students’ ideas meant justifications were not a strong response feature when students might simply agree with the ideas of others or might give multiple expressions of justification all based on proportional reasoning.

This study also aimed to explore how students respond after listening to conflicting ideas of other students. Videos and transcripts of interviews allowed us to review dialogue from a number of perspectives (Lesh & Lehrer, 2000), such as students’ developing mathematical cognitions and also their psychological tendencies in resolving conflicting cognitions. Structured interviews involving prompting (Moritz, 1998) ensured conflicting cognitions were expressed within moments after the students’ responses. Higher- and lower-level prompts could be given systematically to explore students’ abilities to make sense of higher-level ideas, and their susceptibility to lower-level ideas. It was also possible to observe students resolving uncertainty caused by conflicting ideas of other students that do not have the authority status of those of a teacher, and their tendencies for tenacity of their own ideas.

Method

The second author conducted individual interviews with 60 students from four Tasmanian government schools: 10 third- and sixth-grade students from each of two primary schools, and 10 ninth-grade students from each of two secondary schools. Students were selected on the basis that they would be willing to talk in interview and not be threatened by the complexity of listening to other students’ ideas and evaluating them. Hence it might be expected that the students interviewed were more willing and able than would normally be expected for their grade levels.

Students were administered the task shown in Figure 1, presented to them on paper and read by the interviewer. Interviews were during class-time (40 mins.), and the task was the first of five or six protocols involving cognitive conflict; other topics were average, comparing graphs, sampling, and dice outcomes. After the student had responded to the task, one or more video prompts (5-30 seconds duration) were shown using a laptop computer. A hyper-text linked transcript of the prompt was also on screen; hence the prompt could be read during or after viewing the video. Transcripts of the prompts are shown in Figure 2. The prompts were gathered from interviews in a previous study of students' understanding of chance and data. The same tenth-grade girl spoke the prompts of Patti (R_1 level), Patricia (M_1), and Pip (P_2). The prompt of Penni (P_2) was also a tenth-grade girl. Sometimes the interviewer emphasised parts of the prompt: for Patti's prompt, the phrase "but in Box B there is 20 more red" that was not said by Patti; for Patricia's, "more blues"; for Pip's, the phrase "same ratio"; and for Penni's, "timesed it by 10". After each prompt the interviewer asked, "What do you think of her idea?"

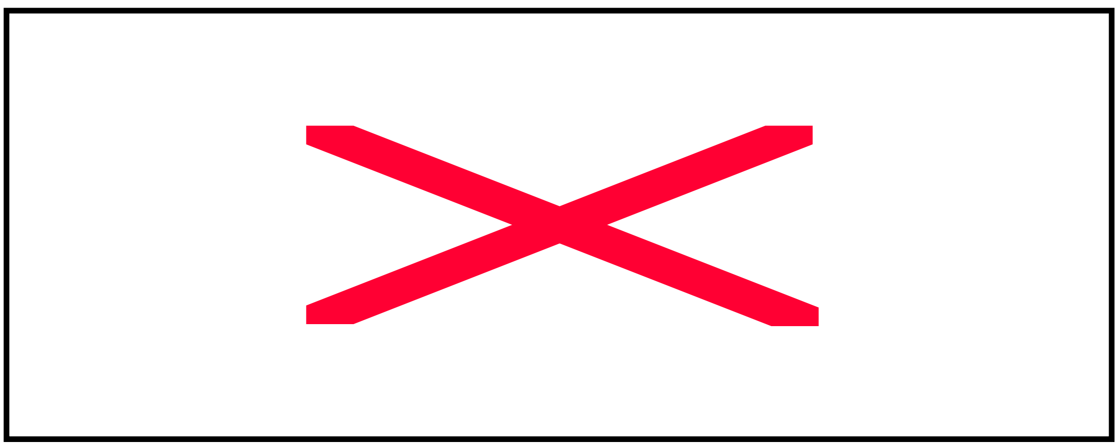

Figure 2. Prompts displayed to students to promote cognitive conflict.

Prompts were shown at the interviewer's discretion, depending on the student's initial response, in an attempt to create cognitive conflict. Students who initially responded at the P_2 level (28) were first shown Patricia's (M_1) prompt, with 1 exception,[1] and only 5 were shown other prompts: A third-grade student was also shown Patti's (R_1) prompt then Penni's (P_2) prompt, a sixth-grade student was also shown Penni's (P_2) prompt, 2 sixth-grade students were also shown Patti's (R_1) prompt, and a ninth-grade student was only shown Pip's (P_2) prompt. Students who initially responded below the P_2 level were shown a P_2 level prompt, with the exception of 3 students.[2,3] Penni's (P_2) prompt was used with all third-grade students and with sixth-grade students who responded at the R_1 level except 2. Pip's (P_2) prompt involved a simpler sentence than Penni's but a more complex word,

[1] Before prompting, one ninth-grade student briefly expressed preference for (B), (A), and then (=) at the P_2 level, and because she seemed to have already expressed a number of ideas, she was shown only Pip's (P_2) prompt, as a means of summing up the relationship in the term "ratio", previously unused by the student.

[2] Two sixth-grade students who responded at the R_1 level that both boxes were equally likely were shown only Patricia's (M_1) prompt. They did not change when prompted.

[3] One ninth-grade student who responded at the M_1 level, preferring Box A, was shown only Patricia's (M_1) prompt, and he then responded at the R_1 level, preferring Box A.

"ratio", and was used with sixth- and ninth-grade students who responded at the U_1 or M_1 levels with 1 exception. For all 16 students who initially responded (=) at the U_1 level, (A) at the M_1 level, or (A) or (=) at the R_1 level, Patricia's (M_1) prompt was also used, usually (14 of 16 interviews) prior to the P_2 prompt. Patti's (R_1) prompt was also used with 6 students who initially responded at the M_1 level.

Each student's responses before and after prompting were assigned to a level according to those of Watson et al. (1997), collapsing responses in the second U-M-R cycle to the level of P_2. Responses were also assigned one of three codes for change in response: *agreement* with a prompt, also sub-coded as either *passive* (without clarifying reasoning) or *active* (paraphrased reasoning); *other*, also sub-coded as either *new idea* (idea expressed not given in the prompt) or *reversion* (agreement with two prompts resulting in return to the student's original idea); and *no change*, rejecting ideas in prompts.

Results

The numbers of students in each grade changing response after prompting are shown in Table 1 by initial response level. For initial responses, 19 third-grade students responded below the P_2 level, and 1 at the P_2 level, whereas 9 sixth-grade students responded below the P_2 level and 11 at the P_2 level, and 4 ninth-grade students responded at the M_1 level and 16 at the P_2 level. It was also interesting to note that for students who responded at the M_1 level, all third-grade students chose Box B, whereas 2 of the 3 sixth-grade students chose Box B, and all 4 ninth-grade students chose Box A. Of the 28 students who responded at the P_2 level, none changed their minds to lower levels, though some reformulated a numerical justification at the P_2 level. Of 32 students who responded below the P_2 level, 16 did not alter their responses, whereas 16 changed their responses. Of these 16, 14 improved (10 to the P_2 level, 3 to the R_1 level, 1 to the M_1 level), often by passive agreement, and 2 changed their minds but then reverted back to their original response.

Table 1
Frequency of Response Change by Grade and Initial Response Level

Response change	Grade and initial response level									
	3 (*n* = 20)				6 (*n* = 20)				9 (*n* = 20)	
	IK	M_1	R_1	P_2	U_1	M_1	R_1	P_2	M_1	P_2
Agreement (Active/Passive)	0	6	0	0	1	2	1	0	0	0
Other (New Idea/Reversion)	0	2	0	0	0	1	1	0	2	0
No change	1	7	3	1	0	0	3	11	2	16

The dialogue for the 16 students who changed their responses after prompting is summarised in Table 2. Many third-grade students who chose Box B at the M_1 level passively agreed with a P_2 level prompt, whereas three sixth- and ninth-grade students who chose Box A at the M_1 level actively responded at higher levels after a prompt.

Table 2
Individual Student Response Changes Sorted by Grade, Initial Response, and First Prompt

Grade	Initial response	First prompt	First change	Second prompt	Second change	Final level	Overall change
3	M_1 (B)	Patti (R_1)	Agree (P)	Penni (P_2)	No change	R_1 (A)	Agree (P)
3	M_1 (B)	Patti (R_1)	Agree (P)	Penni (P_2)	Agree (P)	P_2 (=)	Agree (P)
3	M_1 (B)	Penni (P_2)	Agree (P)	Patricia (M_1)	No change	P_2 (=)	Agree (P)
3	M_1 (B)	Penni (P_2)	Agree (P)	Patricia (M_1)	No change	P_2 (=)	Agree (P)
3	M_1 (B)	Penni (P_2)	Agree (P)	-	-	P_2 (=)	Agree (P)
3	M_1 (B)	Penni (P_2)	Agree (A)	-	-	P_2 (=)	Agree (A)
3	M_1 (B)	Penni (P_2)	Agree (P)	Patricia (M_1)	Agree (P)	M_1 (B)	Other (R)
3	M_1 (B)	Penni (P_2)	Agree (P)	Patti (R_1)	Other (NI)	R_1 (=)	Other (NI)
6	U_1 (=)	Patricia (M_1)	Agree (P)	Pip (P_2)	No change	M_1 (B)	Agree (P)
6	M_1 (A)	Pip (P_2)	Agree (A)	Patricia (M_1)	No change	P_2 (=)	Agree (A)
6	M_1 (B)	Pip (P_2)	Agree (A)	-	-	P_2 (=)	Agree (A)
6	M_1 (B)	Pip (P_2)	Agree (P)	Patti (R_1)	Other (NI)	P_2 (=)	Other (NI)
6	R_1 (A)	Patricia (M_1)	No change	Penni (P_2)	Agree (P)	P_2 (=)	Agree (P)
6	R_1 (=)	Patricia (M_1)	Agree (P)	Penni (P_2)	Agree (P)	R_1 (=)	Other (R)
9	M_1 (A)	Patricia (M_1)	Other (NI)	-	-	R_1 (A)	Other (NI)
9	M_1 (A)	Patricia (M_1)	Other (NI)	Pip (P_2)	No change	P_2 (=)	Other (NI)

Note. Abbreviations: Agree (P) [Passive] and (A) [Active]; Other (NI) [New Idea] and (R) [Reversion].

Passive agreement was observed when students acknowledged agreement with another student's idea (e.g., "a good idea"), but made no attempt to express the idea in their own words. One third-grade student changed response twice, from the M_1- to the R_1- to the P_2-level, however it is not clear whether there was reasoning to accompany this change.

S1 I think Box B... because there's more blues in Box B and there's 4 in Box A. [Video Prompt: Patti] A good idea. [I: So you think she's probably right that Box A is better?] Yes. [Video Prompt: Penni] Yes. [I: Which do you think is the best idea?] The girl who said it doesn't really matter. [Grade 3]

Active agreement was observed when students agreed with the other student's idea and expressed the idea in their own words. One sixth-grade student appeared to have access to a tentative proportional idea before prompting, but only after prompting did she prefer it.

S2 I would choose Box A. [I: Why?] I don't know [...] it is probably because there's less that may be go wrong less times. [I: So that there's less marbles overall?] Yes. [I: Or less reds?] Yes. [I: Does that matter?] Not really. Because compared to these two really the amount of reds is similar compared to the amount of blues in the boxes as well. I find having less marbles in the boxes to be less intimating I suppose. [I: So you think you would be better off picking from Box A?] Yes. [Video Prompt: Pip] Yes I suppose that that does actually work. [pause] Yes, it does work. It probably is actually a better concept if you are thinking about a mathematical concept. If it was me, even though that it doesn't matter which box I choose from, I personally choose from [Box A]. [I: So because it doesn't matter, you feel free to choose either?] Yes. [Video Prompt: Patricia] Yes, it is probably ... I reckon it is just as good as my idea because I said Box A... but then it really doesn't matter, but it really depends on how she will feel if she was doing the thing. [Grade 6]

S2 did not seem to consider the proportional idea dominant for making a decision, rather choosing Box A because of a psychological preference ("less intimidating"). After prompting, the student appreciated the value of the proportional idea when expressed as a ratio, which had authority status as a "mathematical concept".

Some students improved their response levels but in new ways not directly related to the prompts given. One ninth-grade student chose Box A at the M_1 level, but when prompted with Patricia's (M_1) idea for Box B, developed her own ratio idea at the P_2 level.

> S3 Box A because there's less in it [...] [Video Prompt: Patricia]. I don't know. Well there's not much difference because it is the same ratio, 6 to 4 and 60 to 40. [I: So do you think it doesn't matter or would one of the boxes be better?] I don't really know. I guess it doesn't matter. I don't like chance and data at all. [Grade 9]

One sixth-grade student initially chose (=) at the R_1 level, and passively agreed with a prompt at the M_1 level, but then reverted to (=) by passive agreement with a P_2 prompt.

> S4 It doesn't matter. [I: Why?] There's 40 blue in Box B and there's 60 red, and there's 6 red and 4 blue in Box A. So basically there's more red anyway, so it is not going to matter. [Video Prompt: Patricia] Yes, it is a good one. Yes I would go with that. [I: So she made you change your mind?] Yes. [Video Prompt: Penni]. It might be better than the first one. Yes I think it does. [I: Is her reason similar to the reason you were first thinking of?] Yes. [I: That both boxes have more red?] [student nods head] [I: So what do you think... Box B is better or it really doesn't matter?] It doesn't matter. [Grade 6]

Of students who did not change their minds, some expressed interest in the new idea. Others reformulated their original idea, often expanding on a proportional reason, as seen in the following dialogue with a sixth-grade student who responded at the P_2 level.

> S5 In Box B I think that even though there's more you might have a better chance than there [Box A]. But then there's 20 more red [Box B] and there's only 2 more there [Box A]. But I reckon they would probably have the same chance of getting it. [ticks (=)]. [...] Because 6 and 6 and 4 and 4 is sort of what I have done. [Video Prompt: Patricia]. It makes sense more than Box A because [...] you still have the chance ... like if there's 100 [Box B] you have got 60 red, 6 tenths and you did that with 10 [Box A], 6 tenths. 4 tenths [Box A] and that could be 4 tenths as well [Box B]. So it is only like the same [...] you have got the same amount of chance. [Grade 6]

Discussion

Before prompting, proportional reasoning was used by few third-grade students (5%), some sixth-grade students (55%), and most ninth-grade students (80%). Compared to the results of Watson, et al. (1997) and Watson and Moritz (1998), the responses of this study in interviews were similar for third-grade students, but slightly better for sixth- and ninth-grade students. In interviews students were probably more attentive and also, as noted, the sample of students may not be representative. In addition, third-grade students were less articulate at stating arguments, whether written or orally, whereas sixth- and ninth-grade students appeared to benefit from stating arguments orally. The result that, at the M_1 level, third-grade students chose Box B whereas ninth-grade students chose Box A may indicate that many younger students simply reason "how many chances" by attending to numbers of blues. Older students responding at the M_1 level, however, appear to acknowledge that numbers of reds are important in determining the chance of a blue, and these older students are more successful at developing their own proportional ideas when prompted.

When prompted with ideas of other students, improvement in response levels observed in this study (of 32 students who responded below the P_2 level, one-third improved to the P_2 level) within about four minutes was comparable to that observed in four years by Green (1991) and Watson and Moritz (1998). Much of this improvement, however, was by passive agreement, indicating that although some students can appreciate higher level ideas they are exposed to, it is not clear that they consolidate these ideas in their own minds for use in a parallel task some time later, or for transfer to related tasks. Some students understood the prompt and paraphrased it, and others developed new ideas not related to the prompts they were given. These improvements showed promise for this technique involving cognitive conflict, both as a research tool for exploring student reasoning and as a teaching tool for developing reasoning. If students are grouped such that a range of ideas is expressed, the results suggest that students will rarely change their mind to lower levels, whereas many students can appreciate arguments at higher levels, and some consolidate these such that they can express the arguments in their own words.

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