Teaching Elementary Probability: Not Leaving it to Chance

Helen Chick
University of Melbourne
<h.chick@unimelb.edu.au>

Monica Baker
University of Melbourne
<m.baker@unimelb.edu.au>

This paper considers the role of content knowledge and pedagogical content knowledge in the teaching of probability to Grade 5 students. Lessons of two teachers were studied to determine the activities and teaching strategies used to bring out ideas associated with chance, and examine how probability understanding is developed in class. The lessons are shown to be rich in deep probabilistic ideas. The complex interplay between these concepts was sometimes handled well by the teachers, whereas on other occasions gaps in content and pedagogical content knowledge had the potential to cause misconceptions for students.

Since the 1990s much research attention has focused on the pedagogical content knowledge of teachers. At the same time, chance and data have become more significant components of the curriculum, particularly at the primary school level. This report takes a closer look at the way two teachers use both content and pedagogical content knowledge to help Year 5 students develop key concepts in probability.

Background Issues

Pedagogical Content Knowledge and Probability

As is now well known, the idea of pedagogical content knowledge (PCK) was a key tenet of Shulman's 1986 paper on teacher knowledge. This construct, with its focus on knowledge for teaching a particular content domain, is hard to study, not least because it is best exhibited in the dynamic environment of the classroom. It manifests itself in the choices made by teachers about teaching materials and activities, in teachers' responses to student questions, in knowledge of typical student conceptions, in teachers' explanations of concepts, and so on. Another reason that it is difficult to study is because of the degree to which PCK is intimately associated with content knowledge (CK). Marks (1990) illustrates the problems this causes for any attempt to categorise an instance of teacher knowledge as either CK or PCK, and highlights the ambiguities between them.

For the probability content domain, since the place of chance in the curriculum was established only recently and research on students' understanding is still so limited, teachers have had few opportunities to be assisted in their development of appropriate PCK. This is particularly true for primary school, since most research has considered secondary and tertiary level students and probability concepts. Garfield and Ahlgren (1988) provide a comprehensive overview of the issues. They recommend that teachers "introduce topics through activities and simulations, not abstractions; … recognize and confront common errors in students' probabilistic thinking; [and] create situations requiring probabilistic reasoning that correspond to students' views of the world" (p. 48). Garfield and Ahlgren also cite the studies of Green and Fischbein that suggest there can be *declines* in probabilistic understanding over time because of the complex interplay between intuitive understanding and educational experience, implying that experiences that develop appropriate intuitions are particularly important, especially at the elementary level. In order to provide these, teachers require significant content and pedagogical content knowledge.

That this can be problematic is shown in a number of studies (e.g., Carlson & Doerr, 2002; Watson, 2001)

Elementary probability is characterised by limited procedural techniques, yet there are deep conceptual issues to address. Significant among these are the ideas of long-term probability (i.e., probability as a long-term phenomenon), variation and random behaviour, sample space, likelihood, and fairness. Watson (2005) gives an extensive account of issues associated with these and other topics (see also Amir & Williams, 1999). Establishing the idea of probability as something that quantifies likelihood and long-term behaviour requires time, as shown in the extended series of lessons conducted by Brousseau, Brousseau, and Warfield (2002). Teachers may thus view it as a costly procedure given the crowded curriculum (Carlson & Doerr, 2003). The idea of sample space is a central component of quantifying likelihood, and is often a focus of early probability activities. Variation and randomness are central to probability understanding, yet seem to contradict the idea of likelihood, since likelihood quantifies some constant attribute while variation recognises the randomness attached to that constant. As a consequence it can be difficult for students—and teachers—to integrate these two concepts, as shown in many studies (e.g., Watson & Kelly, 2004). Finally, probability is central to understanding about fairness in games, and although students may have an intuitive understanding about this, for some fairness is associated with consistency in behaviour rather than equally likely outcomes (e.g., Watson & Chick, 2001; Watson & Moritz, 2003).

The Present Study

In light of the probability concepts discussed above, and mindful of the challenges inherent in conveying them, it seems timely to look at how teachers use their content knowledge and pedagogical content knowledge in teaching these topics. This paper reports on a study of two teachers who each presented a pair of lessons on chance to their classes of Grade 5 students. It seeks to answer the following questions:

1. What key probability concepts were brought out in the lessons?
2. How did CK and PCK influence the ways in which teaching activities were used?
3. How did CK and PCK influence the teaching of key probability concepts?

Methodology

The present study was part of a larger project examining teachers' PCK, which involved observing and video-taping pairs of lessons for a number of teachers. This report focuses on two Grade 5 teachers, Irene and Greg (names are pseudonyms), who conducted pairs of lessons on chance. The teachers were from the same school and were teaching their own classes; they had also collaborated about some of the things that they planned to do in the lessons. During each lesson, the video-camera followed the teacher, who wore a microphone, and field notes were made by one or both of the authors.

The video data were analysed by the two authors to determine key points at which CK and/or PCK were evident in relation to probability understanding. These instances were grouped using the probability themes above and analysed for commonalities and differences (cf. the method of analytic comparison, Neuman, 2003, p. 456). In particular, the way in which the chosen activities could be and were used to bring out key probabilistic concepts were noted, along with other critical moments.

Results

Despite the fact that only two lessons were observed for each teacher, they were rich in their consideration of chance concepts. We begin by giving an overview of the lesson pairs for each of the teachers, to indicate the chronology of the lessons and the types of activities used. We then look at the key probability concepts and the way in which CK and PCK impact on their development in the hands of the teachers.

Overview of the Lessons

Both teachers involved their students extensively in their lessons. Irene had a very effective and seemingly effortless questioning technique in which she was able to draw out from the students the ideas that she thought were important. Greg used brainstorming and questioning too, but appeared to have to "work harder" to achieve the outcomes he wanted.

Figures 1 and 2 give an overview of the content of the lessons taught by the two teachers, showing the activities used and their approximate duration. Both teachers used the same spinner game and worksheet page, which came from a textbook and its associated teacher resource book (Feely, 2003a, 2003b). The spinner game involved two spinners divided into equal parts, labelled with the numbers 1-9. Students spun both spinners, and if the sum of the two numbers was odd, player 1 won a point, while player 2 won a point if the sum was even. The first to 10 points won the game. Students were instructed to play the game a few times, and then decide if it was a fair game. The "Odds On" worksheet referred to a standard pack of playing cards, asking for the chances of certain outcomes.

Duration	Section	Description
Lesson 1		
10 mins	Brainstorm	Whole class brainstorming session about chance related words and activities in the context of an unlikely weather scenario.
8 mins	Fruit in the hat	Irene puts different kinds of fruit into a hat and discusses likelihood of pulling out certain items, initially using chance words (whole class discussion). Irene introduces writing probability mathematically, using number of desired outcomes and the sample space for fruit in the hat, while linking back to chance words.
35 mins	Coin tossing and spinner game	In pairs, students flip a 20c coin 100 times and record the number of heads and tails. Results are discussed with reference to expectation and variation. Fast finishers start the spinner game.
6 mins	Spinner game discussion	Students begin playing the game when they have finished their 100 coin tosses; later the class discusses whether the game is fair.
Lesson 2		
3 mins	Revision of likelihood	Brief whole class revision of chance terms used in the previous lesson, and of mathematical way of writing down probabilities.
45 mins	"Multo" game	Students write 16 numbers between 0-99 on a 4x4 grid. The times tables from 0x0 to 9x9 had been written on slips of paper and are drawn from a box. The first student with four in a row wins.
15 mins	"Odds On" page	Students work individually while Irene moves around the class.

Figure 1. Overview of Irene's two lessons.

Duration	Section	Description
Lesson 1		
7 mins	Brainstorm	Class brainstorming about chance related words and activities.
15 mins	Least to most likely	Students write down events likely/unlikely to happen before the end of school, and then arrange these from least to most likely.
7 mins	Discussion on 3 spinners	Three spinners, one divided evenly into four colours, the other two with uneven divisions, are discussed in terms of fairness and the likelihood of certain colours being spun.
15 mins	Spinner game	Students play the game, and a discussion follows on the fairness of the game.
6 mins	Game for house points	Numbers 1-9 are drawn, students use these to form two 4-digit numbers. The winner has two numbers with the smallest difference.
Lesson 2		
22 mins	Spinners sample space	Greg guides the class in determining the sample space of the spinner game and the probabilities of even and odd results.
4 mins	Weather probabilities	Discussion of determining probability of certain weather conditions (follows from a student comment in previous lesson).
20 mins	"Odds On" page	Students work individually while Greg moves around the class.

Figure 2. Overview of Greg's two lessons.

CK, PCK, and Chance Concepts

In this section we look at the way probability concepts were developed in the lessons. This discussion is not exhaustive, but is intended to highlight some of the critical events during the lessons where concepts were particularly affected by teachers' CK and/or PCK.

Long-term probability. There were two significant occasions at which the idea of long-term probability was apparent. The first was in Irene's first lesson, in which she devoted half an hour of lesson time to getting pairs of students to toss a coin 100 times. This resulted in 1100 outcomes for the whole class, and was an appropriate activity for talking about "50:50" as the probability of getting heads and for conveying the idea that numerical probability values reflect long-term behaviour. The time spent on this activity, while much less than the time spent drawing samples in the lessons described by Brousseau et al. (2002), was still sufficient for developing these concepts and seemed to reflect PCK about establishing these probabilistic ideas. Indeed, Irene and one of her students had earlier discussed the need for suitable sample sizes in order to establish results. At the end of this activity, unfortunately, Irene placed value on the outcomes that were *exactly* 50:50, and attributed variations from this to flaws in the coins.

The second occasion associated with long-term probability occurred for both teachers, because of the way in which they used the spinner game. A full analysis of this game—which Greg actually did in his second lesson, as described later—shows that the game's bias towards even outcomes is minuscule: 41/81 to 40/81. The students did not play the game long enough for such a margin to be evident, yet in both classes students expressed the belief that the game unfairly favoured even outcomes. This unjustified belief may have been based on the incorrect sample space argument that was proposed by numerous groups: that since odd+odd and even+even both give even sums whereas odd+even does not, then

even is more likely to occur. Having argued this, students may then have been more aware of the even outcomes than the odd ones.

Variation and Random Behaviour. The concept of variation only arose in Irene's lessons. Early in the first lesson Irene asked three students in turn to draw one item from the set of 2 grapes, 2 lemons and 2 apples (with replacement). She elicited from the class the idea of "expecting one of each" type of fruit from the three draws. When the outcome differed from this, with 2 grapes drawn, she highlighted that with chance "we cannot say *for certain* what will happen". This recognition of variation was in contrast to what happened after the coin tossing, where she emphasised that the variations from getting 50 heads and 50 tails with 100 tosses were due to how the coins are tossed and, even more significantly, to the wear and tear on coins.

In her second lesson the students played "Multo" (see Figure 1), and came to realise that choosing numbers with more factors would increase their chances of winning. After a few games Irene asked the class "Is it completely luck whether you win this game or not?" A student responded that it is half luck and half skill, because "you get lucky when you pull out certain numbers, but you can put down numbers with more factors on the grid". One student suggested that if you choose numbers with not many factors then you will not be as successful. Another student suggested that it is almost completely luck. This student explained that although he could increase his chances considerably through his choice of numbers, another student who had not chosen so carefully could still win. This difficulty of reconciling long-term probability and variation was left unresolved.

Sample Space. While conducting the fruit in the hat activity, Irene discussed the sample space—although without using that terminology—and used the concept to develop numerical expressions of probability. This discussion included expressing certainty and impossibility. She also discussed the two options for coin tossing and related this to "half" and "50:50". Later in the same lesson, after the spinner game had been played, one of the students claimed to have counted the possibilities in the spinner game to get 38 even and 35 odd, which suggested that he had been considering the full sample space albeit inaccurately. Unfortunately, perhaps because of time constraints, this was not discussed or explored further. In fact, although there was some discussion of the possibilities for the spinner game, this received only limited investigation in Irene's class.

Greg, on the other hand, prompted by a realisation in his first lesson that it might be useful to write down all the possible outcomes of the spinner game, devoted nearly half of his second lesson to this exploration. He was very directed in his instructions to students, suggesting systematic ways of recording the outcomes so that they would all be noted. This approach seemed to dominate his own thinking, because when a student proposed an alternative strategy, he seemed certain that the student was wrong (which was true), but unsure of how to explain why or develop the strategy into one that would work (which could have been done to good effect). During this exploration Greg asked the students to work out the probabilities of some specific outcomes, which simultaneously helped to highlight the need to enumerate the sample space while it distracted from the original question of determining whether even or odd outcomes were more likely. This seemed to complicate the lesson for a while, but eventually the totals of 40 odds and 41 evens were identified. At this point, the "evens" outcome was stated as being more likely, although there was no discussion of the marginal difference between the outcomes, nor to emphasising that quantifying outcomes helps determine likelihood.

The idea of sample space arose on two other occasions. The "Multo" game in Irene's class led to discussion about numbers that are more useful to use (i.e., those with more factors, which thus have more occurrences in the set of times table cards) and those which are impossible (e.g., primes bigger than 7). Irene encouraged students to consider numbers that occur more frequently, but did not explore any quantitative differences in likelihood. Greg also considered sample space in an informal way, prompted by a student's comment about the probability of certain weather conditions on a Wednesday being 1 in 7 (presumably based on Wednesday being one day of seven). Greg had the students contribute to a "sample space" for weather conditions, which ended up being quite long especially when he allowed for the fact that combinations of conditions are possible. He used this list of "all these possibilities" to highlight why the appropriate value for probability was not 1/7.

Likelihood. Irene introduced her first lessson with a class discussion about the likelihood of it snowing, during which the concept of likelihood was developed with rich and rigorous use of language. Through careful questioning, she encouraged students to justify and clarify their claims, and to be precise about what event was being considered. She then built on this in the activity involving pieces of fruit drawn out of a hat. She began by using the sample space to ascertain the likelihood of certain fruit being drawn, using language such as possible, impossible, and likely. Irene used this as an introduction to quantifying probabilities. A strong connection both to the sample space and to chance language was retained during this activity, and Irene highlighted the importance of one and zero as certain and impossible. Unfortunately, this link seemed to be lost later. In the "Multo" game in her second lesson, Irene encouraged the students to consider numbers that are more likely, such as numbers with lots of factors; however, this was never quantified. Subsequent class conversations suggested that the students may have been left with an exaggerated sense of the likelihood of such numbers.

Greg had a similarly strong introduction to likelihood concepts. The brainstorming session at the beginning of his first lesson, followed by the least to most likely activity, gave students an opportunity to compare events that were very close to impossible or certain. However, Greg's initial attempt to place quantifiable events on a numerical scale was problematic. For example, he concurred with student suggestions that, on a spinner evenly divided into four colours, the chance of spinning a particular colour was "even" or "50:50", and he then placed this event halfway between impossible and certain. In this case Greg seemed to be recording the probability of one event compared to another event, rather than the absolute probability of that event. Later, when discussing the two spinners game, Greg led his students through using the sample space to calculate numerical probabilities of certain numbers, and thus address the question of the game's fairness. However, at no point were these quantified probabilities discussed in terms of the language explored in the previous lesson, so there was no opportunity for students to understand the meaning of the very small difference between the chances of "even" or "odd" winning the game.

Fairness. The complexity of the "fairness" concept was apparent in the lessons. A couple of students in Irene's class felt that a game is fair provided you play by its rules, but Irene highlighted, with the elicited support of other students, that the structure of the game can favour one outcome more than the other thus making it unfair. Towards the end of her second lesson Irene contrasted the spinner game and "Multo" and pointed out to the class

that whereas some games are “more fair” than others it is sometimes possible to increase your chances of winning by having more knowledge, or by thinking carefully.

Greg also incorporated discussion of fairness in his lessons, firstly by considering single spinners coloured in different proportions. Here it was suggested that all colours needed to be present and in equal proportions for the spinner to be fair, because no colour was more likely than any other colour. Later in the same lesson, with the actual spinner game, there was considerable discussion of the fairness of the game, using arguments about what was more likely to occur. As noted earlier, consideration of the full sample space in the second lesson resulted in the final determination of the game’s fairness, with a brief acknowledgment that since “even” was more likely to win the game was not fair.

Discussion and Conclusion

The results depict the complex interplay among concepts (as seen in the fact that some teaching episodes involved two or more probability concepts), the challenges associated with teaching these concepts and the relationships among them, and the importance of content knowledge and pedagogical content knowledge. Both teachers had chosen activities that would be rich mathematically, considering the language of probability, the meaning of likelihood, the quantification of probability, and the idea of sample space. The teachers’ PCK was evident not just in the choice of activities, but in the ways that they were able to link concepts to students’ experience (e.g., in Irene’s extreme weather example, and in Greg’s “least to most likely” activity) and to other mathematical content (e.g., Irene’s use of “Multo” which involved consideration of factors). The teachers’ approaches varied, with Irene giving students greater freedom to think about concepts and holding rich discussions with groups and individuals. Greg provided more direct guidance to students about what to do and, perhaps because he was at the limit of his own CK, had more difficulty exploring students’ ideas when they deviated from his own.

The complexity of pedagogical content knowledge was particularly apparent in the use of the spinner game. This activity came from the teacher guide for the mathematics textbook used by Grade 5 students in the school (Feely, 2003a). When the authors saw the associated activity sheet for the first time it took us a while to realise all that it offered, including opportunities to explore sample space, fairness, long-term probability, and likelihood. Each teacher utilised various aspects of the game, with Greg getting his class to go further with the analysis and the issue of the sample space. In fact, Greg’s decision to do this was made *during* the first lesson, rather than planned in advance, presumably since he now realised the opportunity that the activity afforded to do this. This may well have been a moment during which Greg’s pedagogical content knowledge actually developed. Irene, on the other hand, spent less time on sample space (perhaps because of lack of time), but was more careful in her description of fairness, using words that would help her students understand its meaning. It seemed apparent from the lessons that neither teacher had used the activity before, so it would be interesting to see the way in which they use it in the future. Moreover, the teacher guide gave very little guidance about what the activity offered probabilistically and how to bring it out. It failed to mention two significant problematic aspects: the small difference in the probability of the outcomes and the fact that the most obvious false argument (odd+even=even, and so on) leads to the correct conclusion. The question of how to adapt activities to address problems like these and to meet student and curriculum needs is an important one intimately linked with pedagogical content knowledge; unfortunately this could not be explored further with the teachers.

The important role of content knowledge in PCK was also evident in the lessons. On many occasions this was positive, as when Irene discussed the numbers needed for reliable survey results. There were also occasions when incorrect content knowledge was evident, such as Irene's belief about why an exact 50:50 result does not occur. Greg's discussion about weather probabilities not being "1 in 7" showed his content knowledge, but also revealed the complexity of deciding how much to explain to students (since it is not just that there are many different weather types, but that some are actually more likely than others) and the difficulty of actually explaining it in a way suitable for Grade 5 students.

These lessons show that quite deep probability concepts *can* be considered in Grade 5. They also highlight the complexity of the concepts, and the importance of having teachers with appropriate content and pedagogical content knowledge. The key probability concepts identified above need to be understood by teachers themselves, plus they must be able to recognise which activities will foster such understanding in their students and how to bring this to the fore in their lessons.

Acknowledgments: This research was supported by Australian Research Council grant DP0344229. The authors thank Irene and Greg for allowing this window into their world.

References

Amir, G. S., & Williams, J. S. (1999). Cultural influences on children's probabilistic reasoning. *Journal of Mathematical Behavior, 18,* 85-107.

Brousseau, G., Brousseau, N., & Warfield, V. (2002). An experiment on the teaching of statistics and probability. *Journal of Mathematical Behavior, 20,* 363-411.

Carlson, L., & Doerr, H. (2002). Secondary teachers' attitudes toward probability and their teaching strategies. In B. Barton, K. C. Irwin, M. Pfannkuch, & M. O. J. Thomas (Eds.), *Mathematics education in the South Pacific* (25th annual conference of the Mathematics Education Research Group of Australasia, pp. 171-178). Sydney: MERGA.

Feely, J. (2003a). *Nelson maths for Victoria: Teacher's resource Year 5.* Melbourne: Thomson Nelson.

Feely, J. (2003b). *Nelson maths for Victoria: Year 5 student book.* Melbourne: Thomson Nelson.

Garfield, J., & Ahlgren, A. (1988). Difficulties in learning basic concepts in probability and statistics: Implications for research. *Journal for Research in Mathematics Education, 19,* 44-63.

Marks, R. (1990). Pedagogical content knowledge: From a mathematical case to a modified conception. *Journal of Teacher Education, 41*(3), 3-11.

Neuman, W. L. (2003). *Social research methods* (5th ed.). Boston: Allyn and Bacon.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Watson, J. M. (2001). Profiling teachers' competence and confidence to teach particular mathematics topics: The case of chance and data. *Journal of Mathematics Teacher Education, 4,* 305-337.

Watson, J. M. (2005). The probabilistic reasoning of middle school students. In G. A. Jones (Ed.) *Exploring probability in school: Challenges for teaching and learning* (pp. 145-169). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Watson, J. M., & Chick, H. L. (2001). Factors influencing the outcomes in collaborative mathematical problem solving—An introduction. *Mathematical Thinking and Learning, 3,* 125-173.

Watson, J. M., & Kelly, B. A. (2004). Statistical variation in a chance setting: A two-year study. *Educational Studies in Mathematics, 57,* 121-144.

Watson, J. M., & Moritz, J. B. (2003). Fairness of dice: A longitudinal study of students' beliefs and strategies for making judgements. *Journal for Research in Mathematics Education, 34,* 270-304.