Development of a New Research Tool: The Cognitive Demand Profile

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This paper describes the development of a new research tool, the cognitive demand profile, from Chick, Watson and Collis' task analysis maps and Taplin's cognitive structure of mathematical tasks schema. The cognitive demand profile is to be used for the analysis of applications assessment tasks. Its application to two mathematically equivalent abstract and contextualised tasks is demonstrated.

The term cognitive demand has been used by only a few researchers. Evans (1991), for example, considers the cognitive demand of teaching tasks to consist of "the requirement of specific procedures elicited by particular cues, recall of specific knowledge, development and application of structured conceptual knowledge, or higher order procedures involving interpretation, transfer of rules to unfamiliar materials, or the combination and modification of procedures" (pp. 125-126). Deletion of the developmental aspects in this definition allows its application to the cognitive demand of assessment tasks. A related term, cognitive load, is used by researchers such as Sweller (1988) for the information processing capacity required by a particular solution strategy.

The cognitive demand imposed by an applications task will be taken to mean the demand on attentional resources and working memory imposed by the task. However, these are not considered to be independent but interrelated and constrained by some maximum capacity (Sweller, 1988). Carlson, Khoo, Yaure and Schneider (1990) point out that in addition to the demands of temporary storage and concurrent processing of information in working memory, there is often a further demand for the integration of information. This may involve the integration of various chunks of the given data or the integration of cued concepts and processes and/or interim results during processing. These cues may come from a secondary source such as external memory support in the form of pen and paper recordings, calculator displays or memory, or the secondary production of information from long term memory (LTM) (Fong, 1994).

With applications tasks, the cognitive demand is related to the interaction between the mathematical demand of the task and the extent to which the mathematics needed to model the situation is embedded in its description. Thus, the integration of information is a critical aspect of the cognitive demand of such tasks. To date there are no direct measures of cognitive demand. This paper deals with the development of a new research tool for analysing the cognitive demand of a task.

Tools to Compare Cognitive Demand of Applications Tasks

Two extant analysis tools that may prove fruitful for comparing the cognitive demand of tasks will be examined firstly. Modifications and additions to these tools will then be suggested which will form the basis for a *cognitive demand profile* which will be described and its use demonstrated. The purpose of this cognitive demand profile will be primarily for task analysis prior to the setting of an assessment task rather than response analysis after the task has been set, although it may fortuitously be able to be used in this manner.

Chick, Watson, and Collis's Task Analysis Maps

A mapping procedure developed as an extension of the SOLO Taxonomy by Chick, Watson, and Collis (1988) works both as a task analysis map and, with additions, as a response map. Further refinement of the mapping procedure (Collis & Watson, 1991) has resulted in the model described below, the symbolic components of which are shown in Figure 1.

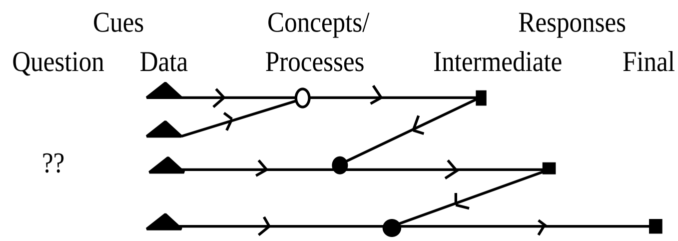


Figure 1. The components of a typical task analysis map.

The map is split into three major parts: *cues* stated in the problem statement, the *concepts/processes* used or intended to be used in formulating a response and the *response* itself. The cues consist of the question being asked (??) and the data (▲) given with the potential to cue a response. These data are linked by connecting lines to the concepts and/or processes used in solving the problem. Concepts, processes and/or strategies expected as part of an understanding of the question are indicated by a filled in circle (·). Abstract concepts, processes and/or strategies additional to those expected as part of the understanding of the question are indicated by an open circle (o). Lines then connect these concepts or processes to the responses (■) they produce which can be intermediate or final. Intermediate responses lead back to further concepts and processes being applied. This is shown by arrows.

Although the mapping procedure proved useful for visually representing the mathematical processing and cognitive demand of two upper secondary applications problems (Stillman & Galbraith, 1998), this procedure is not without limitations. For example, task analysis maps for the mathematically equivalent tasks A and B in Figure 2 show that task B, embedded in the context of bacterial levels in a swimming pool, is higher in cognitive demand as there are more data involved than in Task A (Figure 3). However, the maps do not appear to provide insight into why this context made the mathematical processing virtually inaccessible for a large number of Year 10 students who attempted the task despite previously solving decontextualised problems similar to Task A.

Task A: Given $C = 30t^2 - 240t + 500$ $(0 \leq t \leq 4)$. Solve for t when $C = 140$.

Task B: A swimming pool is treated to control the growth of harmful bacteria. Suppose for the first four days after treatment, the bacteria count, C, per cubic centimetre, is given by $C = 30t^2 - 240t + 500$ $(0 \leq t \leq 4)$ where t is the number of days after treatment. After how many days will the bacteria count be 140?

Figure 2. Two mathematically equivalent tasks.

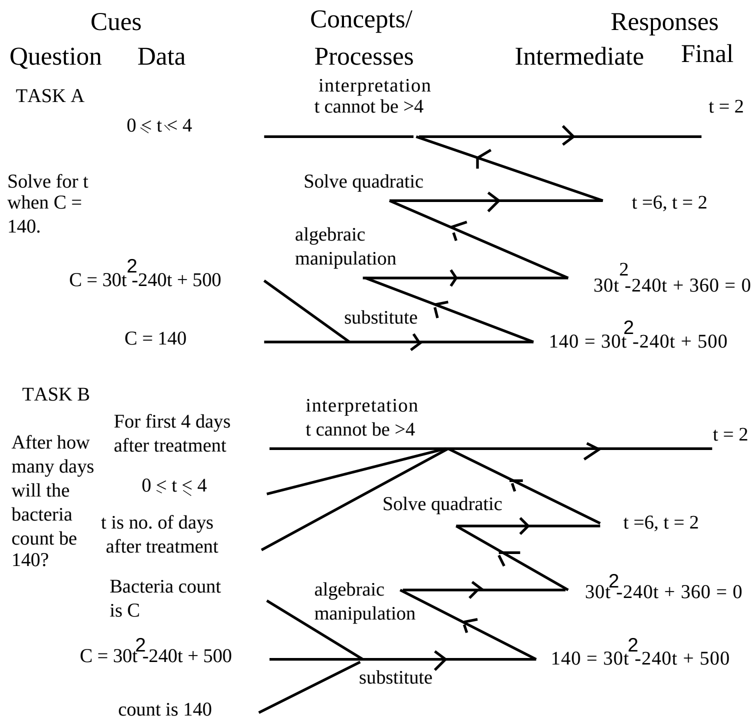


Figure 3. Maps of two mathematically equivalent but contextually different tasks.

Taplin's Cognitive Structure of Mathematical Tasks Schema

Taplin (1995) developed a schema using task analysis maps in conjunction with four other indicators to compare the cognitive structure of tasks. The features Taplin took into account were: the *level of processing* required to solve the task; the *number and type of cues stated* in the problem without having to involve other concepts and processes from the student's previous experience (Collis & Watson, 1991); the *extent of the demand on working memory* and the *complexity of the concepts and processes required*.

The SOLO Taxonomy was used by Taplin (1995) to determine the *level of processing* required to solve the task. As the research tool is being developed for a study of applications tasks in Years 11 and 12, task solvers would be expected to operate at the formal mode. Within each mode, Taplin classified the problems further using the complexity of the "'Structure of a response'...the learning cycle from simple to complex (unistructural to relational) *within* a mode. The five levels of SOLO are implicit within any mode" (Biggs & Collis, 1982, p. 217). Campbell, Watson and Collis (1992) proposed that there may be a number of learning cycles (referred to as U-M-R cycles) in a given mode. Accordingly, Taplin classified problems within each mode as being unistructural, multistructural and relational and in a particular learning cycle as appropriate.

An analysis of the *complexity of the stated cues* was also based on the SOLO Taxonomy. Taplin's classification was as follows:

- Unistructural - cue is applied directly with no need for further interpretation.
- Multistructural - further interpretation is required before application.
- Relational - integration with other cues is required.

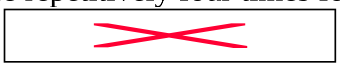
This appears to be a rather odd interpretation of the SOLO Taxonomy. It is not the complexity of the cues that is being captured here, rather the *complexity of how cues are* *applied*. In keeping with an interpretation more compatible with the SOLO Taxonomy *the complexity of the stated cues* will be taken to encompass (a) the number of cues stated; (b) the type of cues involved (i.e., unrelated (À) or related ( )); and (c) how they are applied (i.e., singly in isolation, combined as a group but not integrated, or integrated and then applied as a group). The integration of cues will be indicated by the use of the symbol, Γ.

A notation has been devised by the author which identifies each cue and also shows how all the cues are applied. Stated cues are identified as $d_1, \ldots d_n$ with the ordering of the subscripts denoting the order of appearance of the cues in the task statement. Each cue is specified, for example, $d_2 \rightarrow q = 120$. Specific groupings of cues are also identified, for example, a group consisting of the first two cues appearing in the problem statement is denoted by G_1: (d_1, d_2). with the indicating they are related. An overview of how the cues are applied is also shown. For example, the application of cues in a particular task could be shown as $C \quad \{\Gamma G_1, G_2, d_6\}$. Let us suppose that these cues correspond to the four related cues, $d_1, \ldots, d_4$, and the two unrelated cues, d_5 and d_6, respectively. The four related cues are applied as the group, G_1, which is specified as G_1: (d_1, d_2, d_3, d_4). These cues must be integrated before application and this is shown in the overview by using the symbols, ΓG_1. Secondly, the two unrelated cues, d_3 and d_5, are applied as a group, G_2, which is represented as G_2:À(d_3, d_5). The groups, G_1 and G_2, overlap sharing the common element, d_3. Finally, the remaining cue, d_6, is applied singly.

According to Collis and Watson (1991), the direct application of a single cue requires low *working memory capacity*; the combination and application of several unrelated cues requires medium working memory capacity whilst application of related cues which need to be integrated requires high working memory capacity. These authors define working memory capacity as “the amount of working memory that is required to process the cued data at each of the different response levels” (p. 66) of the SOLO Taxonomy.

Taplin (1995) measured the *demand on working memory* by *the number of interim results* which were required before the final result could be obtained, and *the number of times two or more concepts or processes were combined to produce one interim result*. A necessary addition to this is the *number of times two or more interim results are acted on by another process to produce another interim result or the final result.* This is similar to Fong’s (1994) notion of production of information from a secondary source. Fong placed problems requiring the secondary production of information at a higher level in his Information Processing Taxonomy for assessing problem solving. However, if the interim results are recorded using external memory (e.g., pen and paper or in a calculator display) there is no need for them to be held in memory even temporarily, so even though there is a secondary production of information, it is not necessarily creating an extra demand on working memory storage-wise but there would still be some attentional resources needed for coordination.

The *complexity of the concepts/processes required but not provided* was classified by Taplin (1995) using the same classification as for cues, which has already been indicated as problematic. Despite devising and using modified definitions in keeping with the SOLO Taxonomy, the author felt they did not add any further insight into the nature of the application of specific concepts/processes, so it was considered that a break with the SOLO Taxonomy at this point was warranted.

The previous notation was extended to identify each process and show how the processes are applied. Concepts/processes are identified as $p_1, \ldots, p_m$ with the ordering of the subscripts indicating the order of their first application. Each concept/process is specifically identified, for example, $p_1 \rightarrow$ substitution. Repetitive application of one, or a group, of concepts/processes is indicated by repetition or the use of bracketing (for grouping) and exponentiation. For example, the sequence, $(p_1, p_2)^2, p_1$, means that the two proceses, p_1 and p_2, are applied twice in that particular order followed by the process, p_1, again. If a concept/process is applied to more than one interim result this will be indicated by the use of an asterisk, for example, p_2^*. For each task an overview of how the concepts/processes are applied is given. For example, if there are five different concepts/processes required by a task and three of these, namely, p_1, p_2 and p_3, are used in sequence repetitively four times followed by the other two processes once, this would be shown as  . A further extension of this notation which includes the responses $(r_1, \ldots r_l)$ is used to show the evolution of the solution and the interaction of concepts/processes with the stated cues and the responses.

A further factor, *redundancy*, has not yet been accounted for. Redundancy occurs when two or more chunks of data add no further information than any one given singly, or irrelevant extra information is stated. For example, the addition of the words "for the first four days" to Task B when it is already known that "$0 \leq t \leq 4$" would be considered redundant. The redundancy is only potential, however, as the task solver could just ignore the inequality altogether and integrate the other two cues if the symbols were not understood. Another task solver who understood the inequality would, however, gain no further information by being able to process it as well as the equivalent wording but would use up attentional resources in integrating the two. Thus, redundant information may not have equivalent status for different students. A further measure is proposed - *percentage potential redundancy* which is *the percentage of the given data cues that add no further information*.

Fong (1994), in critiquing the use of the SOLO Taxonomy in mapping tasks and student responses, points out that even though this model identifies all possible kinds of data it does not characterise the cognitive processes which generated those data and it does not acknowledge any difference in the availability for retrieval of data stated in the question or data not stated in the question but subject to initial retrieval from LTM. Unstated mathematical data can be of two types - information related to the current mathematical topic of the problem which has been recently studied or is well rehearsed, called Type A by Fong, and information from related mathematical topics (Type B). Fong (1994) claims that the former is more readily retrieved than the latter. In Task A, for example, the process of interpreting $0 \leq t \leq 4$ requires the recall of Type B information whilst solving a quadratic equation requires the recall of Type A information.

Furthermore, in applications tasks the embedding of the task in its context appears to range from where this is just a separable frame for the mathematics as in Task B, to where the context and the mathematics interact and the interpretation of the problem in a mathematical sense relies on the contextual cues (Stillman, 1998). For example, in a contextualised task the task solver may have to interpret the external cue of a wound healing completely in the mathematical context as having an unhealed area of zero. The ability of the task solver to make this interpretation is dependent on prior experience and the ease with which memories of this are activated and retrieved from LTM and then applied to the task context. Such contextual information will be referred to as Type C information. It is proposed that the concepts and processes elicited from LTM should be classified as requiring Type A, B or C information in line with the definitions just given. In most instances it would be expected that Type A information is easier to retrieve from memory than either Type B or C. Evidence of a proposed fourth type of information (metacognitive knowledge, experiences and strategies), Type D, is not readily observed in scripts but could be revealed in interviews or videotapes. Its retrieval and storage adds to the demand on working memory. Metacognitive knowledge can therefore lead to an increase in cognitive demand by increasing the need for executive control of coordination.

Cognitive Demand Profile

The proposed *cognitive demand profile* for an applications task consists of *a task analysis map* (or several of these if there are alternative solutions expected) together with the following *cognitive demand indicators*:

- Level of processing including the mode of functioning expected and the complexity of the structure of the expected response (as unistructural, multistructural, relational or extended abstract)
- Complexity of the stated cues which will be taken to encompass (a) the number of cues stated, (b) the type of cues involved (i.e., unrelated (À) or related ( )) and (c) how they are applied (i.e., singly in isolation, combined as a group but not integrated, or integrated (Γ) and then applied as a group)
- Percentage potential redundancy of cues
- Number and application of concepts/processes required
- Classification of concepts and processes elicited from memory (as requiring Type A, B, or C information)
- Number of times two or more concepts/processes are combined to produce one interim result or the final result
- Number of times two or more interim results are combined and acted on by another process to produce a further interim result or the final result
- Number of interim results

Using this scheme, Tasks A and B are profiled as in Table 1. The profile not only highlights the increase in number of the cues from three to six in Task B but also the change in complexity of how the cues are applied. In Task A, two related cues, d_1, "$C = 30t^2 - 240t + 500$ " and, d_2,"$C = 140$", are combined, integrated and applied as a group to the first process, p_1, "substitution", and this is followed later in the solution by the single unrelated cue, d_3, "($0 \leq t \leq 4$)", being applied directly to another process, p_4, "interpretation". This pattern of application of cues is indicated as $C \quad \{\Gamma G_1, d_3\}$ in the table. In Task B, two different groups of three related cues, G_1 and G_2, are integrated and applied as separate groups to these same two processes, p_1 and p_4, respectively. The pattern of application is shown in the table as $C \quad \{\Gamma G_1, \Gamma G_2\}$. The other difference highlighted is the percentage redundancy which is zero for Task A but 11% in Task B.

The notation specified in the table can be used to show the interaction between cues, concepts/processes and responses in the two tasks as in Figure 4.

Table 1
Comparison of Tasks A and B Using Cognitive Demand Profile Indicators

Indicator	Task A	Task B
Processing level	Formal - Relational	Formal - Relational
Cues	3 cues: $d_1 \rightarrow C = 30t^2 - 240t + 500$ $d_2 \rightarrow$ C = 140 $d_3 \rightarrow (0 \leq t \leq 4)$ $G_1: \; (d_1, d_2)$	6 cues: $d_1 \rightarrow$ for first four days after treatment $d_2 \rightarrow$ bacteria count, C $d_3 \rightarrow C = 30t^2 - 240t + 500$ $d_4 \rightarrow (0 \leq t \leq 4)$ $d_5 \rightarrow$ *t* is the number of days after treatment $d_6 \rightarrow$ count be 140 $G_1: \; (d_3, d_2, d_6)$; $G_2: \; (d_1, d_4, d_5)$
Application of cues	$C \quad \{\Gamma G_1, d_3\}$;	$C \quad \{\Gamma G_1, \Gamma G_2\}$
Potential redundancy of cues	0%	11%
Specification of concepts/ processes	$p_1 \rightarrow$ substitution (Type A) $p_2 \rightarrow$ algebraic manipulation (Type A) $p_3 \rightarrow$ solution of quadratic equation (Type A) $p_4 \rightarrow$ interpretation (Type B)	$p_1 \rightarrow$ substitution (Type A) $p_2 \rightarrow$ algebraic manipulation (Type A) $p_3 \rightarrow$ solution of quadratic equation (Type A) $p_4 \rightarrow$ interpretation (Type B)
Application of processes	$P \quad \{p_1, p_2, p_3, p_4\}$	$P \quad \{p_1, p_2, p_3, p_4\}$
No. of times ≥ 2 processes combined	0	0
Specification of responses	$r_1 \rightarrow 140 = 30t^2 - 240t + 500$; $r_2 \rightarrow 30t^2 - 240t + 360 = 0$; $r_3 \rightarrow t = 6, t = 2$; $r_4 \rightarrow t = 2$	$r_1 \rightarrow 140 = 30t^2 - 240t + 500$; $r_2 \rightarrow 30t^2 - 240t + 360 = 0$; $r_3 \rightarrow t = 6, t = 2$; $r_4 \rightarrow t = 2$
No. of interim results	3	3
No. of times ≥ 2 interim results combined	0	0

Note. = related; Γ = cues have to be integrated before being applied.

Task A	*Task B*
$p_1(G_1) \quad r_1 \quad G_1: \ (d_1, d_2)$	$p_1(G_1) \quad r_1 \quad G_1: \ (d_3, d_2, d_6)$
$p_2(r_1) \quad r_2$	$p_2(r_1) \quad r_2$
$p_3(r_2) \quad r_3$	$p_3(r_2) \quad r_3$
$p_4(d_3, r_3) \quad r_4$	$p_4(G_2, r_3) \quad r_4 \quad G_2: \ (d_1, d_4, d_5)$

Figure 4. Interaction between cues, concepts/processes, and responses in Tasks A and B.

As can be seen in the above, as well as there being more cues in Task B the application of these cues does not follow their appearance in the problem statement and is much more complex requiring further attentional resources to be invested in sorting out the cues, determining their relevancy, coordinating and integrating them. In line with Collis and Watson's (1991) interpretation of working memory capacity requirements, Task B is also much higher in demand in this respect. For example, the last process in Task A has to be applied to the last interim result in combination with a single isolated cue whilst in Task B, the same process is applied to the same interim result but in combination with a group of three integrated and related cues. The cognitive demand profiles have clearly shown that the integration of information (cf. Carlson et al., 1990) has been the critical aspect of the difference in cognitive demand of the two tasks which may account for differences in success rates in mathematically equivalent contextualised and decontextualised forms of tasks when mathematical and language competencies are adequate.

Conclusion

There is an existing belief amongst teachers that it is possible to set applications like Task B which are of the border type where the student does not have to integrate the contextual and mathematical cues in order to solve the task. As one teacher put it:

> We try to make it so you can walk in without any prior knowledge and if you did have prior knowledge it is not going to make any difference.

Unfortunately, as the analysis in this paper has shown such a belief is not well founded as students do not necessarily see the task context and mathematics to be separable leading them to face what to them is an apparently daunting task of integrating contextual and mathematical information.

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