"Zero is Not a Number": Teachable Moments and their Role in Effective Teaching of Numeracy

Tracey Muir
University of Tasmania
<tracey.muir@utas.edu.au>

This paper reports on the use of "teachable moments" and the role they play in effective teaching of numeracy. Transcripts from three case study teachers' numeracy lessons were examined to identify teachable moments; qualitative descriptions illustrate the nature of these teachable moments and their potential to enhance students' understanding. It was found that teachers "missed" teachable moments, incorporated them into the discourse or actively ignored them. The findings indicate that teachers need to identify when to act upon teachable moments to avoid the likelihood of students forming misconceptions about important mathematical concepts.

Background

Studies such as the the Effective Teachers of Numeracy Study (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997) have contributed much to our understanding as to what constitutes effective teaching of numeracy. Askew et al. identified effective teachers of numeracy in a range of schools in the UK by looking at mean test scores of students over time. Their identification of effective teachers of numeracy was based on rigorous evidence of increases in pupil attainment, not on presumptions of "good practice" (Askew et al., 1997, p. 8), using data collected from over 2000 pupils and evidence gathered from a sample of 90 teachers. According to Stephens (2000), there appear to be no comparable studies of numeracy undertaken on a similar scale in other countries, but a study designed to identify the effective teaching of numeracy by early childhood teachers was undertaken in Australia by Clarke et al. (2002). Like Askew et al., Clarke et al.'s identification of effective teachers was based on data related to students' mathematical growth. Both studies identified a number of characteristics that were common to effective teachers of numeracy.

The characteristics that were common to both studies were identified by the author, with the term "principles of practice" adopted to describe the commonalities: make connections, challenge all pupils, teach for conceptual understanding, purposeful discussion, focus on mathematics and positive attitudes. As an illustrative example of how the principles were derived from the two studies, "make connections" was derived from "used teaching approaches which connected different areas of mathematics and different ideas in the same area of mathematics …" (Askew et al., 1997, p. 1) and "used teachable moments as they occurred and made connections to previous mathematical experiences" (Clarke et al., 2002, p. 13). The other principles were derived in a similar way.

In order to understand the craft of teaching, exemplary cases against which to measure or model performance are needed (Leinhardt, 1990). The studies conducted by Askew et al. (1997) and Clarke et al. (2002) allowed the author to identify "critical elements" (Leinhardt, 1990, p. 19) that characterised effective teaching of numeracy. Reports from the two studies, however, did not detail specific observable teaching behaviours that illustrated these principles in action. Following initial data collection and a further review of the literature, the author identified six teaching actions which could be directly observed in the classroom and which provided illustrative examples of the principles in action. These actions were: choice of examples, choice of tasks, questioning, use of representations, modelling and teachable moments. In this paper, teachable moments are specifically discussed, using examples from classroom observations, and the following research questions are addressed:

- To what extent did the teachers in this study recognise and act upon teachable moments?
- When and how did teachers use teachable moments in the course of their numeracy lessons?

Theoretical Framework

Teacher knowledge and beliefs

According to Askew et al. (1997), understanding why some teachers were more effective than others required an examination of the relationship between teachers' knowledge, beliefs and classroom practice. Shulman (1987) proposed that a teacher's knowledge base was comprised of seven types of knowledge. Although acknowledging that all knowledge types are important and interact with each other to impact on effective teaching of numeracy, teachers' content knowledge and pedagogical content knowledge (PCK) are particularly applicable to the study discussed in this paper. According to Shulman (1987) the teacher has a special responsibility in relation to content knowledge and should possess depth of understanding in order to communicate what is essential about a subject and be able to provide alternative explanations of the same concepts or principles. Having a well-developed content knowledge, however, does not necessarily result in effective teaching. Teachers also require well developed pedagogical content knowledge (PCK) which entails "the blending of content and pedagogy into an understanding of how particular topics, problems or issues are organised, represented and adapted to the diverse interests and abilities of learners, and presented for instruction" (Shulman, 1987, p. 8).

Together with teacher knowledge, teacher beliefs have also been found to significantly influence classroom practice. Thompson (1992) distinguishes beliefs from knowledge in that they can be held with varying degrees of conviction and although independent of their validity, are valid for the individual who holds them. Askew et al. (1997) found that the teachers in their study differed in their beliefs about what it is to be a numerate pupil, how pupils learn to become numerate, and how best to teach pupils to become numerate. According to Askew et al., the implicit beliefs or theories that teachers have, combined with their knowledge, influenced the way that teachers interpreted classroom events. For example, if a teacher believed that being numerate involves "the ability to perform standard procedures or routines" (p. 31), then pupil errors were more likely to be interpreted as the result of pupil carelessness or lack of attention (transmission belief). If however, a teacher believed that pupils were trying to make sense of information, then errors may be interpreted as arising from misunderstanding, rather than carelessness (connectionist belief).

Constructivism

Students will often construct knowledge in unplanned ways, which may not be consistent with the intention of the teacher. Constructivism is based on the notion that learners "construct their own knowledge" (Van de Walle, 2007, p. 22) and assumes that learning takes place as students process, interpret, and negotiate the meaning of new information (Newmann, Marks, & Gamoran, 1995). This is heavily influenced by the students' prior knowledge and their assimilation of new information depends heavily on whether that information helps them explain or extend their past experience (Newmann et al., 1995). The construction of an idea will therefore vary from individual to individual and even with the same teacher and within the same classroom (Van de Walle, 2007). In the classroom, teachers can assist students with constructing accurate understandings through the use of the six principles of practice and the teaching actions previously identified. Teachable moments in particular, can be used to make connections to previous mathematical experiences (Clarke et al., 2002) and to identify possible student misconceptions which can then be explicitly addressed and worked on (Askew et al., 1997).

Teachable Moments

In this context, a "teachable moment" refers to a teacher's simultaneous act in response to a student's answer, comment or suggestion and is utilized to either address a possible misconception or to enhance conceptual understanding. Clarke et al. (2002) used this term in their study, stating that effective teachers used teachable moments as they occurred and made connections to mathematical ideas from previous lessons or experiences. In order to capitalize on a "teachable moment" the teacher may recognize the connections between different aspects of mathematics (Askew, 2005) such as the connection between fractions, decimals and percentages, the opportunity to provide a context or "real-life" situation and the links with previous learning.

Direct reference to teachable moments in the literature is rare, but Arafeh, Smerdon, and Snow (2001) provided an example of how teaching lessons can be analysed to explore teachable moments. In their study, Arafeh et al. used TIMSS videotape classroom data to identify teachable moments, which they defined as "the set of behaviours within a lesson that indicated that students are ripe for, or receptive to, learning because they express confusion, misunderstanding, uncertainty, struggle, or difficulty with a mathematical problem, concept or procedure" (p. 3). Unfortunately when reporting on their findings, the authors focused on the methodological approaches and the evaluation of the software used to code the video footage, rather than providing illustrative examples of actual teachable moments. The study discussed in this paper addresses this "gap" through documenting classroom examples of teachable moments, and examining their potential to impact on students' construction of understanding.

Methodology

A case study approach (Stake, 1995) was used to document the numeracy practice of three upper primary teachers. The teachers were selected using purposive and opportune sampling (Burns, 2000); years of teaching experience ranged from six to eighteen and their classroom practice in general was highly regarded by the Principals in the different schools in which they taught. They were not selected because they were recommended as being particularly effective teachers of numeracy, but were considered "good practitioners"; this was an important consideration as the author was interested in focusing on factors specifically related to developing students' numeracy, rather than generic factors such as behaviour management. There was an assumption, therefore, that the teachers possessed other effective qualities, such as good rapport with students, organisational skills and effective management techniques. The researcher observed and videotaped a total of 17 numeracy lessons. Parts of the lesson involving teacher led discussions were transcribed within hours of observation and field notes were also used to document aspects of the lessons which were not captured on videotape. Following each lesson, the video footage was viewed by the author and the teacher; discussions related to viewing the footage were audio-taped and transcribed within hours. The transcripts of the lessons were analysed to identify the "principles of practice" in action through the teachers' choice of examples, choice of task, modelling, questioning, use of representations and teachable moments. Each of these actions were then further analysed and their effectiveness evaluated, sometimes through the use of specific criteria. For example, teachers' choice of tasks was evaluated through using the same framework as Arbaugh and Brown (2005) to determine the levels of cognitive demand. Lesson and audio transcripts were manually analysed and coded to identify teachable moments – instances when the teacher or a student made a comment that indicated a misconception or there was an opportunity to enhance conceptual understanding.

Results and Discussion

The teachable moments identified from the transcripts were able to be classified in three ways: the teachable moment was "missed", with no acknowledgement made by the teacher; the teachable moment was incorporated into the lesson; the teachable moment was actively ignored. Examples of each of these responses will now be discussed, using excerpts from the lesson transcripts to demonstrate the nature of these responses.

Teachable Moments Missed

The first excerpt has been extracted from a lesson involving modelling of the guess and check strategy used to solve problems. The whole class of grade 5/6 students were seated on the floor in front of the teacher (Sue) and students took turns to volunteer their "guesses" and record their answers on the whiteboard. Students were read aloud the following problem:

> A family set out on a 5 day trek. Each day they travelled 50 kilometres less than the day they had before. Total distance that they travelled was exactly 1500 kilometres. How far did they travel each day? So they went 1500 kilometres for 5 days – each day they travelled 50 kilometres less than the day before, so how could we work it out?

One of the students, Mandy, initially guessed 900 kilometres and recorded this in a table on the whiteboard. The guess was too high and Randall then volunteered to try his guess:

> Randall: 200
>
> Sue: 200, all right
>
> Randall: [starts filling in table, beginning with 200]
>
> Sue: So they're not traveling – whoops – they're not traveling anywhere
>
> on Friday? They're going to stay at home. OK, so is that going to add
>
> up to 1500?
>
> Randall: [indecipherable]
>
> Tr: Is it Randall? What does it add up to? 3, 4, 500?
>
> The discussion with Randall ended there, and Sue then asked another student to try with a different guess:
>
> Sue: Tyler, you're going to have a go? What is your guess?
>
> Tyler: 600

Teachable moment

Sue could have asked Ryan why he selected '200', to make a connection with the first guess being too high

In the above excerpt, Sue could have incorporated the teachable moment, without disrupting the flow of the lesson. As the aim was to develop an understanding of the "guess and check" process, discussion could have occurred around how to decide which number to "guess" next, based on what occurred with previous guesses. Tyler could have been asked to justify why he had selected 600, making the connection that 900 was too high and 200 was too low.

The next excerpt occurred when Sue introduced students to the task of writing their own problems. After some discussion, the following occurred:

> Sue: Well, yes – you see if you can make it appropriate to grade 5 and 6 people. OK, it's got to be age appropriate so it can't be what am I – a number between 4 and 5
>
> [some students laugh]
>
> David: There's not a number.
>
> Sue: No

Teachable moment
Both Sue's comments and the student's response indicates a misconception – that there are no numbers between whole numbers

The above exchange provides an example of when a teachable moment can be used to address misconceptions. It appears that both Sue and the student (and possibly the rest of the class) did not consider fractions and decimals as numbers. The example of choosing a number "between 4 and 5", would actually have been appropriate for this age group and would have enabled students to challenge themselves to think of numbers that would "qualify". Sue's failure to capitalise on this teaching moment indicates both a lack of content knowledge and PCK and the likely construction of a (mis)understanding that there are no numbers between whole numbers.

Incorporating Teachable Moments

At other times, teachable moments were recognised by teachers in the course of the lesson and incorporated accordingly. In the following excerpt, John, another case study teacher, was modelling the process for dividing numbers and expressing the remainder as a decimal. The example used was 764 divided by 15 and John modelled the process on the whiteboard, with students contributing answers at different stages of the process (the correct answer was 50.93, with the "3" recurring). The following discussion occurred part way through the modelling, and occurred specifically in relation to expressing the remainder as a decimal by placing a decimal point at the end of 764 and adding zeroes.

John: So we've got 45 – Monty I reckon that's pretty close to it

Monty: You put down 3 next to the 9 and then you put a little dot above the 3 because it's recurring

Teachable moment

The use of the term 'recurring' is used for the first time and John recognizes that it requires clarification

John: So you talk about the recurring – what does that mean?

Chloe: That there needs to be another zero, and the remainder goes on to it

John: Listen carefully – I think I'll take some of the blame for this – Brad

Brad: You put a recurring dot – after the dot – the two numbers are the same

Teachable moment

John provides an illustrative example to demonstrate the meaning of recurring

John: So if we had – this is an example – how many threes in ten [writes short division algorithm] a million zeros - how many threes in 10, 3 and 3 left over, how many threes in 10, three and three left over [keeps writing 3 next to zeros and 3 along the top] OK and one of my dreams is to be a hermit and live in a cave and just do that for the rest of my life because that pattern continues – and instead of having to write 5 million and zillion and infinity threes, we can just write 3.33 recurring [writes a dot over the second three] – but if the digits aren't the same

Trevor: But they can't be the same again because there's going to be a million zeros again and it's never going to be 93 again

John: Um, it may do – but just for our purposes, if it's the same digit and you can see the patterns going to continue, you can use a recurring dot, but if not, just leave it – if you want to be super safe, don't use it at all. Tamara?

Teachable moment

John recognizes that the concept will need more explanation for some students

Tamara: Would you put a recurring dot if it kept on going 9 3, 9 3?

John: OK – you're testing me – it's something I'll have to find out – but somewhere in the back of my mind from my grade 11 maths, I think you put 2 dots, but I'll have to find out the answer – I'll have to check with the experts – but until I get back to you, don't put anything there – but remember the important thing we're looking at is the process, OK?

While this excerpt also illustrates the need for teachers to carefully consider their choice of examples (see Muir, 2007), it also demonstrates how teachable moments can be incorporated into the teaching exchange. John had probably anticipated through previous teaching about decimal remainders that the term "recurring" would arise and that it had the potential to confuse students. While this indicates that John possessed PCK in relation to this, his final comment reveals that he was not as confident with the content knowledge and felt inadequate with answering Tamara's question. After viewing the video footage of the lesson, John commented:

> I think personally it's really poor on my part not being confident enough to use the specific mathematical terms, which I know is really important um cause we're trying to get the mathematical language through that um so that's not an oversight, but probably lack of planning on my part um and knowledge about using the recurring [decimals] and so it's something that I need to brush up or re-brush up on make sure I know it for next time, so they've got those answers there which they're genuinely interested about …

While John recognized the teachable moments inherent in the above excerpt, it seemed that his lack of adequate content knowledge may have impacted on his ability to address the moment in the most appropriate way.

Actively Ignoring Teachable Moments

Viewing of the video footage shortly after the lessons observed provided the teachers with opportunity to clarify their responses to teachable moments. The following excerpt occurred in a lesson conducted by Sue, and involved a discussion about square numbers. Students had volunteered a number of examples, then one of the students mentioned zero.

> Sue: OK, think of another one. David?
> David: Zero
> Sue: Zero?
> David: 0 times 0 is zero
> Sue: Zero? We've never actually had zero as a square number have we? No.
> Scott: Zero isn't even a number
> Sue: No, it's not
> David: It is, zero times zero

The above exchange highlights two different potential misconceptions: that zero is a square number and that zero "isn't even a number".[9] It is logical that the question arose because it depends on one's definition of a square number. If it is taken to mean a number multiplied by itself, then zero is a square number, however, if the requirement is that the number can be displayed as a square, then zero does not qualify as a square number. These students had not been exposed to the visual representation of a square number, hence they were basing their answer on the definition of a number multiplied by itself. Sue later explained her reluctance to incorporate this teachable moment:

> Didn't want to go there then – would go back and do it later – didn't want to distract from what they're doing…I would have left it at that stage as it would have disrupted the whole flow … it depends on whether or not it's going to matter to what we're doing – if it was going to affect what they were doing, then I would – but this wasn't. I'd probably come back to it and finish – on that day – otherwise David and the likes would use it as a ploy to distract you from what you're doing.

Sue justified her decision to ignore the teachable moment but recognized that it was important in that she would address it later on. While it is debatable about "whether or not it's going to matter", Sue's comment revealed that the decision as to whether or not to incorporate a teachable moment depends very much on the teacher's judgement, and is influenced by factors such as context, stage of the lesson and knowledge of, and beliefs about, students.

Conclusions

In any given lesson, it is likely that several opportunities will arise which could be interpreted as teachable moments. Whether or not these moments are addressed depends very much on a combination of the teacher's knowledge, beliefs and professional judgement. While teachers may be reluctant sometimes to interrupt the "flow" of the lesson, sometimes students' comments reveal serious misconceptions which need to be addressed. If teachers possess inadequate content knowledge and/or PCK, then these moments may not be recognised and misconceptions may not be explicitly addressed and worked on (Askew et al., 1997). In this study, teachable moments provided illustrative examples of the principles of practice in action, such as *make connections* and *teach for conceptual understanding*. Implications from the findings indicate that teachers need to firstly recognise teachable moments and then capitalise upon them to enhance student understanding. This paper has contributed to the current research through describing what "teachable moments" look like in the classroom and accounting for some of the differences in the approaches teachers take to responding to teachable moments.

9 Zero is a number as it denotes the absence of units, and/or the initial point or origin (James & James, 1966)

References

Arafeh, S., Smerdon, B., & Snow, S. (2001, April). *Learning from teachable moments: Methodological lessons from the secondary analysis of the TIMSS video study.* Paper presented at the annual meeting of the American Educational Research Association, Seattle, WA.

Arbaugh, F., & Brown, C. A. (2005). Analysing mathematical tasks: A catalyst for change? *Journal of Mathematics Teacher Education, 8*, 499-536.

Askew, M. (2005). It ain't (just) what you do: Effective teachers of numeracy. In I. Thompson (Ed.), *Issues in teaching numeracy in primary schools* (pp. 91-102). Berkshire, UK: Open University Press.

Askew, M., Brown, M., Rhodes, V., Johnson, D., & Wiliam, D. (1997). *Effective teachers of numeracy*. London: School of Education, King's College.

Burns, R. (2000). *Introduction to research methods* (3rd ed.). Melbourne: Longman.

Clarke, D., Cheeseman, J., Gervasoni, A., Gronn, D., Horne, D., McDonough, A. (2002). *Early numeracy research project final report.* Melbourne: Australian Catholic University.

James, G., & James, R. C. (Eds.). (1966). *Mathematics dictionary.* New York: D. Van Nostrand Co., Inc.

Leinhardt, G. (1990). Capturing craft knowledge in teaching. *Educational Researcher*, *19*(2), 18-25.

Muir, T. (2007). Setting a good example: Teachers' choice of examples and their contribution to effective teaching of numeracy. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice* (Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia, Hobart, pp. 513-522). Sydney: MERGA.

Newmann, F., Marks, H., & Gamoran, A. (1995). Authentic pedagogy: Standards that boost student performance. *Issues in Restructuring Schools*, *8*, 1-15.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, *57*(1), 1-22.

Stake, R. E. (1995). *The art of case study research*. Thousand Oaks, CA: Sage Publications.

Stephens, M. (2000). *Identification and evaluation of teaching practices that enhance numeracy achievement.* Canberra: Commonwealth of Australia.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York: MacMillan Publishing Company.

Van de Walle, J. A. (2007). *Elementary and middle school mathematics* (4th ed.). Boston, MA: Pearson.